PURE THOUGHT
AND THE RIDDLE OF THE UNIVERSE

FRANCIS SEDLÁK

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BY

FRANCIS SEDLÁK

VOLUME I

CREATION OF HEAVEN AND EARTH

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TO MY FRIENDS,

A. AYLIFFE AND G. REINLI,

OUT OF GRATITUDE FOR THEIR GENEROUS ENDOWMENT OF THE PRESENT WORK: A GRATITUDE ONLY DEEPENED BY THE SPONTANEITY OF THEIR OFFER TO HELP, IN SIMPLE DISCHARGE OF WHAT TO THEM SEEMED THE MOST ELEMENTARY DUTY OF EVERY INTELLIGENT HUMAN BEING IN THE INTEREST OF TRUTH. MAY EVERY POOR PHILOSOPHER BE BLESSED WITH SUCH FRIENDS!

PERSONAL REMARK.

"You cannot stand still by the side of any one personality, however great. You must try, however inadequately, to do the work over again for yourself, and thus, notwithstanding the debt which the world owes to Hegel, we have each of us in this generation to try, if we would comprehend the true meaning of his teaching, to think it all for ourselves, in the light he has given us, but still for ourselves. Otherwise we should not really make any progress."

These words of Lord Haldane ("Pathway to Reality," vol. ii., p. 84) came home to me when, after having vainly tried to rouse interest in my translation of Hegel's "Wissenschaft der Logik," advertised in my previous book "A Holiday with a Hegelian," containing a digest, paragraph by paragraph, of the first two volumes of the great work in question, I proposed to convert my translation into a free paraphrase of Hegel's meaning. To cease to adhere to the literal context of the original, means to do the work over again; and in that case the original author must not be made responsible for a possible deviation from his own meaning. So comes it that my primary intention of simply paraphrasing Hegel's Great Logic converted itself finally into the present work.

To discuss this work in advance of its perusal seems to me inappropriate; but a glance over its table of contents will show those who already are familiar with Hegel's system that I have, indeed, aimed at an original elaboration of the whole subject-

matter. The fact that, in my attempt at an *à priori* derivation of the measurable features of the Solar System, I am breaking up fresh ground, may, perhaps, incline my critics to leniency. "There is at least something grand," says Hutchison Stirling in his "Secret of Hegel," p. 594, new edition, "in the way in which Hegel would set up time and space themselves as the co-ordinates that to the divination of Kepler and to the necessity of the notion of Hegel yielded and yield the law $\frac{S^3}{T^2}$ or $\frac{A^3}{T^2}$." Seeing that Kepler's third law is only the merest fringe of the measurable features of the Solar System, I have set myself a task immeasurably grander—at any rate, immeasurably vaster, some will even say, an impossible task, yet a task which, consistently with the standpoint of Absolute Idealism, has to be faced sooner or later. I have done my best and one cannot do more.

FRANCIS SEDLÁK.

WHITEWAY, NEAR STROUD,
GLOUCESTER, *March*, 1919.

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PART III.

INTRODUCTION.

Pure thinking must be distinguished from what ordinarily passes for thinking. The ordinary thinking may be characterised as loose, because it lacks continuity. The objects with which it concerns itself are picked up in a more or less haphazard manner. One turns his attention now to this one, now to that one, for no other reason than that it too is to be included among the subjects which it behoves an intelligent man to know something about. The band unifying them is indeed our common hunger for knowledge; but by knowledge is meant so far simply a mass of information about anything and everything that somehow or other comes to hand.

In distinction, then, from this kind of thinking, the pure thinking is not meant to flit from one object to another, as it were, *per saltum*, but is meant to remain continuous with itself. Its object must no longer simply be "picked up" as what is given beforehand, but must be ushered on the scene by its own activity. The goal is no longer simply to pigeonhole in memory a mass of information about all manner of things, to make of mind a kind of dictionary, a mere collection of current notions, but rather to realize how these notions are connected with one another in the element of Thought as such; to build up the system of Thought from within its own self in revelation of its own spontaneity.

So long as our attention is turned outwards, we are apt to overlook that, in thinking about facts, we superimpose upon them forms of thought which cannot be said to lie in the facts themselves. For instance, when we designate a fact as the *ground* or *cause* of

another fact; when we say that this or that is *necessary*, that this or that is *essential* or only *contingent;* that we must distinguish between *finitude* and *infinitude*, that the universe is governed by *laws:* we are introducing notions originating in the mind itself.

Along with facts we presuppose also our own capacity for interpreting them. In tracing a thing to another thing as to its cause, we do so only because we try, cannot help trying, to grasp the relation between them. But an effort to grasp anything at all involves an act of thought. Therefore, in concerning ourselves with facts, we do not leave them as we find them. We give them a form of thought, or convert them into a thought-form. To do so requires no effort on our part. We cannot help so doing, simply because it is our nature so to do. After all, we are born thinkers. To think is our very nature. It is by thought alone that we are distinguished from beasts. But, then, however evident this truth be to us at first sight when our attention is directed to it, there is yet required a certain degree of rational maturity before we come to draw the stated distinction between the pure and the loose thinking.

Of course, to draw this distinction does not mean to postulate a hostile antagonism between the two kinds of thinking, as though the pure thinking were meant to dispute the *raison d'être* of the loose thinking. But for having thought, and continuing to think, loosely, we would not be able to grasp the idea of pure thought. So far as there is an antagonism between the two kinds of thinking, it is the normal antagonism between the awakened, or already awakening, self-consciousness as a thinker and the still enduring self-oblivion as a thinker. In the first flush of enthusiasm, the pure thinker is apt to treat the loose thinker with contempt, thus provoking him to retaliate with corresponding, and in that case with overwhelming, vehemence. The pure thinker is doomed to remain one of very few, whilst the loose

thinker counts in millions. But the futility of such an opposition must sooner or later become patent on both sides. After all, we grow. What is obscure to-day will be clear to-morrow or the day after. Why, then, excite oneself that a particular individual is not ready to dismiss his prejudices to-day, this very moment, at one's bidding? People do not live in order to please one. The pure thinker realises that pure thinking is none the less satisfying even when pursued by himself alone. We are born thinkers: to become conscious of this fact and to live up to it, by making pure thinking the central interest of one's life—surely, this can give no cause for regretting!

PART I.

QUALITY.

CHAPTER I.

PURE BEING, NOTHING: BECOMING.

THE answer to the question, How to begin to think purely? is implied in the very idea of pure thinking. This is the thinking of itself, i.e. self-conscious thinking. Therefore, it must begin with the least than can be thought.

Inasmuch as to begin means to take something for granted, this presupposed something must not be picked up from the sphere of facts, but must be already a matter of pure thought. And because the beginning implies also extreme simplicity, the pure thought taken for granted must admit of no further simplification. It must be the simplest thought. Therefore, in order to initiate the system of pure thought, we must begin by thinking Nothing. For, surely, no thought can be simpler than that of Nothing.

Such a beginning seems surprising. The loose thinking, not only as voiced by a critic of our industry, but our own loose thinking, protests. What can be made of such a beginning? Nothing is only Nothing. It can lead only to —Nothing. All that can be said of it is—Nothing. By beginning with it, we nip in the bud the very possibility of further progress.

But this protest on the part of the loose thinking must not affect us. All we need trouble about is that the thought of Nothing is the simplest thought. By taking Nothing for granted, we satisfy the primary requisite of true knowledge. In this respect, we must not begin with a prejudice. Prejudice is at home only in the sphere of the loose thinking, because this is the still self-oblivious thinking. As regards the pure thinking, just because it is the self-conscious thinking, it transcends in its very nature that rational blindness which elevates a mere prejudice above the voice of reason.

Surely, it is not sound common-sense that protests

against taking Nothing for granted, if the object is to be on one's guard against prejudice. The prejudice giving rise to the protest against the beginning with Nothing consists in the taking for granted that Nothing is only Nothing, and hence, that it can lead to Nothing. *Ex nihilo nihil fit*, is the classical expression of this prejudice.

True, one cannot bake bread out of Nothing. But pure thinking is not analogous to baking bread. It is because of this lack of analogy that the otherwise sensible statement about the impossibility of making something out of Nothing becomes a mere prejudice, when applied to the *thinking* of Nothing. So far as Nothing is *thought*, it cannot be said to be just nothing, because it is a thought, a determination of pure thinking. To say that pure thinking must begin with the thought of Nothing, that it must take Nothing for granted, obviously means that it must begin with its own Being; with the simplest Being of Thought or the simplest Thought of Being.

To think Nothing means to think unqualified or *pure* Being. All that may be advanced in definition of Nothing directly applies to Being as such. This is the name for that very simplicity or utter indeterminateness of Thought which means Nothing. That Nothing *is*, is granted even in the protest that it *is* only Nothing. In asking, What is Nothing? as well as in answering that it is only Nothing, the loose thinking itself implies a direct identification of Being and Nothing, and hence is really in agreement with the standpoint of pure thought with respect to its beginning. The fact that this its actual confirmation of the nature of a pure beginning passes with it for an irrefutable criticism of the validity of such a beginning, supplies an illustration of its own blindness to what is implied in its own utterance.

In answer to the question, How to initiate the system of pure thought? we must say: By thinking either pure Nothing or pure Being. In so far as the affirmative is generally granted precedent, pure Being should come first. But because to think pure Being means directly and indistinguishably to think simply Nothing, and vice versa, it does not truly matter which comes first. As a matter of fact, the beginning is made by thinking Being—Nothing—Being. . . . And it is, then, this simplest thought that

takes the place of an external object for our immediate thinking. After having initiated the simplest act of thought, we must proceed to make of the simplest thought the object of further thinking, with a view to converting the result of this further thinking into the object of still further thinking.

It is true, by the way, that, in pursuing this task, we cannot avoid availing ourselves of our habitual turns of expression. Unless we help ourselves freely to the contents of our mind, such as it is to begin with, we should have to renounce all elasticity of expression. But it depends on us alone whether the involved introduction of much that must needs go beyond the scope of our proper subject-matter becomes a danger to an adequate accomplishment of our task. Clearly, we must be on our guard as to what belongs to our proper subject-matter and what to our redundant reflection. For instance, in reflecting on the nature of the simplest thought, we are at once tempted to introduce the form of judgment. The reflection that to think pure Being amounts to thinking Nothing, invites the statement that pure Being is the same thing as Nothing, or vice versa. But so we are imposing upon Being or Nothing the significance of Subject or Predicate, whilst such a distinction must not yet be admitted into the scope of the simplest act of thought.

As a matter of fact, the simplest thought annuls the copula *is*. Being is not yet to be grasped as if pushed on one side in opposition to Nothing on the other side. It is the utter indeterminateness of Being that is Nothing, or the utter vacuity of Nothing that constitutes Being pure and simple. Nothing is not to be predicated about Being, as if this latter were Being also in distinction from Nothing. And neither is Being to be predicated about Nothing, as if this latter were Nothing also apart from such a predicate. There is here as yet no question of a particular Being to be distinguished from another such Being as its *not*. The simplest thought can concern only the thought of the utterly unqualified Being, of Being as pure Being, i.e. as Nothing; and similarly of Nothing as the pure Being of Thought. So far, there is nothing to fall back upon for the fixation of Being in distinction from Nothing, and vice versa. These are only two distinct words which are directly

and utterly identified when thought; and how they are thought is all that matters to us. To insist on their distinction as mere words would mean to refuse to think them, when yet the sole object of our industry is to think about thinking, not to play with mere sounds, as words become directly they are emptied of their meaning.

It follows that we must avoid recording the simplest thought in the form of judgment. This form of thought belongs so far only to our loose thinking, whilst our proper task consists in recording what lies in the simplest thought itself. Now, in simply dropping the copula and saying that the simplest thought oscillates between Being and Nothing, we still fail to record the simultaneous blending of these words in one single meaning. What we require is a single word recording the simultaneous arising and vanishing of the distinction between Being and Nothing. And we find such a word in *Becoming*.

CHAPTER II.

ORIGIN, DECEASE: PRESENCE.

THAT which becomes is *not* yet; if it already were, why should it become? Yet just because it becomes, must it not also already *be?* Even though not yet become, it cannot be exclusive of Being as such. Becoming is thinkable only as a contradiction of Being and Nothing. In defining it simply as the passage of Being into Nothing, or of Nothing into Being, we are really making a concession to our habitual mode of expression when speaking of things.

For, we know already that Being, the still utterly unqualified Being, does not go over into Nothing, but is as such directly Nothing; that the distinction between pure Being and Nothing is, precisely, only a distinction of sound, not of meaning.

Nevertheless, we cannot achieve an act of thought without creating a distinction. Now, because Becoming implies only the stated *nominal* distinction, we cannot but restate it afresh. Definable as the passage of Nothing into Being *or* of Being into Nothing, Becoming may be interpreted in the sense of Coming-to-be or Ceasing-to-be; of *Origin* or *Decease.* But since we thus simply repeat one and the same meaning, the created distinction ceases to be in its very origin.

This creating of futile distinctions may seem quixotic. But, then, are we expected to apologise for the nature of pure thinking? There is the fact that, when Becoming becomes the object of further thinking, we can only restate its simple definition as a passage of Nothing into Being *or* of Being into Nothing. Becoming rests on this distinction: take it away and Becoming itself disappears. The result of the further thinking of Becoming is, after all, its own Ceasing-to-be. The disappearance of that on which it rests, involves its own disappearance.

Just because pure Being and Nothing do not go over the one into the other, seeing that the distinction between them is only that of sound, not of meaning, the disappearance of Becoming in result of our further thinking about it simply drives home to us that Being must be grasped as *ab initio* gone over into Nothing and Nothing into Being. In taking Nothing for granted, we have really made our beginning with the *direct unity* of Being and Nothing. This unity neither comes to be nor ceases to be, but simply *is*; yet this its Being is not a *pure* Being, just because it is the Being of the direct unity of Being and Nothing.

In asserting at the very beginning: Being *is* Nothing, Nothing *is* Being—we protested at once that the *is* was out of place, because Being could not yet be opposed to Nothing. Now, however, the emphasis falls just on the *is*; but no longer in its primary sense as copula, but in that of the originally presupposed direct unity of Being and Nothing. Inasmuch as this unity neither comes to be nor ceases to be, one might feel tempted to call it the original Being, or the first Cause, or the Unconditioned, the Infinite, God, or simply Self-subsistence, or still otherwise. But we must remember that it is only the *simplest* Being of the direct unity of Being and Nothing that is to be named; the term chosen ought, therefore, not to carry with it such a wealth of reflection as is at once suggested by the mentioned words. The success of our task depends on our ability to maintain pure continuity of thought. Consequently, in recording the result of an act of thought, we must aim at extreme simplicity of expression, lest, in our further thinking, we find the current meaning of our terms at variance with the meaning which was to have been recorded.

For our own part, we find that the simplest term placed by the English language at our disposal in record of a Being which just simply is, is *Presence.* Whatever other term we choose instead, all that it would suggest beyond the simple Presence, whether of God or the World, would have to be treated as going beyond the scope of our present object for further thinking.

Instead of the pure Indeterminateness of our beginning, we have before us *Determinateness*, but still only as *pure and simple.*

CHAPTER III.

REALITY, NEGATION: ALTERABLENESS.

If we repeat that Presence means to us the direct unity of Being and Nothing, we do so only with a view to the creation of the distinction indispensable to the ensuing act of thought. The only way to create this distinction is to emphasise Presence with respect to either Being or Nothing. Our immediate task consists, therefore, in grasping Presence as what emphatically *is* or *is not*. The simplest terms suggesting themselves in record of this contrast are *Reality* and *Negation*.

That the term Reality implies an emphasis on the Being of Presence, is evidenced whenever we ask what this or that is *really*. Inasmuch as Presence is the Being of the unity of Being *and Nothing*, it is from the very first also open to a definition in terms of Nothing: as the Nothing taken up into Being in such wise that the resulting direct unity of both has the form of Being. This is why, in attempted removal of the original equivalency of its Being with Nothing, our rational instinct prompts the question as to the Realness of a Presence. That which *there* simply *is*, may be only a mirage, only a ghost of Reality—only what comes with us, so far, under the head of Negation.

Seeing that Negation denies Reality, it would seem at first sight that it is only another word for the Nothing of our beginning, and hence irreconcilable with Reality. But that this cannot be so, is already plain from the fact that Negation is unthinkable except as a denial of an affirmation. It is a declaration that *something* is not, and the something stands, so far, simply for Reality.

In denying the reality of something, we are immediately removing only the emphasis on the Being in the unity of Being and Nothing. We are not denying this unity itself. Presence as such cannot be denied, because it supersedes

Becoming. It does not come to bé and neither does it cease to be. It is truly the First with which beginning is made. Just because we must take Nothing for granted, and the Nothing of our beginning proves itself to have the meaning of Presence as such, we must define this latter also in terms of Nothing. But this definition yields only an empty determinateness; a determinateness such that it still amounts also to pure indeterminateness, to a *tabula rasa* of of its subsequent qualification. Negation is, therefore, only another term for Presence as such with respect to its sense as Determinateness as such. *Omnis determinatio est negatio.*

We have realised that Presence as such is indeed only a form of Being. Although ushered on the scene in supersession of the untenable contradiction of Origin and Decease, it does not altogether abrogate Becoming. The contradiction of Being and Nothing reappears in its own Inwardness. Even though it itself does not come to be, it makes its meaning explicit as an unrestful unity of Reality and Negation. Reality is not Negation, Negation is not Reality; but just because either *is not* the other, either just as much also *is* the other. They are distinguishable only as aspects of one and the same meaning. Becoming acquires in this connection the sense of *Alterableness.* And as alterable, Presence itself acquires the sense of *Something or Other.*

CHAPTER IV.

SOMETHING OR OTHER: QUALITY.

When speaking of Something *or* Other, we imply that the distinction between different things, between Something *and* an Other, is of no consequence. We do not pretend to choose between various things, but indicate that *anything and everything* of the same quality will do. But so these current expressions record just what is implied in the meaning of Alterableness: that Otherwiseness remains a mere hint at change; that so far as Something admits of alteration, this change is of no present significance, or is to be viewed only as an instance of the same general meaning.

If of two things one is called Something, the other thing is an Other. But it does not matter to which of them we point as Something. Everything is Something or an Other. We cannot confine the term Something only to *this* particular thing. And similarly the Other. Even if we take this latter in the sense of an Other as such, we simply identify it with Something as such. For, if the Other is not to be contrasted with its Other, but taken as the Other purely in its own self, it acquires the sense of Something as such. This, to be free from all contrast to an Other, is precisely the distinctive meaning of Something as such.

The idea of a particular Something singled out from the totality of the existing distinctions goes still beyond the scope of our present stage. Instead of a distinctive Something or Other, we have before us a self-identical Something or Other. Or the distinction between things remains as it were a matter of a mere pointing at them, whilst they themselves take no part in this distinguishing. In thinking of Alterableness, we cannot help creating the distinction between Something and an Other, but only to find it melting away no sooner has it come to view. Accordingly, Alterableness itself equally disappears.

The self-identical, or, to emphasise the disappearance of Alterableness, we had better say *inert*, Something now before us seems to bring us back to Presence as such. As a Being that neither comes to be nor ceases to be, Presence as such may equally be said to be inert. Indeed, since Alterableness has been said from the very first to concern only *a* Presence and the disappearance of the former would seem to argue the disappearance of the latter and hence a return to Presence as such, we are now apparently meant expressly to put on record that Presence is originally Inertness.

But it is also true that Inertness records the result of our further thinking about Presence. We may be said to have found the first answer to the question implied in the making of Presence as such the object of further thought: the question as to what sort of Being it is. Instead of Presence as such, we speak now properly of Presence as *Quality*. So far from bringing us back to Presence as such, the disappearance of Alterableness brings home to us that Inertness concerns already a sort of Presence. In fact, all the categories involved in the just accomplished act of thought, concern properly Quality: the sort of Being Presence as such becomes as the object of further thinking.

That the Being with which beginning is made cannot remain blank of every definite meaning is clear from the very first. We called it at first *pure* Being, because in order to arrive at the knowledge of truth, we must begin by taking Nothing for granted. In so far as we next came to speak of Presence as such, we were simply restating pure Being, because so far there was only the Nothing of our beginning to fall back upon for its definition. Just because Presence as such is the simplest abstraction of all that is, it remains Determinateness pure and simple, i.e. still only pure Being, but as *thought*.

The distinction between pure Being and Nothing concerns them only as mere sounds. For this reason, the passage of pure Being into Nothing, or vice versa, proved itself devoid of meaning. In defining Becoming as an untenable contradiction, we may be said to have put on record that this term does not concern the *thinking* of pure Being. As thought, this latter is at once Presence as such. And it is only because we insist on producing the object of our

thought, that we have imparted even to the First, with which beginning is made, the significance of the result of an act of thought.

The beginning with Nothing or pure Being, when either is credited with a distinctive meaning of its own against the other, is truly a *thoughtless* beginning. The first act of thought is meant to remove this element of thoughtlessness, and hence ushers on the scene the *thought* of pure Being under the name of Presence or Determinateness as such. And just because it is from the very first clear that we are not meant to stop at the simplest thought, Presence as such is really at once credited with the sense of Quality, as against its further possible specifications.

CHAPTER V.

SOMETHING AND AN OTHER: ALTERATION.

It is to be realized that when we now proceed to draw a *qualitative* distinction between Something and an Other, we do not simply " pick up " a familiar fact. To say that we *create* this distinction may seem at first sight a matter of mere words; especially in the opinion of those who ignore that Being and Thought are distinguishable only in the fundamental unity of both, and that Thought is in that case the positive, Being the negative. The familiar statement that there is nothing in intellect which was not first in sense, simply means that we begin our search after truth long before we become self-conscious or pure thinkers. We must realize, however, that it may be just as much asserted that there is nothing in sense which was not first in intellect. Whilst apparently introducing the present distinction of Something *and* an Other on the ground of our familiarity with facts, we must show that this distinction is necessitated by our further progress in knowing what Quality really is.

To begin with, Quality stands for Something *or* Other. That is to say, Something *and* an Other have immediately the sense of mere aspects of one and the same Quality. Or to avail ourselves of another current expression, we refer so far only to anything and everything of the same Quality—for instance, to anything and everything eatable. That the eatable involves qualitatively distinct articles of food, is obvious even to a starving man; but when one is ravenously hungry, one little cares what one eats, provided there is something or other to eat. Indeed, even the Quality of eatableness proves itself finally to be a mere anticipation, assuming *de facto* the sense of mere chewableness—in illustration as it were that Something or Other not only is not meant explicitly to refer to qualitatively distinct things, but that so far the very thought of Quality remains implicit.

But now we are meant to emphasize Something in distinction from an Other. If, however, we do not take into consideration that a Presence remains in either case the same direct unity of both its aspects, we re-create only the former now superseded contrast between Something as such and Other as such. In order to advance, we must emphasize either of the two aspects of Quality without breaking up their direct unity. And this means that this direct unity must be grasped as related to its own self in the sense of an Other.

Something or an Other is thus no longer a mere restatement of either Reality or Negation with respect to the passage from the one into the other, but a duplication of the thought of that self-identical Something which supersedes the contradiction arising between the two sides of Alterableness, when they are credited with a distinctive meaning of their own. Or rather, instead of a mere duplication of the same self-identical Something, we have now before us the result of its analysis. When we insist on distinguishing Something and an Other with respect to their qualified Presence, we distinctly also negate their interchangeability. The term "duplication" reflects only on the fact that the sides of the present qualitative distinction may be equally viewed as simple Presences and hence from the standpoint of Alterableness. As duplicated, Something retains the same Quality. Still, just as Alterableness already hints at the possibility of a qualitative distinction between its sides, so the present emphasis on this distinction cannot, in turn, altogether dispose of its fundamental unity. Our progress in knowing necessitates that we should create the distinction between Something and an Other, but not that we should render this distinction meaningless by treating its sides as absolutely exclusive of one another. We have simply further specified the object of our present act of thought: instead of Alterableness, there is now before us *Alteration.*

CHAPTER VI.

CONSTITUTION, THE IN-ITSELF: LIMIT.

WHEN we speak of Alteration, we refer to a concrete state of Becoming; to a Becoming having for its sides Something and an Other. But at the same time we presuppose these its sides as given to begin with. We do not profess to witness the Becoming itself, but rather infer it, because in the place of something there suddenly appears something else. As a concrete state of Becoming, Alteration refers to a Being such that it vanishes in its very arising and for that reason comes to our notice only as a result of this its Becoming.

But since we must now presuppose Something and an Other as given to begin with, in the place of the pure Being or Nothing, the disappearance of the concrete state of Becoming no longer results in Presence pure and simple. In fact, we have already realized that, in becoming the object of further thinking, Presence as such acquires the sense of Quality. Accordingly, the result of Alteration implies a further specification of Quality.

At first sight, it would seem that we are meant to put on record that Quality is not mere Inertness. In defining it as the direct or inert unity of Something and an Other, and next proceeding to emphasize this unity with respect to either of its two aspects, we did not seem to affect its Inertness, because the arisen distinction appeared to be of our own making. Nevertheless, this distinction must equally be credited with objective significance, by virtue of our fundamental assumption that Being is a form of Thought and vice versa. Therefore, we must go beyond our primary grasp of Quality as Inertness. Or rather, its Inertness must not be interpreted in the sense of absolute deadness. Susceptible as it is to alteration, it does not exclude Alterableness.

Something and an Other are before us in such wise that the *and* equally has the sense of an *or*. Still, whereas primarily, under the head of Alterableness, we emphasized the *or*, now the emphasis comes on the *and*, because Something and an Other must now be viewed as qualitatively distinct. In a word, we ascribe to them distinct *Constitutions.* In speaking of either, we now at the same time imply that either also *ceased to be* the other. Or, Alterableness concerns now the Constitution of a self-identical Something either *before* or *after* its Alteration.

In so far as Constitution appears to form only the surface of something or other, it is overlooked that anything and everything is a self-identical something only by virtue of its Constitution. Still, since Constitution presents itself as something alterable, and Alterableness results in Quality as Inertness, when Inertness stands for Permanence, it is just as much dialectically necessary that the self-identical Something should be equally grasped in the sense of a permanently fixed background to its constitutional Alterableness. It is in this way that there arises the conception of the In-itself, of what something is *in itself*, in distinction from its Constitution. But that this distinction is purely formal, follows from the very manner of its arising.

Surely, we cannot know what Something or Other is in itself, if it is to exclude that very thing which constitutes its Quality. The In-itself can then refer only to Presence as such and in this way is emptied of all that constitutes our subsequent progress in knowing. Presence as such, the First with which beginning is made, is only the *tabula rasa* of Knowableness. Accordingly, too, the In-itself passes for unknowable. There is still nothing to be known in it. In order that it become knowable, it must imply at least Alterableness. In fact, we must at once reflect upon the contrast between Alterableness and Alteration, because otherwise the distinction between the In-itself and Constitution does not arise.

Just because Constitution emphasizes Something or Other in qualitative distinction from Something else, and hence reflects upon the distinction between Something *and* an Other, Alterableness admits of a double sense—once with respect to the own Ceasing-to-be on the part of Something or Other, another time with respect to its own

self-assertion as a self-identical Something; and it is clearly this latter sense of Alterableness that can really be meant, when we speak of what Something or Other is in itself as against its Constitution. We refer thus to it as if it were free from its liability to becoming something else in consequence of its constitutional Alterableness. We assume that a thing may be treated also independently of its relatedness to everything else; that, in so far as this relatedness cannot absolutely be excluded, its effect need not rob something or other of its self-identicalness; in short, that something or other just as much reacts on its external Alterableness and remains congruous with itself all through the changes to which it is open by virtue of its Constitution.

But we are all the time aware that this kind of self-affirmation has its *limit*. We cannot think of Alterableness to the exclusion of Alteration. The distinction between the In-itself and Constitution may be presented as due to an emphasis on either Alterableness or Alteration in a direct unity of both. In spite of our original insistence on Alterableness, we had to introduce the conception of qualitatively distinct things. The necessity for the creation of this distinction lies in the fact that we have anticipated Alteration already when speaking of Becoming, with the consequence that Presence as such was from the very first pregnant with our present conception of a Something that has ceased to be an Other, i.e. of a qualitatively distinct Something or Other.

Because we insisted on taking literally Nothing for granted, we imparted to the thought of the simplest Being the significance of the result of the first act of thought; the significance of a Being having Becoming at its back whilst yet being the First with which beginning is made. Unless we had done so, we would not have been able to make of Presence as such the object of our further thinking. But in defining Presence as such as the direct or inert unity of Being and Nothing, we have really been foreshadowing our subsequent more concrete grasp of it as the direct unity of Something and an Other. The Becoming at the back of it acquired then the significance of Alteration, whilst the simultaneously implied negation of this its background converted Becoming or Alteration into mere Alterableness. Therefore, in defining the object of our further thinking as

the direct unity of Alterableness and Alteration, we simply explicate the contradiction constituting the very mainspring of our progress in knowing. In our attempt to avoid the ambiguity attaching to Something or an Other in the current expression Something or Other, we introduced the distinction between Constitution and the In-itself; and it is obvious that the In-itself refers to that Being which what becomes must also already be, in explicit record of its character as a Being such that it at the same time remains purely implicit.

It is essential to our progress in knowing that we draw a distinction between Anticipation and Realization. Our task consists in nothing else than a constant endeavour to elucidate to ourselves the meaning which is *ab initio* implicit in the original creative act of our mind. The nature of Thought must reveal itself already in its simplest act, when, however, its import remains still also hidden from view. The original Being implicates the Being of the whole World ; only the simplest thought cannot be expected to explicate this its implicitness or Inwardness. From the standpoint of our progress in knowing, we must distinguish between our anticipation of the complete explanation of the true Inwardness of Creation and the degree of its realized exposition. This is precisely the distinction which we are just now putting on record in speaking of the In-itself and Constitution.

The dialectic past constitutes the dialectic now, whilst the future dialectic remains only at the stage of the In-itself, as a *tabula rasa* of further knowledge. But because the creative act, reproduced at the beginning of every act of thought, also already anticipates the conclusion still to be reached, it is just as much the dialectic future that may be said to constitute the dialectic present, whilst the past dialectic acquires the significance of mere implicitness or In-itself-ness.

Pure thought constitutes our very Being even when we are not aware of it. The idea of Truth, the unity of Thought and Being or Feeling animates us even when we ignorantly dispute it. When we at last come to acknowledge it, our present prejudices fade away or become only matter of memory. In constituting the Present, the Past remains also only an In-itself, whilst the Future, though in

a way still only an In-itself, *de facto* also already is at work in us and hence constitutes us.

Owing to the fact that our progress in knowing does not consist in going away from our beginning, but rather only in widening the scope of the simplest thought, we cannot succeed in breaking our original unity into completely independent constituents. Instead of an absolute discerpibility, the exposition of this unity involves only a constant re-creation of the contradiction constitutive of the simplest act of thought. In order that this re-creation bring about progress in knowing, the extremes of the original contradiction must *pari passu* become more and more definitely outlined; but they must always remain in touch.

Every act of thought creates a distinction only in order to vindicate the fundamental unity in the appropriately widened or narrowed meaning. The increasing definiteness acquired by the fundamental unity in the course of its creative evolution contrasts it with its original indefiniteness; and inasmuch as this indefiniteness is by anticipation, or *in itself*, all-comprehensive, the gradual explication of this its Implicitness may equally be described as a narrowing down of its scope. It is just now, when we seek to record this contradiction by means of a single term, that the narrowing aspect of the original all-comprehensiveness comes to view. For the term suggesting itself in record of the direct unity of Alterableness and Alteration now before us is *Limit*.

We cannot speak of Limit apart from at once granting and denying the distinction between Something and an Other. The conception of a limited Something, of something limited, goes together with the conception of something else which limits it. The Limit which something or other is or has, is at one with its own self as well as an Other to its own self. Therefore, Limit implies the sense of the In-itself as well as of Constitution. It joins and just as much disjoins Something and an Other. In so far as it applies indifferently to either, it implies the standpoint of Alterableness. Yet along with this standpoint, the limited something is equally granted a distinctive meaning of its own against the other one limiting it and forming part and parcel of its own Constitution.

There is, then, before us the conception of a self-identical Something as what has Alteration at once at its back and in front, and whose alterableness, therefore, has a *limit*. This term puts on record what we might, by anticipation, have associated with the result of the previous act of thought. Inasmuch as this result makes us realize that, in making Presence as such the object of our thought, we presuppose a certain restriction on its originally quite inchoate meaning, our present result makes us realize that we have done likewise in making quality the object of our present act of thought. But because the restriction has now been presupposed in connection with what is itself already a Being of a sort, we have further qualified Quality. Instead of being matter of implicitness, only the In-itself, the Limit is now matter of envisagement and hence the object of the next act of thought.

CHAPTER VII.

LIMIT, PRESENCE: FINITUDE.

SOMETHING or Other, Anything and Everything, is necessarily limited, because alteration belongs to its own inwardness. Unless the Otherwiseness implied in its Alterableness becomes distinctly outlined, it remains indistinct. Before we can identify it with a distinct something, we must postulate its Relatedness to something else, i.e. to its own self as another qualitatively distinct Something. Accordingly, too, the loose thinking tries to get hold of its meaning only by means of comparison with everything else and hence presupposes our present grasp of Anything and Everything as necessarily limited.

Instead of the Something as such, we have before us Something such that it is *not* the something which it yet in itself also *is*. The *is* and the *not* are in this respect moments of a direct unity called Limit. Accordingly, Anything and Everything at once is and is not through its Limit. It is through it, because the limit constitutes its very inwardness. But just because the universal Inwardness insists on turning its own self inside out, Anything and Everything just as much also is not through its Limit. We have here the fundamental distinction of Thought and Being, of God and the Universe, with an emphasis on the direct unity of its sides.

But so this direct unity must really be grasped also by way of a *third* to its two sides. Inasmuch as the Limit is either Something or an Other, it is at once both and neither. This is why everything admits of being viewed as at once inside and outside its limit.

Now, if that Something and an Other, in distinction from the Limit as such, forfeit their own distinction of Inwardness and Outwardness and thus lose their own explicitness as a graspable meaning, there is then before us

the distinction between Limit and Presence as such. But that this distinction amounts to an untenable contradiction becomes obvious directly we try to grasp it. Indeed, we cannot help anticipating its nullity in the very act of creating it.

If the limit is to be thought apart from Something and an Other, in the sense as it were of an inert background to Alteration, and *eo ipso* to Alterableness, its meaning loses its characteristic feature and hence is effaced. Instead of the Limit, there is then before us Presence pure and simple, i.e. an empty abstraction, the *tabula rasa* of all meaning. And as Something and an Other themselves disappear in the original blankness or darkness, the distinction between Limit and Presence arises only to disappear. We cannot help making it, but no sooner have we formulated it than it escapes our grasp. Its sides are only aspects of Becoming, or rather Altering; of that concrete state of Becoming which is foreshaded in connection with Becoming as such.

Accordingly, Something and an Other are not Presences lying quiescently side by side. True, they limit one another; but because either is in itself the other, they also do not limit one another. In paying heed to their identity, as simple Presence, we set aside their qualitative distinctiveness; whilst in holding them fast in this sense, we set aside their fundamental identity. This incompatibility of Limit and Presence, with respect to one and the same Something or Other, yields the notion of Finitude. Just because it is limited, Anything and Everything is *finite.*

In agreement with our own realization that the dialectic process carries the result of Alteration at once to the stage of the Finite, the ordinary reflection too signals the standpoint of Finitude as a halting stage in its endeavour to grasp the nature of things. The cognition that everything carries in its very origin the germ of decease, that all is *vanitas vanitatum*, counts as a high-water mark of human wisdom. In order, however, not to succumb to the pessimistic sadness begotten by this insight, we must realize that the vanity in question concerns only the fringe of our Being; its constitutional Outwardness.

True, inasmuch as everything is an untenable contradiction, it is in its *not.* But, then, this its Not-being is not the Nothing of our beginning. Pure thinking is not frightened

at the present disappearance of all meaning in the bottomless abyss of Finitude as such, because it is confident in its power to re-arise, Phoenix-like, to higher self-realization; because it is buoyed by its undying faith in its destiny to attain to full Self-knowledge. But for this faith, we would not at all care to concern ourselves with pure thinking. In the light of this faith, begotten by the Spirit, we instinctively leap to the conclusion that Finitude brings us back to our beginning only in explication of the *ab initio* implicit Infinitude. The original Being must ultimately be ushered on the scene in the sense of that Being which guarantees our own imperishableness even though our body be "dust returning unto dust"; of that Being which remains unaffected even "*si fractus illabatur orbis, impavidum ferieut ruinae*".

CHAPTER VIII.

TRANSITORINESS, TRANSCENDENCE: INFINITUDE.

At first sight, the thought of Finitude does not seem to contribute anything new to our progress in knowing. In calling something or other finite, we may be said simply to put more distinctly on record that its constitution is nothing permanent in itself, and hence that anything and everything is in itself a becoming something else. But, then, our progress in knowing consists in making explicit what lies in the preceding. Inasmuch as the object of the previous act of thought was to establish Quality in the sense of something self-identical, the thought of Finitude remained hidden in the qualitative immediacy of a limited Something or Other. Even though anything and everything was grasped as foredoomed to become something else, our concern was primarily with its self-identicalness only during the interval between its past and future alteration. Consequently, we did not immediately think of its finitude; just as generally we are in the habit of leaving it out of the question. Even our own death remains during the greatest portion of our life a mere word. It is only after the death of a beloved being that the sadness of finitude really assails us—and then, as a rule, not for very long.

Certainly, then, the thought of Finitude marks an advance beyond the thought of something limited. In making this latter the object of our further thinking, we have altered our attitude to it. We ceased to be oblivious of its limits. From our present standpoint, its duration has, as it were, shrunk to a moment, its origin and decease present themselves to us in restless Alternation. In thinking of its finitude, we think of it as of a concrete state of Becoming, but no longer in the sense of a simple, but of an alterable alteration. Precisely, because no thing is truly permanent in itself, yet for all that no mere Nothing, it is a

perennial Becoming something else: *Transitoriness* pure and simple.

But we cannot now really speak of Transitoriness *pure and simple.* As an unceasing Becoming, the Finite just as much keeps *transcending* its own self. Its Ceasing-to-be whatever it happens to be, *eo ipso* also implies its Coming-to-be something else. And as that which it comes to be is a result of its Alteration, it just as much affirms as negates its previous meaning.

Accordingly, too, our common-sense rebels against the treatment of the Finite in the sense of Transitoriness pure and simple. The very sadness begotten by the thought of Finitude is a protest against it. Why should the thought that everything is doomed to pass away fill us with sadness, if it had not for its background our innate, though temporarily disputed conviction that the passing away is not synonymous with utter annihilation? How could we live and put up with the hardships incidental to our striving, if we really believed that all our striving ends in Nothingness?

If it is true, and undoubtedly it is, that our philosophical opinions represent an effort of the whole man within us, then the standpoint of Transitoriness pure and simple represents a very one-sided expression of our undying Faith in our destiny. This standpoint represents at best only the immediate result of an intellectual effort to break up the fundamental unity of Thought and Being or Feeling. However, one need not be a profound philosopher to realize that an effort to *think* Being to the exclusion of all Thinking implies its own *reductio ad absurdum.*

And if we cannot cling to the standpoint of the Finite in the sense of Transitoriness pure and simple, neither can we grasp it exclusively in the opposite sense of an endless surpassing of itself. We fail to realize the meaning of an *endless* progress. We should lose our interest in progress, if it had no end. What sense would be in striving to reach what we at the same time believed to be beyond reach? The very fact that the whole man within us manifests an unflagging interest in our progress in knowing, argues that this progress has a definite end within our reach. The endless progress, or *Transcendence pure and simple*, is at the bottom only another expression for what is meant by

Transitoriness as such. Just because it is endless, it amounts to a simple passing away.

As a matter of fact, then, in distinguishing Transcendence from Transitoriness, we really oppose the two aspects implied in the conception of the Finite as an endless becoming something else, and consequently just as much negate their distinctiveness. Owing to this self-contradictory nature of the distinction on which it rests, the Finite has its Being in its Not only in so far as this its Not just as much *is*. And in so far as we thus emphasize the positive aspect of the fixed Negation of its own self, as which we have defined the Finite as such, this latter is now to be grasped as explicitly become the Other as such of its own self, and hence as the Infinite.

Inasmuch as the immediately occurring correlation of this notion with our own inmost self rouses our interest in an interpretation of the dialectic of Finitude in distinct application to our own human being, it suggests itself, to begin with, to correlate the Finite as such with the animal stage. The gap, by the way, between this and the mineral stage would then obviously refer to the vegetable stage in distinct embodiment of the contradictory distinction between Limit and Presence, whilst the distinction between Something and an Other in application to the mineral stage when this stage is correlated with the thought of something limited, would in turn refer to the chemical distinction between acids and alkalis, chemical process having to be grasped only as a precursor of the vital process displayed by the growth of a plant. The evolutionary past, when beginning is made with a postulated primordial matter, subsequently conceived as taking more concrete shape, primarily in nebulae, then in the solar system, next in our earth, as a background to the ensuing development of mineral, vegetable and animal kingdoms and finally in the appearance of Man—all this is manifestly a record of a development of the same type as is broadly foreshadowed in our own progress in knowing up to the present stage. After all, the nature of thought must reflect itself in every product of human mind.

Our correlation of the Finite as such with the animal stage is obviously to be taken in marked correspondence to the dialetical necessity of ushering the Finite on the scene

in immediate obliviousness of its fundamental unity with the Infinite—an obliviousness equally characteristic of the loose thought, inasmuch as the Infinite remains to it utterly incompatible with the Finite. In so far, however, as the fundamental unity of both is anticipated at once when attention is riveted to the from the very first equally implied character of the Finite as such as an *endless* becoming something else, and the dialectic of the Finite, consequently, is meant to render the said fundamental unity explicit, its applicableness to our own human being is obvious in reflection upon the general character of this being as a self-realizing middle-term between Nature or our animality and God or our Spirit.

As regards, now, the postulated correspondence to the two main aspects of the Finite, we cannot be in doubt that they refer to Body and Soul, seeing that the relation between these aspects of our human being is of the same contradictory nature as the relation between Transitoriness and Transcendence. Even apart from an independent realization of the relation between our Body and Soul, our present correlation of them with the two aspects of the Finite would lead us to postulate that they are distinguishable only in embodiment of their fundamental unity and hence can be opposed only in such wise that each at the same time reflects upon the other as something implied in its own Inwardness. Just as Transitoriness cannot be contrasted with Transcendence except as in unity with it, and vice versa, the contrast concerning only the emphasis on either in the fundamental unity of both, we would not feel justified in dissolving the living or negative unity of Body and Soul into Body as such and Soul as such. To draw such a contrast between them would mean to revive the already superseded contrast between Something and an Other in preference to our present higher attitude to objectivity.

But since the Body and Soul all the same also are distinguishable, whilst yet the distinction can refer only to an accentuation of either in the fundamental unity of both, it would follow that along with their main distinct connotation, Body in reference to our physical nature and Soul in reference to our transcendental nature, each acquires also a secondary significance in reference to its inseparable unity

with the other, when it must simultaneously reflect the accentuated original meaning of this other. Accordingly, when viewed in the sense of a mere aspect of the physical body, the Soul's Transcendentalness sinks to the animal level; whilst, when the Body is in turn converted into an aspect on the Soul's side, its material grossness is lost.

Now that the restatement of the Soul in the sense of a mere aspect of our physical nature refers to the system of Vitality is surely obvious. Conformably with the standpoint of modern physical science that all bodily functions are accompanied by electric action, and that whenever electric action occurs ether must be present, the aspect of the Soul on the side of the physical Body is spoken of in the theosophical literature as the "etheric double". This term indicates that the vehicle of Vitality—in Rosicrucian literature the "vital body"—becomes visible to the trained sight as a perfect duplicate of the physical body which it "ensouls". Being physical, a slight tension of the nervous system renders vision sufficiently acute to discern it, in which case it is found to be violet-grey in colour and to vary, as to its texture, with the quality of its gross counterpart. The fundamental unity of both shows itself in the fact that even after they are finally separated through death, the vital body remains hovering over the grave, decaying synchronously with the dense vehicle. During life, it admits only of a partial extrusion, in which case the physical body is reduced to a state of lethargy.

Conformably, too, with the postulated sense acquired by the body on the side of the Soul, Occultism credits the Soul with a body of her own under the name of the "astral" or "desire" body, according as we use the theosophical or rosicruscian term. In any case, we distinguish our soul from our body in a double sense. We are not the body, yet we cannot altogether disown it. It is *our own* body that we refuse to identify ourselves with. As a matter of fact, then, we continue to implicate a reference to a body even on the side of the soul.

But we must, further, equally postulate for the soul the possibility of functioning independently of the physical body, just because the In-itself acquires in connection with the Finite a double significance. Along with being directly united with its own Transitoriness, the Finite at the same

time transcends its transitory self. This transitory self is in one and the same respect excluded as well as included. Or, the unity of Transitoriness and Transcendance is an unrestful unity and for that very reason admits of being envisaged in a double way; and even though the distinction thus created no longer concerns two qualitative somethings, it cannot be denied its *raison d'être.*

Qualitativity attaches now only to the side of Transitoriness, and that too only in so far as the Finite is held fast in the sense of a limited something. And since then, Qualitativity is, strictly speaking, transcended already on the side of Transitoriness, it follows that on the side of Transcendence we get qualitatively Nothing pure and simple. That means, of course, that the Soul-aspect of the fundamental unity of Body and Soul is not an object of our ordinary attitude to objectivity; an object of sensuous perception. Nevertheless, to deny this aspect its *raison d'être* would all the same be absurd. Our progress in knowing within the present act of thought is concerned with the grasp of a Being such that it is only in its Not-being; and this Not-being is precisely the questioned Being of the Soul: that kind of Being which is familiar to us as Feeling.

The postulated possibility for the Soul of a function independent of the physical body manifestly concerns our capacity for *originating* Feeling. Of course, in that case we at the same time imply that she herself just as much transcends her Feelings. As thus distinguished from the Soul within her own self, when she herself is already distinguished from the physical Body, Feeling represents the aspect assumed by this latter on her side. And just as the Soul shows forth her original meaning even when reduced to an aspect of the physical or animal Body, so this latter should, in turn, equally retain its original significance, as a limitation of the Soul even when reduced to an aspect on her side. Accordingly, it is the surviving habit of bodily feeling that constitutes Soul's bodily aspect and comes to be felt by her as an obstacle to her innate craving for complete freedom from limitations.

We are aware that this craving is ultimately the desire for Self-knowledge. But this goal cannot be reached so long as the Soul finds herself divorced from physical existence. The fact, by the way, that our capacity for

originating Feeling should properly come home to us only after bodily death, is implied in the very nature of the distinction now under discussion. During *post-mortem* existence, Self-realization on the part of the Soul cannot go beyond her self-assertion as the In-itself of the inner life she has within the circle of her own feelings. What she tries, and must try, to do during her physical existence with respect to her dense body : to remove its seeming Reality or to prove to herself that it is *her own* body—that very thing she equally does during her *post-mortem* existence with respect to her surviving body of feelings. Owing to her persistent endeavour, the clash between her spontaneity and those habits of feeling which ceased to be congruous with her new mode of Being gradually lessens, until she realizes herself as the In-itself of her world of spontaneously generated feelings and thoughts. But because Truth implies the unity of Thought *and* Being, so long as objectivity shapes itself readily conformably to Soul's own conceptions, there is no check on its truthfulness. Therefore, it is in the nature of truth that the Soul should at last begin to miss again those very limitations which she strove so hard to get rid of.

Any one who tries to live an exceptionally holy life is familiar with a similar revulsion of feelings. Gradually the fascination of the ideal begins to fade away, and the resolve to persevere weakens. One even begins to doubt, whether it is really wise to persevere, whilst the temptation to return to the flesh-pots of Egypt grows apace. Now, it is that very necessity which finally, or perhaps we had better say only occasionally, makes an extremist in asceticism throw himself headlong into a fit of debauchery, that also makes the Soul crave a new birth. The implied perplexing contradiction that a particular individual, who to his knowledge has not lived before, and apparently neither can live again, is yet meant to identify himself with a whole series of distinct individuals who lived in the past or are yet to be born in future, serves only to remind us that the term "individual" stands in this respect only for the reincarnating Soul as the In-itself of our finite Being on the side of Transcendence. That is to say, we do not thus reflect as yet upon the true meaning of our Self as Spirit, but only as a middle term between our explicit Finitude and the

still only implicit Infinitude. We call this middle term our Character. For, as these particular individuals, we own ourselves to be at once dependent on, and independent of, our Character. On the one side, we have a particular character by birth, yet on the other side we are also meant to fashion our character, to prove ourselves as masters of our destiny, not to remain simply what we are by birth, but to reach the dignity of a "self-made" man; and Progress remains unexplained, so long as it is interpreted only in the sense of a mere transmission of what is to be found as given to begin with. Along with Heredity and Environment we must postulate also *Epigenesis*. In short, then, our Character is only a link between the experience we acquire in our successive lives and our *pari passu* growing capacity to grasp this experience *sub specie æternitatis*, or in the light of our true Infinitude, to be rendered explicit in the course of our next discussion of the concluding significance of the In-itself of the Finite as the unity of Transitoriness and Transcendence.

CHAPTER IX.

THE INFINITIZED FINITE AND THE FINITIZED INFINITE: THE TRUE BEING.

ALTHOUGH the Finite is found to be *in itself* transcendental, this its character does not become fully explicit in its own sphere, but presents itself immediately only in the shape of an endless becoming something else. The thought of Infinitude begins to loom on our mental horizon only when we bring home to ourselves that the distinction implied in the said Becoming is really transient. Inasmuch as the Finite rests on this distinction, it is doomed to being superseded, so that it does not merely keep transcending its own finite self, does not merely progress to an elusive goal, but seeks and finally reaches a definite end, when its Becoming alters into a positive Being-become and the Finite, as thus become an Other of its own self, reveals its Infinitude.

But just because the Infinite is thus ushered on the scene primarily with an emphasis on its distinction from the Finite, the simultaneously from the very first implied unity of both remains still in the background and hence continues to play the rôle of an implicit middle term between its explicit extremes as now before us. That, however, the grasp of the Infinite in the sense of a real or qualitatively distinct Other to the Finite would only entangle us in a vicious circle, follows from the preceding. For, the conception of Reality is transcended already in the sphere of Finitude, even though Transcendence remains still also transitory. In speaking of the Finite as something real, we properly degrade it to the rank of something limited. Now, it is true that, inasmuch as the Infinite is to be immediately grasped as the Other as such of the Finite, it apparently also brings us back to the superseded ordinary conception of Reality; for a limited Something or Other too may be characterized as the Other as such of the Finite. But this our return to the standpoint of Limit is only meant to

bring home to us that the Infinite has been implicitly presupposed already at the stage of Limit—already, in fact, at the very beginning of our task.

In removing our previous emphasis on the nature of the Finite as nothing permanent in itself, we are really meant to rise beyond the sadness begotten by the thought of Finitude. Objectivity is finally to be grasped as an embodiment of the Infinite. Knowledge of what the Finite truly is in itself disposes of the crude evidence of our ordinary senses as a final court of appeal in reference to the true nature of Reality. Since Reality presents itself to us then as idealized, we propose to call its true Being *Ideality*, i.e. Reality, but as viewed in the light of pure thought, which itself may thus be said to occupy the standpoint of *Absolute Idealism*. However, consistently with the fact that the unity of the Finite and the Infinite remains, first of all, only implicit or direct, its formal explication requires that we should try to oppose its two sides as two mutually exclusive spheres—in which way, we shall, of course, only trace out the dialectical background, or *raison d'être*, of the vicious circle characteristic of the loose thinking in its attitude towards the true nature of the Infinite.

Now, it is true that even in its immediately assumed incompatibility with the Finite, the Infinite is granted the character of imperishableness. In fact, at first sight it would seem that, but for this its presumable incompatibility with the Finite, the Infinite could not at all be credited with its undoubted imperishableness. On closer consideration, however, it becomes just as much obvious that, so far from securing for the Infinite its distinction from the Finite, this attitude towards it achieves the very reverse. For, if the Infinite is opposed to the Finite as to a Being altogether excluded therefrom, this excluded Being becomes its Limit and hence converts the Infinite into something limited, thereby equally finitizing it. Therefore, the Infinite is not to be conceived as if pushed out beyond the sphere of Finitude, or as extra-finite, but only as the *infinitized Finite*, i.e. as the Finite itself, but as grasped *sub specie aeternitatis*. After all, we have not picked it up as a Being lying side by side with the finite Being, but, on the contrary, have ushered it on the scene as the end of the otherwise *endless* progress peculiar of the sphere of Finitude. This kind of

endlessness is really a witness of the Infinite itself within the sphere of Finitude. Quite conformably with the nature of our progress in knowing, in ushering on the scene the thought of the Infinite, we are simply endorsing what the Finite itself has proved itself to be in itself, i.e. the *finitized Infinite.*

Consequently, the distinction created for the purpose of the present act concerns the double aspect of the fundamental unity of the Finite and the Infinite. In this way, we are at once intercepting the perplexity voiced in the question: How does the Infinite finitize itself? The assumed irreconcilableness of the Finite and the Infinite is an absurdity having its origin in a thoughtless use of these terms. One has only to think what one says, in order to realize that the Infinite is thinkable only as in unity with the Finite, and hence that the distinction between them is idealistic.

It is now that it should be plain that the shock which we receive on our awakening from the immediate still naïve attitude towards objectivity, when we find ourselves forced to admit that all that has been part and parcel of our Reality is a mere passing away into nothingness, and consequently feel as if the mainspring of all our striving is utterly broken—that this wave of pessimistic sadness and despair is indeed only a witness of our instinctive Idealism. That which crushes us is not really the parting with our sensuous Reality, but our mistaken impression that the immediately resulting Nothingness engulfs the Infinite itself. Quite unbeknown to ourselves, we have from the very first idealized our Reality, and it is the supposed burial of our ideals, of our Ideality, that we truly bemoan. At the bottom, we are inveterate Idealists, even when the nature of our attachments appears to argue the very opposite. There is no man degraded enough to have no ideals, simply because man, even the worst man, is a born thinker.

But if even the parting with our sensuous Reality is not without its pang, how much more difficult must it be, when we are next expected to supersede the immediately arising identification of our true Self with the Soul? It is quite currently granted that our Self consists of Body, Soul and Spirit. But to how many of us does the term Spirit mean anything beyond a vague hint at our fundamental unity

with God, while the term God remains only an empty word? If, however, we bring home to ourselves that Spirit stands for that very unity of Thought and Feeling which constitutes the element of pure thinking, it becomes plain that the clinging to the stand-point of Feeling, of that Transcendentalism which still falls into the sphere of Finitude and to which, consequently, the notion of Spirit still remains an unreachable beyond, our Self remaining identified only with the re-incarnating Soul—it becomes plain that the clinging to this stand-point may equally be viewed in the light of a formidable obstacle to our true progress in knowing. For, so far, one does not yet simply ask what the Truth is, but in the first place only how it will fare with one's own Soul; and this exclusive concern only too easily sharpens itself into a violent antagonism to the search after pure Self-knowledge in the realm of pure Thought. The labour of Thought is at times very hard—"sour," Hegel calls it—but the Soul wants to be before anything else only *edified* and *comforted;* so comes it that the philosophical search after Truth has even been declared by her the height of human arrogance and impiety.

So long as our capacity for pure thinking is left out of the question and Knowledge is sought with the help of Clairvoyance alone, the eternal Now of God remains apprehended only spatially and temporally. And even though Space and Time are then also already transcended, the perpetually occurring relapse into the sphere of Finitude assumes the appearance, as if God insists on remaining hidden behind them, and consequently, as if He intended to make Himself known only until His objective Self-manifestation comes to its end. So comes it that, although Occultism grants the fundamental unity of the Finite and the Infinite, it opposes its two aspects in the shape of two periods of Time, during the one of which the Infinite is said to descend into Finitude, whilst the other period concerns the returning movement. The possibility of the attainment to pure Knowledge is not denied, but only postponed to a distant future. Time is treated as God's own time, but, as regards the individual, it remains an insuperable limitation to his present grasp of what God is in His own Essence.

Now, that the infinitization of the Finite does take time,

must be conceded. In time, every one of us may hope to reach anything at all reachable. But because it is not an alien power that infinitizes the Finite, the time needed for the reaching of our goal must also be realized as *our own time.* We must not fancy ourselves in the image of puppets passively swept on by the tide of time. True, so far as we are part and parcel of the World, we are in a sense controlled by its time. Time becomes, however, in this way only one of our limitations which we are transcending in our very Self. We are not simply an appendage of the World; only one of the wheels in its mechanism. We oppose ourselves to the World at large and convert its existence into a mere condition of our own self-development. That which time accomplishes for us, is indeed, accomplished by our own effort. Apart from the own progress of the Finite, Time is an empty abstraction. There is no time as such. We can speak of Time only in terms of some happening or other. For purposes of comparison, we must, of course, have fixed units of time; but when we proceed to speak of our own progress in terms of these units, we surely do not assume that our development is of the nature of a clock running down with mechanical precision.

To us, the distinction between the finitized Infinite and the infinitized Finite is no longer a mere restatement of the infinite progress peculiar of the sphere of Finitude. On the contrary, the dialectic of this distinction concerns only its falling to the ground, because it can be upheld only by relapsing into the simultaneously also already superseded standpoint of Finitude. That which from the standpoint of the reincarnating Soul remains a matter of future development, belongs now already to our dialectic past. The ultimate goal of this future development is anticipated already at the very beginning of our task, because it consists in getting rid of the idea that pure Knowledge can be reached only by virtue of endless experience in the sphere of Finitude. Facts are only so many problems calling for solution, and Occultism simply extends the region of facts *ad infinitum.*

Guided by our instinct of Reason, we realized from the very first that the only way to come to *know* what we *are*, is to refuse to take anything for granted. Nevertheless, it is only now that the wisdom of this our beginning is

properly plain to us. Quite conformably with our present realization that the Infinite is unthinkable except in fundamental unity with the Finite, we did not pretend to draw a rigid line between our thinking and being. However much we insisted on taking nothing for granted, it was obvious to us that our own Being, the Being of the world inclusive, must remain presupposed. Consequently nothing was further from us than a wish to quarrel with facts. Our taking nothing for granted did not imply a denial of anything familiar to us as a matter of fact, but had for its background the distinction between Knowledge and mere Familiarity. It is just because our study of various sciences resulted only in a keener sence of our Ignorance, and especially because it became plain to us that our hunger for pure Knowledge, for a fully satisfying and conclusive solution of the Riddle of the Universe, cannot be satisfied by a mere tabulation of facts, a task to which we saw no end, that it occurred to us to seek this solution independently of our familiarity with facts. The Nothing taken for granted was the Nothing of our Knowledge, a record of our instinctively logical grasp of the finitude of facts when apprehended only externally.

Just because we at once intuitively assumed the standpoint of pure thinking, we were at once also aware that a fact of any kind is *de facto* only an embodiment of our own thinking—an *embodiment*, because it did not occur to us to dispute its objectivity. Therefore, we acknowledged to ourselves, however implicitly at first, that thinking is not merely something purely subjective, but a happening having equally an objective significance as a matter of fact. This is obviously the simple import of our present distinction between the finitized Infinite and the infinitized Finite. But as this distinction was at first in our mind only implicitly, the fundamental unity of Thought and Being presented itself to us only abstractly. The transition from Thought into Being, or *vice versa*, remained a problem to be solved in the ensuing progress in knowing, and the Nothing of our beginning had perforce to be interpreted also in the sense of a negative attitude to the necessarily presupposed sphere of facts.

The import of the contradiction which we thus ushered on the scene is now equally plain. It is the contradiction between our ordinary conception of Reality and our present

grasp of it as Ideality. Inasmuch as the loose thinking makes itself dependent on given facts, it never can become conclusive. No matter how much it widens its sphere, there is always a beyond. It cannot be sure that its inferences have universal validity and hence goes together with scepticism as to the reachableness of Absolute Truth. It does not occur to it that, since facts are, after all, a form of Thought, and since problems originate in the realm of Thought, Absolute Truth is to be found through pure thinking alone.

Thus, in conclusion of the present act of thought, we are meant to put on record that pure thinking concerns itself with the true Being even when it reflects upon what ordinarily passes for Reality. In so far as it presupposes the sphere of facts, it views this latter only in the sense of the finitized Infinite and, in proof of this its attitude towards it, proceeds to re-create it from within its own self, or in the form of Thought. Just because it no longer ignores the share of Thought in Empiricism or Occultism, it puts past the fancied gulf between the Finite and the Infinite. In distinguishing between them, it remains conscious of the idealistic nature of this distinction. The alternation between Transitoriness and Transcendence, and the resulting infinite progress, is in this respect grasped as ended. In short, pure thinking concerns itself only and solely with what is eternally *true* and hence neither affected by, nor dependent on, anything but its own self. In so far as the true Being is in itself alterable and hence also becomes something else, this its alteration remains an appearance concerning only its spatial and temporal display. At the bottom of this display, there is emphatically only the One true Being: the present meaning of the original Being as an object of our further thinking.

Our previous progress in knowing appears to us thus also only in the light of an introduction meant to free us from the lingering sway of our ordinary attitude towards objectivity, so that we may at last approach our proper task with full consciousness of its true import. And, surely, it is appropriate that this introduction should itself fall within the system of pure thinking, because the assertion that pure thinking concerns itself only and solely with the true Being, would otherwise have the character of something simply or blindly taken for granted.

PART II.

QUANTITY.

CHAPTER I.

ONENESS, EMPTINESS: MANIFOLDNESS.

JUST because the One Being idealizes the finite Being, its Presence is a higher kind of Reality than the one belonging to the finite or qualitative Being. In order to indicate that this its Presence is not within the range of ordinary perception, we may now also call it *Emptiness*, in more specific restatement of the general meaning of Ideality.

We know already that the true Being insists on laying bare its meaning also in such wise, that its Outwardness appeals to the inner vision alone, to the so-called clairvoyant vision, whose actuality is for the present to be taken for granted. Therefore, in defining Emptiness as the Quality of the One Being, we simply wish to convey the idea that its clairvoyantly perceived Outwardness is from the standpoint of our immediate attitude to objectivity as though it is *not*. Nevertheless, Quality is thus reflected upon already in its true sense as Ideality. Consequently, Emptiness must not be interpreted in the sense of mere Nothingness. The One Being is emptiness in such wise, that this its Emptiness also *fills* it. In fact, we are thus referring to the finite Reality as of what the One Being has originally emptied itself, in order to secure by means of this its own Self-avoidance its own Self-fulfilment.

In so far as the One Being and Emptiness may be also viewed as the present restatement of pure Being and Nothing, when, instead of with the original pure indeterminateness, we are beginning with the Limit as become null and void, Becoming acquires now the sense of an unrestful unity of the One and Emptiness. Now, this is a self-annulling distinction. Emptiness is not a positive Limit to Oneness. It is outside the One only as at the same time belonging to its Inwardness. Therefore, the One Being is itself Emptiness, and *vice versa*. No matter which of the previous distinctions we predicate of them, the

distinction remains purely idealistic. When we say that their unrestful unity amounts to a Becoming of Many Ones, to a Self-manifolding, we must at once protest that the notion of Becoming cannot properly be applied to the One on account of its Changelessness. The one true Being cannot really become *another* One, even though it is *in itself* alterable and hence just as much subject to Alteration. We have already realized that Alteration can now only concern its own self-discernment into a Manifoldness of Ones, whose Becoming means truly Self-differentiation pure and simple.

The Many Ones stand for that Alterableness without which the One Being would remain at the stage of the Unknowable. For this reason, they must not be credited with qualitative distinctiveness. Just because Reference to Otherwiseness is now directly converted into pure Self-reference, there cannot here be a question of a realistic exclusion between the Many Ones. In excluding another One, each one would exclude its own Self and hence lose its own *raison d'être*. The conception of Separateness would entangle us here in that vicious circle which we are supposed to have already superseded in our grasp of the nature of the true Infinite. The One Being requires for its Self-representation Many Ones, but each One is only a Being-for-One; i.e. its Reality is Ideality.

It is really because we would fain credit the One Being with ordinary Qualitativity, or grasp it, as it were, in the image of a particular existing One, and, in this way, make of it only *a* One among many separately existing Ones, that the original *one* One presents itself also as a *plural* One. In this way, however, we are only resuscitating the contradiction between the finitized Infinite and the Infinitized Finite. But, then, we are no longer blind to the proper import of this contradiction. We know that our present inability to identify the One Being with any particular One, is simply due to the fact that our task to *qualify* the One Being is still at its very beginning. We are still only busy with a clarification of our grasp of the original Being as Presence pure and simple, and hence still a long way from a clear grasp of the how the One Being becomes present in the form of a particular One there before us as a tree, a dog, a man.

In so far as Occultism speaks of the One Being as differentiated into infinitely many Gods, it conceives each God as presiding over a World of his own. At first sight this standpoint appears quite in agreement with our own notion of the present many Ones. On second thoughts, however, we detect in this standpoint the already noticed defect of Occultism—a defect which it cannot avoid, because it represents the true Being still only with respect to its transitory aspect, and for that reason still also remains distinguished from pure Thought even while already having the same goal: the solution of the Riddle of the Universe. Namely, the present Ones are not thought *sub specie aeternitatis*, but only conceived, visualised, *sub specie temporis*. The truth forced on us by the dialectic movement, that each One is affirmatively present only as the Limit rendered null and void, and hence that each has its Presence only in Emptiness and its own Emptiness: this insight is in that visionary standpoint conspicuous by its absence. Each God, along with the solar system which he proceeds to create in Time and Space, is there presented purely affirmatively, and so we are asked to take for granted that there is an incalculable number of solar systems or Worlds, in the image of, or also different from, our own World. Whether such a conception is indeed justifiable, cannot, of course, be decided conclusively at the present stage of our progress in knowing, but that it does not illustrate our present notion of the many Ones is sufficiently plain.

Our solution of the Riddle of the Universe is truly concerned with the derivation *à priori* of the categories regulative and constitutive of what is given in experience. We do not seek a point in time, at which the World came to be. Such a quest must needs end in utter nebulosity of conception; in mere hypotheses transcending the region of observable facts without reaching the element of pure thought. From the standpoint of such a quest, the origin of the World has its explanation only and solely on the ground of "visions" presenting themselves to a seer. But whilst fully admitting the genuineness of the visionary faculty, as a necessary mode of our mind, the fact that this alternative to our own standpoint constitutes the transitory fringe of pure Intelligence, implies that even though already concerned with the world in its Absoluteness, the grandest

visions do not really fully transcend the standpoint of Finitude. The true meaning of the Origin of all that is, is still hidden from them in the inmost mode of Intelligence ; in the element of pure thought. Consequently, the highest flight of Visionarism stops at the vicious circle of the Finite and the Infinite. The Infinite is not yet grasped as what is eternally changeless, but only as what temporally progresses *ad infinitum*. Periods of manifestation alternate with periods of self-absorption, and what was in a previous manifestation at the mineral, vegetable or animal stage, advances in the next manifestation to the vegetable, animal or human stage respectively. Hosts of creative Hierarchies themselves the results of past development, build up the new Cosmos, until they themselves reach Cosmic Planes above our own, concerning which the deepest seers can say no more than that they are the fields of great Hierarchies of Beings of indescribable splendour. And neither can they explain how the original host of creative Hierarchies came to be there to build up the first Cosmos. The problem of Creation remains in this way a problem. The contradiction of Becoming is not solved, but only perpetuated *ad infinitum*. The original Being remains only an "It," or a "That," concerning which Nothing can be said. As a matter of fact, then, this "It," "That," "Aum," "Parabrahm," or however else it is called, is nothing than the Nothing or pure Being of our beginning. Pure thinking begins where Occultism breaks down. The postulated Cosmic Planes beyond the reach of the deepest Seers are truly the realm of Pure Thought, whilst the Hierarchies of Beings of indescribable splendour are the categories regulative and constitutive of what is given in experience.

For the time being we have, then, before us only the bare possibility of Self-limitation. Instead of referring to many Worlds existing separately in boundless Void or Space, the plural One stands at best for the possibilities of many Egos taking up into themselves the One World and in this way alone making it indefinitely self-differentiated. Each Ego represents the One Being in that very manner, in which we are meant to grasp the latter's finitization in a present One. We are many whilst holding that we are one. Every religion warns us against the heresy of Separateness. He who acts without the least consideration for his fellow-men,

is *de facto* denied the right to live at all; in illustration, surely, of the verity that a present One loses its *raison d'être* directly it is credited with a Being *sui generis* in distinction from them. The Many are singly and collectively only in so far as their qualitativity is singly and collectively negated. In that their Presence re-awakens the standpoint of Finitude, it shares at once the fate of the Finite as such: its Being is in its Not, in the original Void. Just as the Finite is only because the Infinite is, so the many Ones are only because the One Being remains unalterably *one*. Along with its immediate sense as *Self-differentiation*, the One Being simultaneously implies the sense of *Self-integration*.

CHAPTER II.

SELF-DIFFERENTIATION, SELF-INTEGRATION: QUANTITY AS SUCH.

SELF-DIFFERENTIATION presents itself as the original or immediate feature of the *one* One only in so far as we perpetuate the standpoint of the loose thinking, the standpoint that the Finite comes first and the Infinite afterwards, in connection with the One Being. On further thought, however, it becomes plain that the One Being must be credited from the very first also with the sense of Self-integration. For this reason the question as to the priority or posteriority between the plural or one One is really pointless. Such a question amounts to a relapse into the already superseded vicious circle of the Finite and the Infinite. Therefore, we have already ruled it out of question; and we may now equally comment upon the irrelevancy of the further possible question: Into how many ones does the One differentiate itself? The answer to such a question is so obvious that it should not be necessary to point it out at all. The Many of the plural One cannot, surely, be conceived as limited to a particular number, because the plural One is the eternally self-differentiating One. Conceived *sub specie temporis*, there is no end to the Many; the task of counting them would require an infinite time.

The distinction of Self-differentiation and Self-integration is a more definite restatement of the previous distinction of Self-avoidance and Self-fulfilment. That means to say that the distinction amounts to an untenable contradiction. If we take Self-integration by itself, we presuppose the plural One as given; but as we have just realized, the self-integrating One has the same claim to originality as the self-differentiating One. The distinction between them cannot be maintained, because either presupposes in the other its own self, in consequence of the fact that the stand-

point of qualitative Otherwiseness cannot survive in the sphere of absolute Idealism. The sole purpose of our present discussion may be said to supply a clearer elucidation of the meaning of Ideality, as a Being-for-One. And since Ideality is to be grasped as expressly negative of the sphere of Quality, and we have just been forced to introduce the notion of Self-differentiation or Self-integration, the proper name for the present offspring of the dialectic process, for the idealistic Immediacy of the Infinite, of the one Being as the unity of the self-differentiating and self-integrating One, is *Quantity* pure and simple.

But while thus availing ourselves of a term current in the sphere of mathematics, we ought at once to intercept the arising impression, as though the object of our further thinking is exclusively of mathematical significance. That the fundamental mathematical categories which are bound to crop up in further specification of Quantity as such, equally assume the significance of symbols typifying the stages of the dialectical development of the one true Being in its immediate Ideality with respect to *any* of its existential aspects, follows at once from the fact that the one true Being, which we are now symbolizing by the term Quantity, is immediately self-differentiated Oneness.

It is true, by the way, that reference to particular existential aspects of the one true Being must rest, so far, only on our familiarity with facts; but as such references are useful in helping to remove the drawback of a purely abstract exposition of our proper subject-matter, we proceed, after already having applied our previous progress in knowing to our own human Being, to draw attention to the exact correspondence between the dialectic past of our present stage and the historical past of the Christian era.

Just because this era historically ushers on the scene the standpoint of Absolute Idealism, the grasp of God in Spirit and in Truth; the dialectic by means of which we have reached this standpoint must prove itself to delineate the same type of development as is historically observable in the present civilization up to the establishment of Christianity, when this establishment is viewed in correspondence to the stage reached by our dialectic process in the notion of Quantity as such. Indeed, just as the dialectic culminating in the notion of the true Being presented itself to us in

the light of an introduction, meant to free us from the lingering sway of the ordinary attitude towards objectivity, so the history of the Oriental World culminating in the Greek World,[1] may well be envisaged in the light of a preliminary disentanglement of the human consciousness of Freedom from its primary involution with Nature or Necessity. The correlation of the Chinese civilization with the sphere of the ordinary Reality; that of the Indian civilization with the sphere of Finitude and its endless progress; and, finally, that of the Persian, Phoenician, Semitic and Egyptian civilizations with the direct unity of the Finite and the Infinite, leaps to the eye. The Greek civilization with its rather intellectual and artistic development is, therefore, ushering on the scene the true Infinite, but still only by way of a contrast between Reality and Ideality, seeing that the true Being is, so far, meant only to hint at its full import, becoming the object of further thinking only as the One. Accordingly, too, the Greek World is succeeded by the Roman Empire.

That the phase which the World-spirit is here passing through exactly corresponds to the dialectic of the One and Many, is brought home to us by the divided state of Roman community; its breaking up into perfectly equal and abstractly free individuals who yet remain without any political rights. On the one side there is here the abstraction of Universality, and on the other the sphere of private individuals, whose individual idiosyncrasy is, however, ruthlessly immolated on the altar of that abstraction. That is to say, the many are still devoid of Qualitativity in opposition to the central One. The Emperor cannot yet be said to *rule*, because no other will is imagined that is not subject to his will. Instead of the relation between the Sovereign and Subject, there is here the relation of absolute despot to absolute slave; so that, quite conformably with the supersession of Quality in the One, the concrete element in the character of both the Emperor and private individual is of no moment whatever; with the consequence that all that involves fixed condition, hence also thought, becomes conspicuous by its absence, the springs of action being reduced to mere lust, passion, caprice absolutely unfettered. Precisely, subjective

[1] Our references are based on Hegel's "Philosophy of History".

individuality is, so far, meant simply to avoid itself. Accordingly, too, Philosophy of this time counsels striving after perfect indifference to life and anything which the real world has to offer—a counsel of despair to a world devoid of all stability.

Yet this universal misery, what is it but an awakening to the need of a reconciliation of the contradiction lying at the base of the Roman World? But just because this contradiction—the aspect of the immediately self-differentiated One as against its Self-integration—forms here the inmost Being, is an integral part of the Roman Being, the postulated Self-integration, or *Salvation*, can at first be sought only subjectively in opposition to the existing state of things. There arises the at first universally execrated sect of Christians; of those who widen the notion of absolutely abstract Oneness to its full meaning, as reached in Quantity as such, and thereby posit the unity of Man with God, putting in the place of the Emperor's caprice the Will of God as revealed in their own conscience.

Just as our dialectic of the One presents itself to us only in the light of a clearer elucidation of the meaning of Ideality, so the soulless discipline, undergone by the World-spirit under the Roman sway, may be envisaged as having for its purpose a clear realization of individual nullity on its natural side. Because the natural is the unspiritual, the only way to usher on the scene the standpoint of Absolute Idealism is to give up one's natural Being. Hence, in so far as the Roman World compels people to realize their own personal Nothingness, it submerges them in the despair of Finitude only in order to give birth to the consciousness of the nature of God as pure Spirit, and herewith also to the consciousness of individual Freedom.

And that the trend of the subsequent development of the Christian world too could be further paralleled with the ensuing dialectic development of the notion of Quantity, comes to our notice at once in the fact that, in contrast to the Greeks and Romans, who directed their energies outwards only *after* completing their inner development, the successors to the Roman Empire began their career by at once deluging the world and only afterwards settling down to develop inwardly through a gradual assimilation of the conquered element in their midst. For, inasmuch as our

present transition from Quality into Quantity postulates the fundamental unity of both, and hence leads at once to the anticipation that the trend of the ensuing dialectic of Quantity must consist in a gradual self-recovery of the immediately simply negated Quality, the correspondence of this trend of development with the stated reversal of the original oriental mode of development in the western World is obvious.

Nevertheless, since the ensuing development of the notion of Quantity is typical of the development of any particular representative of the one true Being primarily only with respect to the observable quantitative distinctions, and consistently too with a systematic unfoldment of our intended solution of the Riddle of the Universe, we had better restrict the objective significance of our present subject-matter, to begin with, only to those aspects of objectivity in which the one true Being may be said to be embodied still purely idealistically in immediate negation of what ordinarily passes for Reality. Therefore, the simplest examples of the objective significance of the notion of Quantity are to be sought, so far, in the idea of pure Space; or more definitely still, in that of Time, seeing that every trace of qualitative meaning, which in a sense still continues to cling to the idea of Space, as the first or immediate determination of Nature, should now become wholly conspicuous by absence.

CHAPTER III.

CONTINUOUS AND DISCRETE MAGNITUDE: COUNTING AS SUCH.

In saying that Quantity negates Quality, we do not deny that it is a Presence. For, inasmuch as Presence as such neither comes to be nor ceases to be, and consequently is to be viewed in the light of the alterable In-itself of the one true Being, Quantity is only a further specification thereof in supersession of its immediate connotation as Quality. From this their correlatedness it at once follows that their distinguishedness is at the bottom only another instance of the vicious circle between the Finite and Infinite. Just because the true Being of Quality is the Infinite as Quantity, this notion may be said to stand for the infinitized Quality, when Quality in turn assumes the significance of the finitized Quantity—or vice versa. But as this obviousness of the fundamental unity of both is, so far, based only on a formal application of the results of our dialectic past, we must, in reflection upon the dialectic future of the notion of Quantity, treat its fundamental unity with Quality only in the sense of an anticipation calling for its due realization.

Although, therefore, we have a good reason to anticipate the return from Quantity into Quality, in order to vindicate this return from within the standpoint of Quantity itself, we must initiate its dialectical development by emphasizing its initial lack of Qualitativity. Accordingly, it is only as an object of our further thinking that Quantity as such comes to be properly identified with the fundamental category of the science of Mathematics. Its initial lack of Qualitativity finds its appropriate record in the term *Magnitude*. This is a Being admitting of limitless increase or decrease, and hence stripped of all qualitative significance whatever, there being, apart from the notion of Limit, no qualitative distinction thinkable.

Inasmuch, now, as Magnitude as such stands for the notion of Quantity in reference to its simplest Immediacy, the distinction which we must create for the purpose of the present act of thought, obviously concerns a suitable restatement of the distinction belonging to the own Inwardness of the notion of Quantity, as the unity of the self-differentiating and self-integrating One. Because the accent falls now primarily on the aspect of Self-integration, and Magnitude, therefore, is to be immediately emphasized with respect to its inner undividedness or compactness, its increasing or decreasing remains a matter of pure *Continuity* with itself. And in so far as Continuity restates the aspect of Self-integration, the term restating the opposite aspect of Self-differentiation is *Discreteness.*

Of course, inasmuch as we are thus really contrasting the same fundamental unity of both the aspects of the One of Quantity, the arisen distinction between the Continuous and the Discrete Magnitude is a purely vanishing one. Owing to the utter lack of Qualitativity in connection with the Alterableness characteristic of Magnitude as such, it is impossible to conceive either of its stated specifications apart from the other without entangling oneself at once in a hopeless vicious circle. As is well-known, this vicious circle is one of the four cosmological antinomies adduced by Kant in his "Critic of Pure Reason" in vindication of his assertion that our Reason is incapable of finding the truth—an assertion not really at variance with our own standpoint, because what Kant calls Reason refers to the analytical intellect, and hence to a standpoint coming with us under the head of loose thinking, to which, we ourselves assert, the truth must needs present itself only by way of an unreachable beyond.

"These Kantian antinomies," remarks Hegel in his "Science of Logic," of which, as has been pointed out in our Preface, the present work was to have been, but for our own predilection for original self-expression, only a paraphrase, "always will be an important part of the critical philosophy. It is they in especial which brought about the downfall of the preceding metaphysic and thus may be viewed as an essential transition to the more recent philosophy. It is they which more particularly helped in establishing the conviction of the nullity of the categories of finitude, by demonstrating the nullity of their *content,*

which is a more correct way of disposing of them than the formal one of a subjective idealism, which traces their defect only to our subjective attitude to them, not to what they are in their own self. But with all its great merits, Kant's exposition is very imperfect: partly impeded and twisted in its progress, partly false as regards its result, because he presupposes that cognition has no other forms of thought but finite categories. In both respects these antinomies call for a more exact critic, with a view to throwing more light on their standpoint and method, as well as freeing the main point in question from the unnecessary form superimposed upon it . . . On closer consideration, Kantian antinomies are found to rest on the quite simple categorical assertion of each of the two contrasted moments of a determination on its own isolatedly from the other. But this simple categoric, or properly assertoric, statement is at the same time hedged in by false and rambling scaffolding of reasoning, meant to produce a semblance of proofs and to conceal, or to secure from detection, the merely assertoric nature of the thesis."

Lest this comment on Kant's reasoning power appear unjust, Hegel proceeds to discuss the very antinomy which arises when the moments of Continuity and Discreteness are hypostatized apart from one another. The notion of Quantity being at the same time applied by Kant to Matter the antinomy comes to concern the so-called infinite divisibility of Matter:—

"The *Thesis* of the same is, according to Kantian exposition, as follows:

"*Every composite substance in the world consists of simple parts and ultimately there exists nothing but the Simple, or that which is composed from it.*

"There is here opposed to the Simple, to the Atom, the *Composite*, which is a very backward determination in comparison with the Constant or Continuous. The substrate given to these abstractions, namely, substances in the world, means here nothing further than things as objects of sensuous perception and has no bearing on the nature of the Antinomy itself; one could just as well identify it with Space or Time. Now, in putting Composition in the place of Continuity, the thesis amounts properly only to an analytical or tautological proposition. That the Composite

as such is not pure Oneness, but only something externally joined and consisting of something else, is its immediate determination or meaning. The something else is, of course, by contrast, the Simple. For, when it is asked of what something or other consists, one asks for the indication of something else. If ink is taken to consist of ink again, the sense of the question as to the something else of which it consists is missed; the question is not answered but only repeated. In fact, there arises then the further question whether the thing under discussion is supposed to consist of anything at all. The Composite, however, is necessarily something meant to be combined and to consist of something else. If this something else is taken to be only *relatively* simple, the question remains after as before. To the ordinary conception there suggests itself perhaps only this or that Composite, as regards which this or that something or other might also pass for its Simple, while being itself something composite. But here it is the Composite as such that is in the question.

"Now, as regards the Kantian Proof of the Thesis, it makes, like all the Kantian proofs of the rest of antinomial propositions, the *détour* of being apagogical; which *détour* will show itself as most superfluous.

"'Assume [the proof begins] that composite substances do not consist of simple parts; now, were all Composition thought away, there would be left over no composite part, and neither, as (according to the just made assumption) there are no simple parts, a simple one; therefore, nothing at all; consequently no substance would have been given.'

"This inference is quite correct. If there is nothing but what is composite and we think away all composition, there is nothing left over. This is quite granted, but such a tautological inanity might have been omitted and the proof made to begin at once with what follows, namely:

"'Either it is impossible to think away all Composition, or, when it has been thought away, there must be left over something implying no Composition, i.e. the Simple.

"'In the former case, however, the Composite would not again consist of substances (*because with these, Composition is only an accidental relation of substances,*[1] *without which*

[1] In addition to redundancy of proof, there is here also redundancy of language: because *with these* (namely, the substance) Composition is only an accidental relation of *substances.*

they must persist as on their own permanent beings). Now as this case clashes with the presupposition, there is only the alternative to fall back upon, namely, that the substantial Composite in the world consists of simple parts.'

"As is seen, the very reason which constitutes the nerve of the argument and which renders all that has been said so far quite superfluous, is adduced in brackets by the way. The dilemma is this: Either the Composite persists or else the Simple. If it were the former, the permanent would not be substances, for *to these, Composition is only an accidental relation;* but substances are permanent, consequently that which persists is the Simple.

"It is clear that the adduced reason, that Composition is merely an *accidental* relation of substances, and hence does not concern them, could be annexed as proof immediately without any apagogical détour. If it is correct that Composition is something accidental, there is, of course, at the bottom the Simple. This Accidentality, however, that alone matters, is not proved, but just taken for granted—and in passing, too, in parentheses—as something self-understood, or of secondary importance. True, it goes without saying that Composition has the sense of Accidentality and Externality; but were the point at issue only an accidental putting together instead of Continuity, it would not be worth while to convert it into an antinomy; or rather, none could be formulated. Therefore the assertion as to the simplicity of parts is only a tautology.

"Since the apagogical détour contains the very assertion which is meant to result from it, the Proof amounts concisely to this:

"Let us assume that substances do not consist of simple parts, but are only composite. But now all Composition can be thought away (for it is only an accidental relation). In that case, however, we do away with substances, unless we grant that they consist of simple parts. Still, substances we have must, for we have assumed them; everything is not meant to disappear, something must remain. Having presupposed that there persists that which we have called substance, we must grant that substance is simple.

"To go the full length, let us consider also the concluding inference. It is to this effect:

"'From this it immediately *follows* that the things of the

world are ultimately simple essences; that *Composition is only an external state of the same*, and that Reason must think elementary substances as simple essences.'

"Here we see the externality or accidentality of Composition adduced as *consequence* after its previous parenthetical introduction and employment in the proof.

"Kant protests very much against any supposition that in the contradictory propositions of his antinomies, he is availing himself of sophisms for the purpose of special pleading. The defect of the Proof under discussion is not so much that it displays mere sophistry, but that it is unnecessarily and painfully twisted, for the only purpose of producing an external semblance of a proof and not to let it be patent in its full transparency that what ought to result as conclusion is parenthetically the nerve of the proof; that there is no proof at all, but only an assumption.

"The Antithesis is to this effect:

"'*No composite thing in the world consists of simple parts, and there is altogether nothing simple in it.*'

"The Proof has equally an apagogical turn and in another way is just as objectionable as the former one.

"'Suppose,' it begins, 'that a composite thing, as substance, consists of simple parts. As all *external relation*, consequently also all Composition of substances, is possible only in Space, the space occupied by the Composite must consist of as many parts as the Composite consists of. But Space does not consist of simple parts, but of Spaces. Therefore every part of the Composite must occupy a space.

"'But the absolutely primary parts of all that is composite are simple.

"'Therefore, the Simple occupies a space.

"'Now, seeing that all that is real and occupies a space contains a manifold whose parts are external to one another and consequently is composite and in that case consists of substances, it follows that the Simple must be a substantial composite. Which is self-contradictory.'

"This Proof can be called a whole *nest* (to use a frequently employed Kantian expression) of faulty procedure.

"To begin with, the apagogical turn is a groundless illusion. For, the assumption that the *Substantial is spatial*, but that *Space does not consist of simple parts*, is a direct assertion which is immediately laid at the bottom of

what is to be proved and constitutes the whole of the Proof.

"This apagogical proof subsequently begins with the propositions 'that all Composition from substances is an *external* relation,' but peculiarly enough immediately forgets it again. Namely, it is further inferred that Composition is possible only in Space, but that Space does not consist of simple parts, and hence that the Real occupying a space is composite. But once one assumes that Composition is an external relation, Spatiality itself, in which alone Composition is supposedly possible, becomes on that account an external relation to substances which does not concern them, nor touches their nature any more than anything else that might be additionally inferred from the determination of spatiality. For that very reason, substances ought not to have been put into Space.

"It is further assumed that Space, into which substances are here transferred, does not consist of simple parts, because, from Kantian standpoint, Space is an Intuition; a conception which can be given only through a single object and is not a so-called discursive notion. As is well-known, this Kantian distinction between Intuition and the Notion has given rise to much fuss about the former; and in order to save the effort at proper comprehension, the value and the sphere of the distinction has been extended over all Cognition. Nevertheless, the nature of Space, as well as of Intuition, must also be grasped; if, that is, we really aim at pure Knowledge. Thus there arises the question as to whether Space, even though, as Intuition, it be simple Continuity, is not to be grasped as consisting, in its Notion, of simple parts, in which case Space would admit of the same antinomy which is formulated by Kant only about substances. As a matter of fact, if the antinomy is grasped abstractly, it concerns Quantity as such, and hence just as much Space and Time.

"As, however, Kant's proof alleges that Space does not consist of simple parts, this ought to have been a reason against transferring the Simple into such an inadequate element for its representation. In this respect there arises also a clash between the Continuity of Space and Composition, because the two are confused with one another, the former being substituted for the latter (a fact giving in the

conclusion a *Quaternio terminorum*). Space has with Kant the express determination of being *a single one;* its parts are said to be due only to limitations which do not precede the single, all-embracing Space in the sense, as it were, of constituents. Here, Continuity is quite correctly and definitely predicated of Space, in denial of its Composition from component parts. Nevertheless, in the argument, the transferring of substances into Space is made to carry with it 'a manifold whose parts are external to one another,' and more particularly 'consequently a composite'. Yet, as adduced, the form which manifoldness has in Space, should expressly exclude all Composition from presupposed component parts.

"In a remark to the Proof of the Antithesis, the otherwise fundamental conception of Critical Philosophy is additionally emphasized, i.e. that we have a Notion of objects only as Appearances, but that they, as such Appearances, necessarily presuppose Space, as Condition of the possibility of all external appearance. Now, if substances are meant to stand only for objects as they are seen, felt, tasted, and so on, there is properly no question of that which they are in their Notion. The subject-matter is only what is sensuously perceived. The proof of the Antithesis ought, therefore, to have been briefly: The whole experience of our seeing, feeling, and so on, shows us only what is composite. Even the best microscopes and the most delicate knives have not yet enabled us to *stumble* against anything simple. Hence, neither Reason must expect to discover something simple.

"Therefore, if we submit the contrast between this thesis and antithesis to a more exact analysis and free their proofs from all unnecessary redundancy and distortion, the proof of the antithesis is seen to contain, in connection with the transference of substances into Space, the assertoric assumption of Continuity, whilst in the assumption of Composition, as an external relation of substances, the proof of the thesis contains, in turn, the assertoric assumption of the accidentality of this relation and, therefore, the assumption of substances as *absolute Ones.* The whole antinomy is thus reduced to a separation and direct assertion of the two moments of Quantity; but of them as absolute separate. From the standpoint of mere Discreteness, Substance, Matter, Space, Time, etc., are absolutely divided; their

principle is a One. From the standpoint of Continuity, this One is only a Being-for-One, so that the dividing remains divisibility. There remains possibility of dividing, as possibility, without ever reaching the atom. Now, even if we stop at the determination given of these antitheses in what has been said, there lies in the Continuity itself the moment of the atom, because Continuity necessarily implies possibility of division. By contrast, the said Dividedness, or Discreteness, does away with every distinction between the Ones, just because the simple Ones are the one what the other is. Consequently, Discreteness just as much contains their equality and therefore their Continuity. In that each of the two opposite sides contains the other as one with itself, it follows that neither of these determinations, taken by itself, has truth, but only their unity. This is the truly dialectical consideration of them, as well as the true result."

As is seen, in conclusion of the present act of thought we are meant to put on record the notion of Magnitude as such in supersession of its immediate distinguishableness in terms of the two aspects of the One of Quantity. If we draw a parallel between Magnitude as such and Quality in its primary sense as Determinateness pure and simple, as Quality is implied at the outset of our discussion of the nature of Presence, it will be seen that, with respect to the dialectical whole of Quantity, the distinction between the Continuous and the Discrete Magnitude corresponds in the dialectical whole of Quality to the distinction between Reality and Negation. Inasmuch, then, as Reality and Negation had to be grasped finally only in the sense of mere aspects of the notion of Becoming, after its fresh reappearance in the own Inwardness of Presence as such, when, however, Becoming became Alterableness, and the notion of Becoming acquires at the back of Quantity as such the significance of Self-differentiation, it follows that Magnitude is to be finally viewed from the standpoint of the meaning acquired by Self-differentiation after its analogous reappearance within the own Inwardness of the one of Quantity, and hence from the standpoint of *Counting as such.*

CHAPTER IV.

UNITY, AMOUNT: ARITHMETICAL MAGNITUDE.

In so far as Something or Other is now before us in the sense of something *countable*, its qualitative nature is of no consequence. That Counting does not yet reflect on qualitative distinction between various things, follows from the very import of its Inwardness, as the *self-differentiating unity* of the Continuous and Discrete Magnitude. Since these two aspects of Magnitude do not admit of qualitative opposition, their unity implies the distinction between them in the sense of Self-differentiation pure and simple. Accordingly, in making of this unity the object of our further thinking, we may be said to refer to a restatement of our immediate grasp of the One Being. Of course, in reflection upon the advance which we have since made in our progress in knowing, the Many Ones of Self-differentiation become now *numbered* Ones. The plural One is now before us in the explicit sense of a numerical One having its Presence in Counting alone.

Owing to its absence of Qualitativity, the Countable concerns us simply in the sense of *a* Magnitude or *Number*. Since we thus reflect upon the unity of the continuous and discrete Magnitude, with an emphasis on the aspect of Discreteness, we must treat this unity, or Counting, only as an implicit representative of the equally purely continuous flux of Quantity, whose simplest objective significance refers to the traceless passing away of Time. In so far as the unity in question cannot be broken asunder, Counting must imply an endless continuity of Numbers. But in so far as the said unity is immediately to be emphasized with respect to its aspect of Discreteness, the aspect of Continuity must so far lack an adequate embodiment.

Unable as we are to postulate qualitative distinction between Numbers, whilst yet being already under the necessity

of treating them in the sense of further specified present Ones of Self-differentiation, we can base the distinction between them only on the nature of this further specification. Even though, as objects of Counting, i.e. as Numbers, the present Ones of Self-differentiation continue to lack qualitative *raison d'être*, they are yet no longer entirely devoid of distinguishedness against one another. The reason why we could not at once credit the present Ones with Discreteness, lies in this, that they are ushered on the scene *before* the self-differentiating One is explicitly recognized as the simultaneously self-integrating One. But since the unity of Self-differentiation and Self-integration belongs now to the Inwardness of any and every Number, even though for the purpose of dialectical deepening of this Inwardness the accent falls now again only on the aspect of Self-differentiation, the reason for the original insistence on utter indistinctiveness of the present Ones no longer obtains. In so far, however, as the already necessary distinction between Numbers must all the same remain devoid of qualitative character, it cannot be treated as arising from within their own self, but only in the sense of a record of an external comparison between them. This external comparing of them is precisely the Counting itself.

It becomes thus obvious that Counting as such is indeed the present correspondence to the stand-point of Alterableness. In so far as Alterableness does not yet affect the Quality of a given Something or Other, but concerns only its external shape, the change presents itself in the sense of something countable. The distinction remains purely quantitative and therefore is put on record in terms of Number. The fact, by the way, that no science is said to be established on a firm basis, unless its generalizations can be expressed in terms of Number, shows why Universe remains scientifically an insoluble Riddle. It is only because Number is in itself wholly meaningless that it admits of being applied to anything and everything, without yet throwing any light on its essential meaning. Applied Counting does not alter the nature of Counting as such, and we are just realizing that Counting is not a matter of pure thinking, which alone can solve the Riddle of the Universe.

In so far as a Number is a discrete Magnitude, it may

equally serve as an exemplification of the notion of Limit. In that case, of course, the exemplification must take a form conformable with the quantitative character of this limit and hence remain purely abstract. The qualitative distinction between Something and an Other, at once joined and disjoined by the Limit, can now only refer to two distinct Numbers, one on this side, the other on the other side, of the given Number. Since the notion of Limit may be identified with any Number, every Number admits of being either increased or decreased. Accordingly, Counting may be interpreted in two senses, with a view to either the increasing or decreasing of the Number already reached, the flux of Counting admitting in neither sense of a final stop. In this way we are implying that every Number is a particular *Amount* or *Sum* of the endlessly added or subtracted *Unit of Enumeration.* That the Unit and Amount or Sum represent the present restatement of the two aspects of Magnitude as such, is obvious ; but because every Number stands for the self-differentiated unity of the continuous and discrete Magnitude, along with a simple Unit of Enumeration, the flux of Counting necessitates also a unit in the express sense of Unit and Amount, when, of course, we refer to the *Radix* of a System of Notation.

CHAPTER V.

RATIONAL AND IRRATIONAL NUMBER: GEOMETRICAL MAGNITUDE.

WHEN we reflect upon the import of the modification, to which the original meaning of Number becomes liable in immediate supersession of the purely external relation between its moments, when it comes to be emphasized as Continuous rather than Discrete Magnitude, we find that Number ceases to concern us as a positive integer, or rather in that sense which it has in the System of Rational numbers, seeing that this system includes also fractional and negative numbers without superseding the original meaning of Counting as such. Although *fractional* numbers bring already to notice the *unity* of Unity and Amount, this unity retains the significance of a *rational* number. In order to acquire the express sense of a continuous rather than of a discrete Magnitude, the immediate meaninglessness of Numbers must be superseded, and inasmuch as this meaninglessness is due to a restatement of the self-differentiating or self-avoiding One, its immediate supersession invests the Number with the sense of the self-integrating One.

Our familiarity with the Science of Mathematics makes it at once plain to us that a Number becomes thus transformed into a *Sum of an Infinite Series*, whose purely analytical definition is as follows:

"Let $$u_1, u_2, u_3 \ldots$$

be any set of numbers, positive or negative, formed according to same given law by means of which the n^{th} term is known and is a finite single-valued function of n; and let

$$S_1 = u_1,$$
$$S_2 = u_1 + u_2,$$
$$S_3 = u_1 + u_2 + u_3, \text{ etc.}$$

"When the sequence $S_1, S_2, S_3, \ldots$ is convergent, the series

$$u_1 + u_2 + u_3 + \ldots$$

is said to be convergent, and the limit of the sequence

$$S_1, S_2, S_3, \ldots$$

as n is increased indefinitely, is called the Sum of the Infinite series

$$u_1 + u_2 + u_3 + \ldots .$$"[1]

If the law, according to which the implied set of numbers is formed, is simply identified with Enumeration, we find that, whilst, on the one side, still sharing the nature of an ordinary arithmetical magnitude, the Sum of the Infinite Series is, on the other side, to be interpreted in the express sense of a beyond of infinite approximation. For continues our text-book :—

"It should carefully be noticed that what we have called the *Sum of the Infinite Series is not the sum of any number of terms*, however great; and that it is the *limit of the sum of n terms*, as n is increased indefinitely."

The stated qualification of a particular Number follows directly from the dialectical necessity of bringing to the front the primary implicitness of Continuity as such in connection with the flux of Counting. Inasmuch as this latter concerns the unity of the continuous and discrete magnitude, in concrete restatement of the standpoint of Self-differentiation, or with an emphasis on the aspect of Discreteness; the system of arithmetical magnitude remains totally inadequate for the representation of the same unity, when emphasis is laid, by contrast, on the complementary aspect of Continuity in more concrete restatement of the standpoint of Self-integration.

Just because the system of rational numbers is built up by an endlessly repeated annumeration of a countable One, which itself equally shares the character of an endlessly alterable number, the bond linking numbers together remains external to them, their continuity into one another being constituted only by the mind of him who counts. In explicit embodiment of their continuity, the unit of enumeration must cease to be the thought of an empty One, or its

[1] "Fourier's Series and Integrals," Carslaw, 1906, § 17.

Alterableness must not remain a mere hint at Alteration, but explicitly become an Other to its primary meaning. Accordingly, its numerical significance must be negated; or if we persist in embodying its real meaning in terms of number (in reflection upon the fact that a superseded standpoint invariably turns up within the Inwardness of the next higher standpoint), we must at the same time explicitly efface the primary character of Number.

Quite appropriately, too, in introducing the new kind of number into the domain of Arithmetic, Pure Analysis calls it an *Irrational Number*, because it negates that *notion* of Number which alone constitutes its *raison d'être* or rationality. Nevertheless, since the Irrational Number is truly *no* number at all, and its substitution for the unit of enumeration, consequently, altogether disposes of the very standpoint of Counting, the opposite moment of Amount becomes a Sum of an Infinite Series, its primary arithmetical significance being equally negated, whereby it becomes converted into an Irrational Number.

But we have only to remind ourselves that the standpoint of Counting represents Quantity, as immediately utterly stripped of its from the very first anticipated qualitative character, in order to realize that the Irrational Number represents only an arithmetical mirage of the necessarily resuscitated Quality under the head of Quantity. True, in so far as the element of Quality is just only reappearing on the scene, and we are consequently still far from having properly established its fundamental unity with Quantity, the object of our further discussion necessarily still also retains the character of Magnitude; but because the accent falls now on the aspect of Continuity as such, the Magnitude becomes now *geometrical*. Or in reflection upon the fact that our present result stands also for a more concrete restatement of Quantity as such, we may say that it is only now that we shall properly concern ourselves with *a* Quantity, this term being indeed used synonymously with geometrical magnitude.

CHAPTER VI.

EXTENSIVE AND INTENSIVE MAGNITUDE: ARITHMETICAL CONTINUUM.

As a finite representative of the One of Quantity, any and every Quantity is at once continuous and discrete. But since Continuity concerns now pure self-reference only within given limits, we propose to call it *Extension*, in which case Discreteness becomes, by contrast, *Intension*. Accordingly, in present restatement of the continuous and discrete Magnitude, we get the Extensive and Intensive Magnitude; and it is at once obvious that the greater concreteness of the stage we have now reached in the dialectical whole of Quantity, is meant to come to the front more particularly in the significance acquired by Counting, when it is no longer the original simple unity of the continuous and discrete Magnitude that is emphasized with respect to the aspect of Discreteness, but this unity, as what is just as much in its turn as well continuous and discrete; with the consequence that we are then emphasizing the aspect of Discreteness in the continuous and discrete unity of the continuous and discrete Magnitude, i.e. in the unity of the extensive and intensive Magnitude.

The stated circumstantial definition of this unity serves to force upon our attention the otherwise equally obvious fact that in reference to the present restatement of the self-differentiating unity of the continuous and discrete Magnitude, in a word of Counting, the unity of the extensive and intensive Magnitude just as much implies the already explicated distinction between the arithmetical and geometrical Magnitude. Seing that this distinction concerns us now in explicit correspondence to the contradictory relationship between the In-itself and Constitution of Something and an Other, the standpoint of Alterableness is now no longer to be associated with the arithmetical magnitude and the

standpoint of Alteration with the geometrical magnitude alone. The use of these terms becomes now interchangeable, because the distinction between the implied meanings is now just as much to be affirmed as negated. Obviously, we are implying that the unity of the extensive and intensive magnitude does not admit of either purely arithmetical or purely geometrical exemplification, but that such an attempt in either direction just as much depends on what it seeks to exclude.

For instance, the contradictory character of the distinguishedness between the extensive and intensive magnitude appears at first sight to have a purely arithmetical exemplification in the distinction between the Cardinal and Ordinal Number. But unless we associate with a Cardinal Number a fixed break in the immediately endless flux of Counting, it remains only an ordinary number. We refer then to a particular given amount of countable Ones in the sense of a Two, a Three, etc., and thereby efface the character of a simple Number, as what has its Presence in Counting alone. That is to say, we are then concerned with an application of Counting and in this way have it before us as qualified. The unit of emuneration, consequently, equally loses then its simple primary significance and, as an ordinal number, puts on record that it is constituted by a reference to that particular number, at which Counting as such is to be broken off. We cannot speak of the tenth, etc., unless we refer to a given whole of ten, etc., countable Ones. In so far as every unit of the given whole shares the same sense, the externality of the units composing this number is just as much superseded. In short, the distinction between the cardinal and ordinal number may indeed serve as an exemplification of the qualitative character of the distinction between the extensive and intensive magnitude, but just so the purely arithmetical character of this exemplification is effaced. Qualified as cardinal or ordinal, Number excludes from itself the indifference and externality of its own Amount, and, as what just as much excludes what it includes, or *vice versa*, has the character of a qualitative Quantum which, as we have already seen, has its explicit embodiment in the so-called Irrational Number.

In this way it becomes equally plain that an attempt to

secure a purely arithmetical embodiment of the distinction between the extensive and intensive magnitude in terms of the Irrational Number is equally futile. Even though the conception of the irrational number admits of being ushered on the scene in consequence of an endeavour to supersede the alterable character of the unit of enumeration, owing to which the system of rational numbers remains only an external colligation of units, this conception remains *per se* meaningless, until it is exemplified geometrically. As has been already seen, the irrational number negates the nature of number, and in so far as this negation is not meant to bring us back to nothing pure and simple, but refers to the necessity of superseding the numerical significance of the unit of enumeration, at the back of the modern (Dedekind's) theory of the irrational number there is really the geometrical axiom of Continuity. After having defined the irrational number as a mode of separation of the system of rational numbers into two classes on the understanding that—

"(1) Every rational number belongs to the one class or to the other;

"(2) Every number belonging to the lower class is smaller than every number belonging to the upper;

"(3) The lower class has no maximum, and the upper class no minimum";[1]

the Pure Mathematician "in whose eyes pure analysis is a scheme which deals with numbers only and has *per se* no concern with measurable quantity,"[2] finds himself faced with the question as to whether to any given irrational number, that is to say, to any given mode of separation of the rational numbers satisfying the stated conditions, there will correspond a definite point upon a straight line which will give the same mode of separation.

"It was reserved for the mathematicians of the nineteenth century—notably Weierstrass, Cantor, Dedekind and Heine—to establish the theory on a proper basis. Until their writings appeared, number was looked upon as an expression for the measurement of a line by another which was regarded as the unit of length. To every segment, or with the natural modification, to every point, of a line corre-

[1] "Fourier's Series and Integrals," Carslaw, § 4. [2] Ibid., § 1.

sponded a definite number, which was either rational or irrational; and by the term irrational number was meant a number defined by an infinite set of arithmetical operations (e.g. Infinite Decimals or Continued Fractions). The justification for regarding such an unending sequence of rational numbers as a definite number was considered to be the fact that this system was obtained as the equivalent of a given segment by the aid of the same methods of measurement as those which gave a definite rational number for other segments. However, it does not in any way follow from this that conversely any arbitrarily given arithmetical representation of this kind can be regarded in the above sense as an irrational number; that is to say, that we can consider as evident the existence of a segment which would produce by suitable measurement the given arithmetical representation. Cantor has the credit of first pointing out that the assumption that a definite segment must correspond to every such sequence is neither self-evident nor does it admit of proof but involves an actual axiom of Geometry. Almost at the same time Dedekind showed that the axiom in question (or more exactly one which is equivalent to it) first gave a meaning which we can comprehend, to that property which, so far without any sufficient definition, had been spoken of as the continuity of the line."[1]

Of course, we anticipate at once that the supplied definition of the geometrical axiom of Continuity is nothing more than a geometrical version of the definition of the irrational number. In words of Dedekind:

"If all the points of a line can be separated into two classes such that every point of the one class is to the left of every point of the other class, then there exists one and only one point which brings about this division of all the points into two classes, this section of the line into two parts."[2] That this statement is further said not to admit of proof, "the assumption of this property of the line being nothing else than an axiom by which we assign its continuity to the line," is consistent with the standpoint of mathematical knowledge, because, as demanded by our intuition of the straight line, the stated axiom is provable only *à priori.* Namely, just because the conception of the

[1] Ibid., § 8. [2] Ibid., § 5.

irrational number is prompted by the dialectical necessity of superseding numerical asunderness, and this supersession alters the alterableness of the unit of enumeration, so that this unit is then ushered on the scene in the explicit sense of the limiting One, or is invested with the qualitative character of Limit, it follows that since the given irrational number has no longer its presence in Counting alone, its *real* meaning must take the shape of the simplest spatial embodiment of the notion of Limit and hence the shape of a Point which, however, in reflection upon the contradictory character of Limit, as what in itself just as much negates its own Presence and thereby is in truth only something finite, must be grasped at once in the sense of an element of a straight line, the conception of this latter, as what is produced by the movement of a point, being precisely a record of the dialectic implied in our stated attitude towards the Point as a spatial Limit. The axiom by which Dedekind assigns its continuity to the line is plainly an analytical way of stating that a point is an element of a straight line.

In order to show that the asserted possibility of founding mathematical analysis upon the integral number, refers to this number only as negatively preserved in that concluding unity of the extensive and intensive magnitude which corresponds in the dialectical whole of Quantity to the stage reached in the dialectical whole of Quality in the notion of Limit; and consequently that the simultaneously reached restatement of Counting amounts to an attempt to follow arithmetically all the properties of the straight line, we have only to draw the necessary inference from the fact that the purely analytical definition of the irrational number involves the geometrical axiom of continuity.

"In adopting this axiom we can now say that to each point of the line L corresponds a number, rational or irrational, its abscissa measured from O, and that to each irrational number (i.e. to each mode of separation of the rational numbers into two classes satisfying the conditions enumerated above) corresponds a point of the line of which this number is the abscissa. Thus not only do these irrational numbers which are given to us by Geometry appear as the measures of definite lengths, but the general irrational number, although given to us in what may seem an unreal

way, corresponds also to the measure of a definite quantity. The indistinctness which we may feel in our conception of this number, when it is defined by its upper and lower classes, disappears when we regard it as the measure of the segment of L to which it corresponds. The correspondence between the points of the line L and the arithmetical continuum is now perfect. The points are able to be taken as the images of the numbers, and the numbers as the signs of the points. In consequence of this perfect correspondence we may in future use the terms number and point in this connection as identical.

"The system of numbers, rational and irrational, which forms the arithmetical continuum, is called the system of Real Numbers." [1]

[1] Ibid., § 5.

CHAPTER VII.

THE VICIOUS CIRCLE BETWEEN QUANTITY AND QUALITY.

IN making the arithmetical continuum or the so-called system of real numbers the object of our further discussion, it at once occurs to us to reflect upon it in the sense of a higher restatement of the original system of rational numbers built up by simple ennumeration of the same constant unit. In doing so, we formally follow the necessary trend of our progress in knowing within the dialectical whole of Quantity. Since the explication of the standpoint of Counting in the preceding three chapters assumed the significance of a higher restatement of the subject-matter dealt with in the first three chapters, and the dialectical whole of Quantity is at the same time proving itself as a systematic recapitulation of the import characterising the progressive development within the dialectical whole of Quality, it is now evident to us that we might have treated this dialectical whole under three main heads having the same relation *inter se* as the three component parts coming under each head and finally as the three distinct moments implied in every simple act of thought. So comes it, then, that our present subject-matter presents itself to us in the sense of a higher restatement of the standpoint of Counting, as explicated in the system of rational numbers.

If we now reflect closer on the meaning to be associated with Number at our present stage, we find that we are now manifestly concerned with an explicit record of what, at the stage of Number as such, had to remain only implicit. In the course of our subsequent discussion of the standpoint of Counting, we became aware that an adequate embodiment of Continuity as such necessitated an alteration in this standpoint; and we have now arrived at the stage, at which this alteration is duly established. That a number has its meaning only in reference to its place in the system of

rational numbers, as a particular amount, cropping up in the endless flux of Counting, is from the very first obvious; but whereas the continuity of this counting, not only with respect to its progress towards the infinite, but also with respect to the continuity of one number into another one, remains external to the system of numbers, now we have the number before us as what must by its own nature force itself beyond itself and become endlessly another; conformably with the fact that the present restatement of the standpoint of Counting equally recapitulates the contradiction between Limit and Presence in transition into the standpoint of Finitude.

It is, however, easily seen that, since this transition has now at its back the identification of numbers with points lying on a straight line, its proper exemplification refers to the already mentioned grasp of a point as an element of a straight line produced *ad infinitum.*

As ushered on the scene in correspondence to the immediate significance of Limit, a Point may well have presented itself in the simple sense of a point dividing all the points of a line into two classes such that every point of the one class is to the left or right of every point of the other class, because in that case we dealt with the unity of the extensive and intensive magnitude in higher restatement of the original unity of the continuous and discrete magnitude with respect to the emphasis on the aspect of discreteness. Although, therefore, Magnitude was already before us in the sense of geometrical magnitude or quantity, the attempt to embody its continuity purely analytically introduced the fundamentally false conception as if a line *consists* of points. But, conformably with the further necessity of embodying the unity of the extensive and intensive magnitude also with respect to the aspect of continuity, we must now banish this false conception and grasp the point in its true sense as an element of extension.

The fact, by the way, that we are now all along also already presupposing the idea of Space is, of course, due to the significance we are attaching to this idea as the objective element of Quality. If we have not so far particularly troubled about this correlation, is again due to this, that the task of systematically tracing the objective sig-

nificance of our dialectical progress in knowing can properly begin only at the stage of Quantity. Accordingly, whatever bearing the dialectic of Quality may have on the idea of Space, this idea can be ushered on the scene only in correlation with the initial grasp of the One Being as Self-avoidance. Until the dialectical movement reaches this stage, Space could not be referred to as the simplest Self-representation of the true Being. It is only conformable with the absolute Idealism of pure Thought that the objective import of our progress in knowing properly begins at a stage, when Immediacy is grasped as pure Ideality. But just because Qualitativity is in that case immediately conspicuous only by its absence, and the dialectic of the One serves only to emphasize the untenability of purely qualitative distinction, the beginning is then really made with the one One or Quantity as such, whose simplest objective significance refers to a tracelessly transient Immediacy, i.e. to the idea of Time, with the consequence that the idea of Space crops up properly only in connection with the reappearing Quality, and hence at once in reflection upon its fundamental unity with Time, which latter equally comes to the front properly only after the initial insistence on the aspect of discreteness in the progressive development of Quantity is superseded. It is just because the pure continuity of Quantity as such remains, to begin with, implicit that the science of Arithmetic is not called the science of Time, in contrast to the designation applicable to Geometry, as the science of Space.

In so far, then, as the element of Time can enter upon the scene only in connection with the simplest objective significance of the unity of the extensive and intensive magnitude, when this unity is emphasized with respect to the element of extension, and hence in explicit supersession of the purely analytical attitude to the line, as what consists of points, the line is to be grasped in the sense of a purely continuous One Point, whose simplest objective significance is Movement. As the simplest spatial negation of Space, a Point implies the contradiction between Limit and Presence, and in exemplification of this contradiction, implicating a reference to the element of Time, acquires the explicit sense of an element of a straight line produced *ad infinitum*. In conceiving this production in the sense of

Movement, we, of course, do not at once refer to an externally originated movement, but rather imply that the conception of Movement refers to the simplest objective exemplification of the vicious circle between Quantity and Quality, in the sense of which their anticipated fundamental unity is now before us, Quality assuming with respect to this objective exemplification the significance of the idea of Space, and Quantity that of Time.

But the production of a straight line is before us only as an object of further consideration. In ushering this conception on the scene, we are only putting on record that the unity of the extensive and intensive magnitude is now to be dealt with in reference to the emphasis on the aspect of linear continuity in supersession of the standpoint of numerical discreteness, remaining in evidence, so long as linear continuity is followed arithmetically in the arithmetical continuum. Now, since our present grasp of this continuum is to be envisaged also as the present restatement of the flux of Counting, as dealt with at the outset of the second main subdivision of the dialectical whole of Quantity, i.e. in Chapter IV, and we are now just as much implying the whole of this second subdivision, the further treatment of the infinitely produced straight line obviously concerns the further inwardization of the remaining trend of the previous qualification of the standpoint of Counting. Accordingly, the infinitude of the straight line must next be grasped also in negation of the immediate impression, as though the line could really extend in a straight direction *ad infinitum.*

Whereas the progress towards the infinitely great or small in connection with Counting as such is in place because of the immediate absence of any real reason for breaking it off at any particular number, the straight line now before us presents itself as subject to being produced *ad infinitum* only so long as we do not make of it an object of further consideration. For, in that case, it becomes plain that, in so far as the infinite production of a straight line concerns a purely self-referent movement of the point, and hence a movement such that it is only turning upon itself, to progress just as well meaning to retrogress, and the point itself, consequently, just as well implying the sense of a neutral third to its stated self-discernment—it

becomes plain that the infinitely produced straight line has its true exemplification in a Circle.

As is seen, the element of Alteration, whereby a number loses its numerical significance, has its present embodiment in the bending of the straight line into the simplest closed curve and therewith equally in the implied transition from the first into the second spatial dimension. Therefore, conformably with the fact that geometrical magnitude is now implied in the sense of an area rather than a length, the meaning previously emphasized in connection with a point is now to be associated with a straight line. Just as a point is only an element of a straight line, so this latter, in turn, is only an element of an area. And in so far as the straight line refers now to the radius of the Circle representing the true meaning of the infinitely produced straight line, it becomes obvious that, in so far as the area of the circle concerns us as the present restatement of the arithmetical magnitude in simultaneous inwardization of the standpoint of arithmetical continuum, the moment of unity refers to an infinitely small sector of the area and the moment of amount to the circumference in the sense of an infinitely many-sided polygon, in which case the polygonal side is meant ultimately to be identified with an element of the circumference, in record of the already mentioned fact that in connection with a two-dimensional line its element no longer remains at the stage of a simple point, but must be given the explicit sense of an element of a straight line, i.e. of that straight line which has a common point with the two-dimensional line or curve in question and hence forms a geometrical tangent to it. In so far as the resolution of the circumference into its elements comes thus to mean its conversion into the sum of the infinitely many points, of which its rectified length is analytically conceived as consisting, the area of a circle presents itself finally as equivalent to the area of a triangle having the circumference for its base and the radius for its height.

The fact that the present explication of the infinite progress is to be also envisaged in marked correspondence to the transition from Limit into the Finite as such within the dialectical whole of Quality, comes home to us when we turn our attention to the element of infinite approximation in connection with the implied alteration of the po-

lygonal side. Just because the conception of a polygon is introduced only in the sense of a middle term to the rectification of the circumference, the Alteration of its sides no longer rests simply on *à priori* grounds, as was still the case in connection with the Alteration whereby arithmetical magnitude assumed the significance of a geometrical magnitude, but becomes a matter of demonstrable necessity. After having immediately dealt with Quantity only with respect to its dialectically postulated subjection to Alteration, we have it now before us, in the polygonal side, established as what *must* alter and hence established adequately to its immediate notion. That its Alteration *must* take the shape of an infinite approximation towards an equally present beyond (i.e. the given Limit, whether in the area of the circle or simply in the circumference) follows from the fact that the necessity for the Alteration is now due to an explicit reference to its own Quality, upon the proviso that this its own Quality nevertheless still just as much retains the character of qualitative otherwiseness.

The from the very first anticipated fundamental unity of Quality and Quantity is, precisely, before us still only in anticipation of that mediation, by means of which alone its two sides come to be fully established as necessarily transient into one another. In so far as the said unity is established only in the sense of a direct unity, its demonstration can take the shape only of a vicious circle between Quality and Quantity which immediately corresponds to the contradiction between Limit and Presence, but, from the standpoint of the inwardization of this contradiction under the head of the Finite, comes now to represent this latter in the sense of an infinite progress. And it is obviously also only on account of the still obtaining vicious circle between Quality and Quantity that the now already also demonstrable necessity for the alteration of a Quantity does not yet necessarily involve the existence of a naturally given fact, but falls within the domain of pure analysis, where our mind is still communing only with its abstract constructions.

Seeing, by the way, that it is just the demonstrable necessity now in question that more particularly forms the subject-matter of pure analysis, it is appropriate that its fundamental theorem should simply concern the necessary

and sufficient condition for the convergence of an infinite sequence.

"The sequence $u_1, u_2, u_3 \ldots$
is said to be convergent and to have the limit A, when by indefinitely increasing n the difference between A and u_n can be made as small as we please. In more exact words, the sequence is said to be convergent and to have A for its limit, when any positive quantity ϵ having been chosen as small as we please, there exists a finite positive integer v such that

$$| A - u_n | < \epsilon \text{ for } n \geqq v."$$

In so far as this test of convergency or infinite approximation cannot be used in cases when the value of the given limit A is unknown, the required criterion is given the following general form:—

"A necessary and sufficient condition for the existence of a limit to the sequence

$$u_1, u_2, u_3 \ldots$$

is that a finite positive integer v exists such that $| u_{n+p} - u_n |$ becomes as small as we please for $n \geqq v$ and any positive integral value of p.

"In other words, and more precisely: A necessary and sufficient condition for the existence of a limit to the sequence

$$u_1, u_2, u_3 \ldots$$

is that, the positive quantity ϵ having been chosen as small as we please, there must exist a finite positive integer v such that

$$| u_{n+p} - u_n | < \epsilon \text{ for } n \geqq v$$
$$\text{and } p = 1, 2, 3, \ldots"$$

In order to show that the reasoning necessarily resulting from the attempt to follow the properties of geometrical or generally qualitative magnitude purely arithmetically, becomes itself an instance of the vicious circle now under discussion, seeing that the reasoning must then attempt to exclude what alone constitutes its intelligibility and thus to assume a negative attitude towards its own subject-matter, we proceed to quote equally the proof of the stated theorem.

"We shall first show that *this condition is necessary;* that is to say, that if the sequence is convergent this condition

is fulfilled; secondly, that if this condition is satisfied the sequence must converge; in other words, that *the condition is sufficient.*

"1. *The condition is necessary.*

"Let the sequence converge to the limit A.

"Then $\quad | A - u_n | < \frac{\epsilon}{2}$ for $n \geqq v.$

"But $\quad (u_{n+p} - u_n) = (u_{n+p} - A) + (A - u_n).$

"Therefore $| u_{n+p} - u_n | \leqq | u_{n+p} - A | + | A - u_n |,$

$$< \frac{\epsilon}{2} + \frac{\epsilon}{2}$$
$$< \epsilon,$$

whatever positive integer p may be, provided $n \geqq v.$

"2. *The condition is sufficient.*

"We must examine the two cases: first, when the sequence contains an infinite number of terms equal to one another; second, when this is not the case.

"Let there be an infinite number of terms equal to A. Then if

$$| u_{n+p} - u_n | < \epsilon \text{ for } n \geqq v,$$
$$p = 1, 2, \ldots$$

we may take $u_{n+p} = A$, for some value of p, and we have

$$| A - u_n | < \epsilon \text{ for } n \geqq v.$$

"Therefore the sequence converges and has A for its limit.

"If there is not an infinite number of terms of the sequence equal to one another, it contains an infinite number of distinct terms which form an aggregate (U), say. This aggregate is bounded, since if we suppose $n \geqq v$, all the terms of the sequence except the first $(v - 1)$ are contained in the breadth 2ϵ with u_v as centre. Now (U) must have at least one limiting point within this interval $u_v \pm \epsilon$ (cf. § 13[1]). There cannot be two such limiting points A and

[1] *An aggregate (E) bounded on the right and left must contain at least one limiting point in the closed* interval mM, where m and M are its lower and upper boundaries.*

All the points of (E) lie in the interval mM. Putting aside the case in which m is a limiting point, let a be any point of mM other than m. There

* The interval $a < x < b$ is called an open interval and $a \leqq x \leqq b$ a closed one.

A′, since the distance between these two points can be at most 2ϵ and ϵ can be chosen as small as we please. Indeed, by choosing a value of ϵ less than $A - A'$ we could make all the terms of (U) except a finite number lie upon a segment of breadth less than $\left|\frac{A - A'}{2}\right|$, and thus make its limiting points lie with in an interval smaller than $|A - A'|$.

"Therefore the aggregate (U) has one and only one limiting point. Let this point be A. By the definition of a limiting point of an aggregate, however small ϵ may be, there will be an infinite number of terms of (U), and therefore of the sequence, in the interval $(A - \epsilon, A + \epsilon)$, The points of (U) outside this interval are finite in number; since if they were infinite, there would be another limiting point of the aggregate, and this we have seen to be impossible. Therefore there must be a finite positive integer v such that

$$|A - u_n| < \epsilon \text{ for } n \geqq v,$$

and the sequence is convergent and has A for its limit.

"It has therefore been proved that *it is a necessary and sufficient condition for the convergence of the sequence*

$$u_1, u_2, u_3, \ldots,$$

that corresponding to any positive quantity ϵ, *chosen as small as we please*, there shall exist a finite positive integer v, such that

$$|u_{n+p} - u_n| < \epsilon \text{ for } n \geqq v, \quad \text{and } p = 1, 2, 3, \ldots.$$[1]"

may be an infinite or a finite number of points of (E) in the interval ma. There is a finite number if a is sufficiently close to m, provided m is not a limiting point. There is an infinite number if a coincides with M. Let (A) be the aggregate of the points a such that the interval ma contains a finite number of points of (E). Then (A) is bounded to the right. Let α be the upper boundary of (A), $m\alpha$ not being greater than mM. However small ϵ may be, there is only a finite number of points belonging to (E) in the interval $(m, \alpha - \epsilon)$, while there must be an infinite number of such points in the interval $(\alpha - \epsilon, \alpha + \epsilon)$. If this were not the case α would not be the upper boundary of (A), which would extend certainly as far as $\alpha + \epsilon$. Therefore there is an infinite number of points of (E) in the interval $(\alpha - \epsilon, \alpha + \epsilon)$, and α is a limiting point of (E).

Thus every aggregate bounded on the right and left has at least one limiting point in the interval between its boundaries.

[1] Fourier's Series, Carslaw, § 15.

CHAPTER VIII.

THE QUANTITATIVE RELATION.

IN so far as our previous discussion of the quantitative progress *ad infinitum* presents itself to us in the light of an explication of its original implicitness in connection with the standpoint of arithmetical magnitude, and it follows from the circumstantially systematic or systematically circumstantial nature of our progressive development of the notion of Quantity, that the import of our further discussion must be the further explication of the original inwardization of the infinite quantitative progress under the head of the geometrical magnitude as a Sum of an infinite Series: we are obviously concerned with the form in which the infinite quantitative progress is now to be ushered on the scene, in order that the present correspondence to its original transformation into a Sum of an infinite Series becomes explicit also in reference to its immediate meaning which, as we know, already also implies the result of the original transformation.

Now, the original transformation of a Number into a Sum of an infinite series is traceable to the dialectical necessity of superseding the one-sidedness committed in immediately emphasizing the original unity of the continuous and discrete magnitude only with respect to its aspect of Discreteness. Therefore it suggests itself that the present restatement of this transformation must similarly be traced to a need to supersede the remaining survival of the original emphasis on the element of Discreteness. That such a survival there still must be in connection with the to be transformed form of the quantitative progress *ad infinitum*, *à priori* follows from the fact that this progress has so far been envisaged in the sense of a vicious circle between Quality and Quantity. In so far, namely, as the own Quality of Quantity had, so far, to be treated also in the sense of an Other, and Continuity as such is to be

viewed in correlation with the from the very first anticipated Qualitativity of Quantity, it follows that even though already also emphasized as purely continuous, in supersession of the remaining evidence of Discreteness in the Arithmetical Continuum, the quantitative infinite progress remains still under the shadow of Discreteness with respect to its insuperable discontinuity in reference to the Limit to be reached. Seeing that this Limit is bound to remain an unreachable beyond of an infinite approximation, the latter's pure continuity is all along just as much self-negative and for that reason still continues to share the nature of the arithmetical continuum.

Accordingly, the infinite progress is now to be ushered on the scene in explicit supersession of the immediate necessarily purely analytical attitude towards it. Since that means that we must transcend the standpoint, at which Quality and Quantity are only in immediate unity and hence just as much immediately negative of one another; the Limit of the infinite progress must no longer be emphasized mainly in the sense of an unreachable beyond, but rather as what contains the infinite approximation to its own self within its own self. That is to say, its immediate externality to the infinite progress is to be superseded, with the consequences that the progress comes to be envisaged in correspondence to its original inwardization.

The change of attitude which we are trying to convey is clearly of the same kind as when Truth ceases to be envisaged in the sense of a never to be reached beyond of our thinking capacity, but comes to be realized as having itself prompted our immediately so futile appearing search for it. Or, if we remind ourselves that our next discussion of the infinite progress in the middle section of the concluding main portion of the dialectical whole of Quantity is equally to be envisaged in the sense of a restatement of the trend of the dialectic of Finitude in the dialectical whole of Quality, it becomes plain that the now to be emphasized aspect of the infinite progress is meant to bring to the front the present embodiment of the Finite at once in explicit reflection upon the result of its dialectic and hence in the explicit sense of the finitized Infinite, seeing, precisely, that the notion of the Infinite is now from the very first included in the Inwardness of the quantitative Finitude; which fact, by the way, does not, of course, mean that we are relieved

from the necessity of finding later on the proper quantitative restatement of the stage, at which the notion of the finitized Finite is originally ushered on the scene.

Seeing, therefore, that in connection with the present conversion of the quantitative progress *ad infinitum* into an object of our further discussion, in higher restatement of its original inwardization in a number standing for a geometrical magnitude, we must explicitly envisage its endlessness from within the own Inwardness of its just as much also already given Limit, and thus put on record that the element of Negation, conspicuous in the immediate correlation of the infinite progress with its Limit, so long as this Limit is emphasized only in the sense of an unreachable beyond, is in itself just as much implied as already negated; the original distinction between the rational and irrational number is now to be grasped as due to the own Self-discernment of a fixed magnitude into two magnitudes which now take the place of the original two moments of a magnitude, whether arithmetical or geometrical. That is to say, as superseding the immediate contrast between the two aspects of a given magnitude, the Limit of infinite approximation is next to be grasped as containing this contrast in its own Inwardness in the shape of a *Ratio.*

The fact, by the way, that we are now correlating two magnitudes in the explicit sense of self-differentiated aspects of that magnitude which, in supersession of its immediate sense as the Limit of infinite approximation, assumes now the significance of the *Exponent* of the Ratio between the two magnitudes in question—this fact reminds us that the emphasized qualitative aspect of the Exponent must not be permitted to become submerged in its immediately purely quantitative aspect. It is only as torn out of its context in the dialectical whole of Quantity that the Quantitative Relation, i.e. the equivalency of a given Ratio with its Exponent, appears to concern itself only with mere magnitude, arithmetical or geometrical. For, viewed in its proper light, the standpoint of the Quantitative Relation refers to the stage, at which the already implicitly established fundamental unity of Quality and Quantity becomes the object of our further consideration with a view to the establishing of the element of mediation, whereby its immediately only direct character is deepened into that negative unity, in

which the dialectic whole of Quantity finds its conclusion in transition into the next higher dialectical whole. Consequently, even though the needed mediation is meant to exemplify itself in the form of the Quantitative Relation, we must not forget that Numbers are now before us as charged with qualitative significance, in anticipation of the final objective import of the fundamental unity of Quality and Quantity.

For instance, aware as we already are that Quality has its objective correspondence in Space and Quantity in Time, and consequently that the immediately established implicit unity of Quality and Quantity has its objective illustration, first of all, in the spatial Here and Now of a point and more explicitly in the conception of Movement, it is at once plain to us that the forms of the Quantitative Relation might be interpreted in the sense of an abstract record of the forms, in which Movement may be displayed. And it is, therefore, only because we are not meant explicitly to refer to such an objective display of the unity of Quality and Quantity before its full establishment in correspondence to the true Being, that the Quantitative Relation immediately appears to be of a purely mathematical consequence and that the qualitativity of the Exponent consists, to begin with, only in its Fixedness as a particular magnitude, arithmetical or geometrical.

As regards, now, the forms in which the notion of the Quantitative Relation is to be embodied, in marked correspondence to the stages of its progressively developed dialectic, we at once anticipate that this its dialectic development must present it to view in three main forms; and that, whilst, on the one side, representing the present correspondence to three main stages of the dialectic of Finitude, (1) the notion of the Finite as such, (2) the distinction between Transitoriness and Transcendentalness, and (3) the transition into the Infinite, the three main forms of the Quantitative Relation must, on the other side, be equally envisaged in the sense of another higher restatement of the three main stages of magnitude as discussed in the second main portion of the dialectical whole of Quantity.

A. The Direct Relation.

Inasmuch as the immediate form of the Quantitative Relation brings again to the front the standpoint of the

arithmetic magnitude, the fixed magnitude, which, on the side of the Exponent, represents the qualitative element of the Relation, is to be grasped as distinguished into two magnitudes taking the place of the two simple moments of a number. We have again before us a particular magnitude as an amount of a constant unity, but upon the proviso that this unit is itself a numerical One and consequently that it may be any number whatever. Inasmuch as the moment of amount in the ratio is thus implied in the sense of a product of the number representing the side of unity into the fixed value of the Exponent, it becomes obvious that the immediate form of the Quantitative Relation simply puts on record the abstract possibility of an endless restatement or Alterableness of any number in terms of a *Direct Ratio.*

In this way, we are only bringing home to ourselves the way in which the now fundamentally qualitative nature of the Exponent, as the reached Limit of infinite approximation, comes at once to the front, even when the Exponent is, to begin with, identified only with an arbitrarily fixed number. The arbitrary breaking off of the flux of Counting as such at a particular number no longer rests on the purely abstract ground that any number may serve for the representation of the conclusive nature of the One of Quantity, in connection with the original inwardization of the infinite flux of Counting ; the breaking off of this flux is now traceable to the qualitative element now to be associated with any number, in so far as it stands for the Exponent of a Direct Ratio.

In order, however, to bring to the front the fact that the infinite quantitative progress is now implied as belonging to the own Inwardness of the Exponent, we must associate its given value with a geometrical rather than arithmetical magnitude. The simplist illustration of the Direct Relation is then at once found in the ratio between the circumference of a circle and its diameter. Or we may refer to trigonometrical ratios of an angle as functions of the sides of a right-angled triangle. But the fact that the Sum of the infinite series, as which the Exponent should be established, remains *de facto* an unreachable beyond, is bringing home to us that the supersession of the vicious circle between Quality and Quantity, ushering on the scene the standpoint of the Quantitative Relation, does not yet explicitly come to view at its immediate stage.

That the Direct Ratio does not lend itself to an adequate embodiment of the qualitative element attaching to the Exponent, is really from the very first obvious. In so far as the two magnitudes, into which the Exponent of the Ratio is to be grasped as *self-discerned*, are only as sides of a Direct Ratio, the qualitative element of their Continuity into one another remains conspicuous by its absence, or rather remains submerged in the once again emphasized aspect of Discreteness. As already pointed out, the Direct Relation simply explicates the standpoint of Counting in connection with the moment of unity of a particular fixed magnitude, arithmetical or geometrical. In so far, however, as the Numerator is thus bound to remain the same fixed amount of the Denominator, quite irrespective of the value assigned to this latter, or, by contrast, the Denominator stands for the same fixed fraction of the Numerator irrespective of its value, it is either the Numerator alone or the Denominator alone that is really variable. That is to say, so far as the sides of the Direct Ratio may be identified with any and every number, this their Alterableness does not involve any Alteration in their relation and for this reason *negates* their simultaneous distinguishedness as *two* magnitudes. This is how the vicious circle of before makes itself still noticeable. The two sides of the Ratio are implied in the explicit sense of two distinct magnitudes, yet the Direct Ratio still strips the distinction between them of its presupposedly qualitative nature. Instead of being in the relation typified by Something *and* an Other, they remain correlated only in the sense of Something *or* Other. From the standpoint of either, the other's alterableness is of no consequence.

Obviously, in a more adequate embodiment of the qualitative nature of the Exponent, we must negate the immediate negation of their qualitative distinguishedness as two magnitudes in the Direct Relation. The one of them must no longer be simply the same *fixed* amount of the number standing for unity, or the same fixed fraction of the number standing for amount, but become always *another* amount or *another* unity of the other. That, in this way, we postulate a higher form of the Quantitative Relation, is obvious from the fact that each side of the Ratio comes then to stand for an endless series of numbers

in such wise, that to every given value on the one side there corresponds a certain value on the other side in supersession of the immediately fixed relationship between them. We are now correcting our immediate attitude to the Ratio by insisting on an interchangeability of its sides. In so far as these sides are from the very first given the character of distinct numbers, each implies Amount and Unity, and for that reason their immediate relation in the sense of either Amount alone or Unity alone implies a one-sidedness whose correction amounts to a transition from the Direct into the *Inverted Relation.*

B. The Inverted Relation.

Inasmuch as under this head we are negating the immediately implied negation of the distinguishedness of the two magnitudes forming the sides of the direct Ratio, and the two magnitudes, consequently, are no longer simply degraded to the rank of mere moments of one and the same fixed magnitude; the Exponent presents itself now as also negative of itself as a moment of the Ratio. Whereas under the head of the Direct Relation the moment of Amount remains identical with the amount represented by the number on the side of the Exponent, the distinction between them concerning primarily only the alterableness of the moment of unity on the side of the Ratio, now this alterableness involves also Alteration of the fixed amount represented by the Exponent.

The Inverted Relation no longer simply explicates the from the very first implicated possibility of restating one and the same number in terms of an endlessly alterable numerical one, but is meant to bring to the front the fact that, if the possible endless restatement is to represent the aspect of infinite approximation to the fixed value of the Exponent, the Alterableness of the number on the side of unity must just as much continue itself negatively into the number standing on the side of amount. For, according to the notion of the Quantitative relation, the Exponent has also the character of qualitatively fixed Limit forming a third to the two magnitudes on the side of Ratio, in so far as they are distinguished in the sense of Something and an Other. Or rather, seeing that the qualitatively fixed Limit has now the sense of the finitized Infinite, in

explicit correspondence to the Finite as such, the distinction between the two magnitudes concerns the two senses of infinite approximation, either to the infinitely great or small, in present correspondence to the two senses of the qualitatively infinite progress, Transitoriness or Transcendence.

In so far, then, as it is only now that we are properly concerned with the self-discernment of a fixed magnitude into two qualitatively distinct magnitudes, we reflect upon the Exponent as both *in itself* identifiable with the ratio representing its Inwardness as a qualitative Limit, as well as negatively distinguished therefrom. The fact that we are thus still reproducing the vicious circle between Quality and Quantity, shows that not even the present form of the Quantitative Relation is fully adequate to its notion; but, then, the final form must be ushered on the scene only after a circumstantial exemplication of the import of the present form in restatement of the three main stages of magnitude now equally to be summarily recapitulated in connection with every form of the Quantitative Relation, in which respect, of course, the stage corresponding to the form under discussion receives special emphasis.

Obviously, since the Exponent is to be grasped as self-discerned into two magnitudes of which it forms the third in the sense of a Limit to their mutual limiting, its arithmetical significance refers to the abstract possibility of treating any number in the sense of a fixed product of two variable factors. If in connection with a given number c, we choose to identify the moment of unity with the number a, then, seeing that the moment of amount b is to comply with the condition $c = ab$, its corresponding value is found by dividing a into c. As is seen, the determination of the moment of amount on the side of the inverse ratio is no longer a matter of multiplying, but of *dividing*, the Exponent by the number identified with the moment of unity.

The contrast from the Direct Ratio comes also to the front in the fact that now it does not matter which side we choose to call unity or amount. Just because the two sides are no longer treated as mere moments of one number, but are each asserted in the sense of a unity of unity and amount, the distinction between then becomes a matter of emphasis on opposite moments in the fundamental unity of

both. Therefore, if it is the moment of unity that is emphasized in connection with the number a, the number b comes to be emphasised with respect to the moment of amount, and *vice versa.* In any case, either side increases or decreases as *many times* as the other side decreases or increases.

As regards the geometrical import of the Exponent, its elementary exemplification is obviously found in the mensuration formula for the area of a Parallelogram, when the base is symbolized by a and the height by b. That this formula is at the back of every mensuration formula for areas came to our notice already in the previous transformation of the circular area into a triangular one. In so far as the area of a triangle is equal to the product of its base into one half of its height, it is implied as transformed into a rectangled parallelogram or a Rectangle.

In so far, now, as this area is represented by the product of its two sides and this product is to be emphasized as fixed, it is obvious that the represented area admits, in turn, of an endless transformation, provided that the one side increases or decreases as many times as the other side decreases or increases. The qualitative character of the distinction between the two variable factors comes now to the front in the fact that their magnitudes are contrasted in the sense of the arithmetical and the geometrical magnitude. For, in order that the product of two numbers represent the area of a rectangle, one of them must, to begin with, be emphasized in the sense of a strip of the total area, this area being implied as subdivided into as many strips of the same breadth as is indicated by the numerical one on the other side of the ratio. In so far, of course, as each strip is itself in turn subdivided into as many unit areas as is indicated by the numerical one on its own side, the multiplication of the one factor by the other comes to carry with it the significance of a counting up of the unit areas into which the total area is conceived as subdivided.

But by virtue of the implied contrast between the geometrical and arithmetical magnitude, the Counting up of the unit areas must next present itself also as submerged in Arithmetical Continuum. As a higher restatement of the geometrical magnitude in simultaneous inwardization

of the transition from the standpoint of magnitude into that of the Quantitative Relation, the fixed Product is now also implied in the sense of an unreachable beyond of the infinite approximation to it. In this respect we are obviously referring to those cases when the sides of the parallelogram equivalent in area to the space bounded by a particular curve, say, an ellipse, cease to be determinable as a round number and hence remain arithmetically sums of an infinite series (as is in fact also already the case with the base of the parallelogram representing the area of a circle).

In such cases, the alterableness of the unit area taking the place of the unit of enumeration, is explicitly advanced to the stage of Alteration and hence comes to correspond to the Irrational Number, whose geometrical significance, however, becomes now a line. Just as a straight line is purely analytically conceived as consisting of points, so an area is said to consist of lines, with the consequence that the determination of areas in question ultimately takes the shape of a summing up of the infinitely great number of the infinitely narrow strips, as which the lines are then really conceived, in reflection upon their true significance as elements of the given area.

As is well-known, the summing up now in question goes by the name of Integration, whose symbol "$\int$" is in fact originally the initial letter of the word Sum. "*Integration is essentially a method for obtaining the sum of an infinite number of infinitely small quantities*. This does not mean, as some writers have it, 'if enough nothings be taken their sum is something'. The integral itself is not exactly what we usually understand by the term 'sum,' but it is rather 'the limit of a sum when the number of terms is infinitely great'."[1]

However, for the time being we are not yet concerned with the method employed for obtaining the sum of an infinite number of infinitely small quantities, in a word with Integration, but only with the fact that the qualitative nature of the Exponent is exemplified on the side of the Ratio only with respect to the implied process of infinite approximation. That is to say, the also already affirmatively present Limit of this process is not yet to be considered

[1] "Higher Mathematics for Students of Physics and Chemistry," F. W. Mellor, D.Sc., Longmans, Green & Co., 1905, p. 188.

as reached on the side of the Ratio, because the characteristic feature of the second form of the Quantitative Relation is that the Exponent is negatively distinguished therefrom.

In reflection upon the already mentioned correspondence of the dialectic of the Quantitative Relation with the dialectic of the Finite, it becomes obvious that the already arising need of also bringing the process of infinite approximation under discussion to its due end, and thus explicitly to embody the whole notion of the Quantitative Relation on the side of the Ratio, really anticipates the stage reached in the dialectical whole of Quality in the alternating determination of the infinitized Finite and the finitized Infinite, in transition into the true Being. In order to advance thus far, we must preliminarily usher on the scene the present explicit correspondence to the notion of Infinitude as such, in which case we come, of course, to deal with the third and last form of the Quantitative Relation, which, in reflection upon the simultaneously effected supersession of the Inverted Relation, we propose to call the *Involved Inverted Relation.*

C. The Involved Inverted Relation.

Just as the Inverted Relation explicates what remains implicit in the Direct Relation, namely, that the two magnitudes on the side of the Ratio are *qualitatively* distinct from one another, so the Involved Inverted Relation is meant to bring to the front what, in turn, remains still implicit at the stage of the Inverted Relation. The notion of the Quantitative Relation, as the reached beyond of the quantitative progress *ad infinitum*, is from the very first implied on the side of the Exponent ; but on the side of the Ratio this notion is being ushered on the scene only in dialectically progressive accentuation of the aspects of its Inwardness in simultaneous restatement of its dialectic past. So comes it that, even though the Exponent is from the very first pregnant with the true meaning of the quantitative Infinite, in supersession of the spurious aspect of this Infinite in the infinitely great or small, the explicit embodiment of our previous realization that the quantitative infinite is *de facto* nothing else than the notion of the fundamental unity of Quality and Quantity comes up for treatment only in the concluding form of the Quantitative Relation.

In calling this form the Involved Inverted Relation, we associate with the term Involution primarily only the sense of Supersession. As meant to render explicit what in the Inverted Relation remains still implicit, the concluding form must at the same time supersede the middle form; that is to say, it must still imply or *involve* this form in its own Inwardness, but no longer in its original sense; just as is also the case in the transformation of the Direct into the Inverted Relation. It is only after we bring home to ourselves the form assumed by the Inverse Ratio in consequence of the supersession of its characteristic feature as a double-sided progress *ad infinitum* that the term Involution is found to have equally a bearing on the immediate arithmetical meaning of the concluding form of the Quantitative Relation.

Namely, in negating the infinite quantitative progress connected with the sides of the Inverse Ratio, we are at the same time explicating the qualitative character of the distinction between them. It is only now that we properly realize that this qualitative character has not, after all, received its full explicitness. In our endeavour to correct the one-sidedness characteristic of the Direct Relation, we committed a one-sidedness of the opposite kind by over-emphasizing the aspect of dualism on the side of the Ratio. Not that we simply blundered along. The now to be superseded one-sidedness is necessarily of the type asserting itself in immediate embodiment of Alteration, as a middle term between Alterableness as such and the contradictory unity of both in Limit. After having insisted on the aspect of qualitative distinction between Something *and* an Other in supersession of their simple unity in Something *or* Other, we had to realize that along with their qualitative distinctiveness the two nevertheless in themselves remain the one what the other is, or that their qualitative distinctiveness is after all only traceable to a double way of envisaging their fundamental Oneness upon the proviso that this Oneness is identifiable with either and neither, or, in other terms, that it refers to them as at once inside and outside its own self. It is because the Exponent of the Inverse Ratio had to be emphasized as negatively distinguished from it, that the qualitative character of its sides took the shape of a progress towards the infinitely great or small.

And since, therefore, this progress, as always, only forces on our attention that, so far, the qualitative background of the sides of the Ratio remains implicit, and that, in order to render this background fully explicit, we must postulate them as indistinguishably distinguished, or in such a relation that their Alteration remains also mere Alterableness and vice versa; we must finally restrict their interchangeableness to a value common to both of them, in which case, of course, we are equating them with the root of the number on the side of the Exponent.

As is seen the arithmetical import of the Involved Inverted Relation refers to the abstract possibility of treating every number as either a Square or the Root of a Square, i.e. as subject to Involution or Evolution. These two terms refer to ordinary Multiplication or Division, upon the proviso that the amount by which a number is to be multiplied is identical with its own amount, or that the amount of parts into which a number is to be divided is identical with the moment of amount of the number representing one part.

As transformed into the equation $a^2/a = a$, the Involved Inverted Relation is presented to view with respect to its implied return to the form of the direct Relation, when it may be also called Relation of Powers. But that this its form is due to a transformation of its original form $a^2 = a \cdot a$ is brought home to us in connection with the operation of Evolution. In this respect we are simply retracing backwards the operation of Involution. Seeing that on squaring the number, say, $a \cdot 10^2 + b \cdot 10 + c = 123$, we get $a^2 \cdot 10^4 + 2ab \cdot 10^3 + (2ac + b^2) \cdot 10^2 + 2bc \cdot 10 + c^2 = 15129$, the way in which this expression is to be handled in order to get back to its root is generalized into the operation of Evolution.

Owing to the qualitative character of the distinction between the Square and its Root, if the Square is identified with the consecutive terms in the system of rational numbers, the values corresponding to the Root form an arithmetical continuum. This fact, consequently, serves also as a background to the precise mathematical definition of irrational numbers.

"Arrange the integral numbers, and the squares of these numbers, in two horizontal rows so that the squares are underneath the numbers to which they correspond. Since

the square of a fraction in its lowest terms must be a fraction whose numerator and denominator are perfect squares, we see that there do not exist rational numbers whose squares are, 2, 3, 5, 6, 7, 8, 10, etc.

1			2					3								4 . . .
1	2	3	*4*	5	6	7	8	*9*	10	11	12	13	14	15	*16*	. . .

"There do, however, exist rational numbers whose squares are as near these numbers as we please. For example, the set of numbers

1, 1·4, 1·41, 1·414, 1·4141 . . .
2, 1·5, 1·42, 1·415, 1·4142 . . .

form a lower and an upper class in which the squares of the terms in the lower are less than 2, and the squares of the terms in the upper are greater than 2; and however small a positive quantity ϵ we may choose, rational numbers may be found whose squares differ from 2 by less than this number. There exists such numbers in the lower class and also in the upper.

"Arrange in the lower class all the negative rational numbers, zero, and the positive rational numbers whose squares are less than 2; and in the upper class all the positive rational numbers whose squares are greater than 2. Then every rational number belongs to one class or the other. Also every number of the lower class is less than every number of the upper; and the lower class has no maximum number, nor has the upper class a minimum. We have thus obtained a method of subdivision of the rational numbers satisfying the conditions of § 4.[1] In other words, we have defined the irrational number $\sqrt{2}$.

"With this method of defining the irrational number we can now proceed to its approximate arithmetical determination. Consider the division of the rational numbers by means of which any irrational number A is defined. If ϵ is any positive rational number as small as we please, the terms of the sequence of rational numbers

$$\ldots -2\epsilon,\ -\epsilon,\ 0,\ \epsilon,\ 2\epsilon \ldots$$

fall into one or other of the two classes of the number A.

"Let $n\epsilon$ be the greatest of the terms which belong to the lower class, n being an integral number, positive, negative,

[1] See Dedekind's "Theory of Irrational Number," Chap. VI.

or zero. Then $(n + 1)\epsilon$ will be the smallest of the terms of the sequence which belong to the upper class. The value $n\epsilon$ may be taken as the approximation to A to the limit ϵ by defect, and the value $(n + 1)\epsilon$ the approximation by excess. We might then proceed to subdivisions of ϵ and obtain such closer approximation as is desired.

"We may apply this most simply in the decimal notation if we agree to write any given number with the fractional part positive and the integral part positive or negative. The approximations by excess and defect with a margin of error unity will give the number as lying between two consecutive integers. Then proceeding to the closer approximations in which the error may be ·1, ·01 . . . respectively, we shall obtain a determinate sequence of which the mantissae, will contain one, two, and three figures, and so on. The terms of the sequence which form the approximations by defect never diminish and may increase. If we add a unit to the last figure of each term of this sequence, we get the approximation by excess. The terms of this sequence never increase and may diminish. We thus arrive at the decimal representation of this irrational number A, formed by an integral part, positive, zero, or negative, and a succession of decimals of which each is determinate, since the method of division of the rational numbers which defines A is given. In stopping at the pth decimal place we obtain the approximation to A with a margin of error $\frac{1}{10^p}$ by defect, and in adding unity to the last figure of this number we obtain the approximation by excess to the same order. To this method of representation of a number we are already accustomed in the decimal representation of those rational fractions whose denominators contain factors other than 2 and 5. The difference between the unending decimals in the two cases is that for the rational number the decimal is bound to be a recurring one, and in the case of the irrational number such cannot be the case.

"These two infinite monotonous[1] sequences are the foundation of Cantor's method of defining the irrational numbers."[2]

[1] We use the term monotonous or monoclinic, for a sequence which either never increases and may diminish, or never diminishes and may increase.

[2] Fourier's Series, §§ 6, 7.

CHAPTER IX.

THE RELATION BETWEEN THE ORIGINAL AND THE DERIVED FUNCTION.

ALTHOUGH the Involved Inverted Relation embodies the notion of the Quantitative Relation, as a conclusion of the fixed Magnitude in its self-external otherwiseness with itself, the thus implied infinitization of the immediate finitude of the self-discerning Quantum remains still implicit. The fact that every Number is subject to Involution or Evolution does not yet make the reason for its Fixedness objectively real. The Something or Other ultimately to be identified with the Qualitative Quantum is still conspicuous only by its absence.

That the return from Quality into Quantity is not yet fully established, immediately follows from the correspondence of the Involved Inverted Relation to the notion of Infinitude, as ushered on the scene in conclusion of the dialectic of Finitude. Just as this notion had to be additionally vindicated in regard to the claim immediately made on its behalf, that it is the only true Being in complete idealization of the Finite as such ; so now, in so far as the Infinite means the complete conversion of Quantity into Quality, this its recovered qualitative meaning must equally be additionally vindicated in regard to the claim immediately to be made on its behalf, that it no longer simply rests on a mere form of the Quantitative Relation, but on its objective Reality, in identification with a qualitative Something or Other.

That the full establishing of the return from Quantity into Quality cannot be reached within the standpoint of the Quantitative Relation as such, is obvious from the fact that Quality remains thus correlated with Quantity only in an alternating unity of both. That is to say, Quantity presents itself still as only externally united with Quality, not yet as belonging to the latter's own Inwardness. Even

though every number is the Root of a Square or the Square of a Root, it does not thereby cease to be a mere number. Consequently, the full establishing of the return from Quantity into Quality necessitates supersession of the very form of the Quantitative Relation. And the latter's supersession must, of course, be the outcome of its own dialectic development.

Now, when we reconsider the trend of this development, we find that the Involved Inverted Relation establishes the Qualitativity of Quantity primarily only in the sense of the Finite as such; or rather, of the Infinite in its immediate connotation, as what is negative of, and for that reason also directly identical with, the Finite. This is, precisely, why the formally or implicitly already established unity of Quality and Quantity continues to imply an element of Alternativeness. The completion of the dialectic of the Quantitative Relation must, therefore, concern the deepening of the already established direct unity of Quality and Quantity into a negative unity. The supersession of the form of the Quantitative Relation must be sought in connection with the Alteration suffered by the Involved Inverted Relation through the full embodiment of the element of mediation, implicitly belonging to its own Inwardness. And this means that our remaining task consists in rendering the so far only implicit Involution of the Inverted Relation properly explicit; in which case we refer, of course, to a further restatement of the expression on the side of the Ratio, seeing that it is primarily on this side that the element of mediation is being put on record.

Obviously, since the simultaneous progress towards the infinitely great and small has now for its background the standpoint of the Involved Inverted Relation, the sides of the Ratio can no longer be correlated in such wise, that either of them increases or decreases *as many times* as the other decreases or increases. Just because the standpoint of the Inverted Relation is now to be regarded as also already superseded, either of the sides of the Ratio can no longer be determined from the Exponent by means of the other side, when its magnitude counts as unity. The values x and y on the side of the Ratio must now cease to be treated as mere factors of the Exponent, because this latter is no longer merely a fixed Product, but a Square.

As a matter of fact, then, the *explicitly involved* Inverted Relation has for its Exponent a middle term between the Exponents of the Inverted Relation as such and the Involved Inverted Relation as such, i.e. the *implicitly involved* Inverted Relation. Whilst implicitly sharing the meaning of these two Exponents, the Exponent of the explicitly involved Inverted Relation is explicitly distinct from them. It has the sense of an affirmatively present beyond of infinite approximation, without yet being either a fixed Product of two variable factors or a Square. It suggests itself, therefore, to treat it in the sense of the Sum of an Infinite Series, in so far as the double sense of the infinite progress, implicitly belonging to the own Inwardness of this Series, may well take the place of the simultaneous progress towards the infinitely great and small now to be restated. In fact, we are at a loss to discover any other interpretation of the Alterableness to be emphasized in connection with the sides of the explicitly involved Inverted Relation except that either increases or decreases, no longer as many times, but only *by as much* as the other decreases or increases. And inasmuch as the common value, which is the starting point of the double infinite progress in question, undoubtedly is to be identified with the Root of the Square, the Exponent of the explicitly involved Inverted Relation becomes thus equal to the Sum, instead of Product, of the two sides of the implicitly involved Inverted Relation.

That, however, this explicit embodiment of the notion of the Quantitative Relation also on the side of the Ratio amounts to its supersession, is obvious. For, the reached beyond of infinite approximation can now no longer be identified with the Exponent of any of its fundamental forms. In so far as we are still concerned with the self-discernment of one and the same Quantum, we are preserving the standpoint of the Quantitative Relation as such. But since the self-discernment takes now the shape neither of Involution, nor of the sides of either the Direct or the Inverted Relation, the Exponent of the explicitly involved Inverted Relation is really no longer the Exponent of a Quantitative Ratio. It explicates the notion of the Quantitative Relation with respect to that aspect which necessarily remains implicit, so long as the Exponent has the character of a *given* beyond of infinite approximation.

Seeing that immediate discussion of the Quantitative Relation puts past the element of infinite approximation, it reflects upon it only as already inwardized, with the consequence that the qualitative character, claimed for the Exponent only by virtue of the implied negation of the standpoint of Magnitude, remains still also submerged in its quantitative aspect.

The Quantitative Relation as such may also be said to concern simply the abstract possibility of treating a magnitude either as the Exponent of a Direct Ratio, or as a fixed Product, or finally, as subject to Involution or Evolution; and although the magnitude under treatment is thus equally invested with a qualitative character, this character appears as imposed on it *ab extra*, because the magnitude itself is indifferent to operations performed upon it. But since we are aware that the qualitative character of the number under treatment is not to be traced simply to an arbitrary resolve on our part to treat it as the Exponent of a ratio, but to its own Inwardness as the reached beyond of infinite approximation, whereby precisely its apparently purely quantitative aspect is just as much negated, we are now also at pains to bring this its own Inwardness explicitly to the front.

It will be seen that just as the concluding form of the Quantitative Relation as such implies a return to its immediate form (though, of course, the direct Ratio becomes then a ratio of Powers), so the full establishing of its qualitative character brings us back to the standpoint occupied in the first subdivision of the third main section of the dialectical whole of Quantity. We find ourselves once again entangled in the Vicious Circle between Quality and Quantity. The notion of the Quantitative Relation is before us as split up into its two aspects. On the one side, there is the implicitly involved Inverted Relation, whilst, on the other side, there is the explicitly involved Inverted Relation. That these Relations hang together is obvious from the fact that they concern the self-discernment of one and the same Quantum. But owing to the distinct import of this self-discernment the standpoint of the Quantitative Relation as such, i.e. its apparently still simply quantitative aspect, is just as much explicitly negated in the explicitly involved Inverted Relation. For this reason, too, we can really speak of an Exponent in connection with this Relation

only in a negative sense; in the same sense in which the term Number is retained, even when its use becomes irrational; as it does in application to the points, of which a line is conceived as consisting. Just as in this case a number becomes really only a symbol, and this its meaning is put on record in the term Irrational Number, so, in so far as the Sum of the two sides of the Square is given the form of the Exponent of a Direct Ratio, the purely symbolical nature of this its characterization is analogously put on record in the term Differential Coefficient or the Derived Function, the exponent of the implicitly involved Inverted Relation being called, by contrast, the Integral or the Original Function.

A. Differentiation.

The conception of the Differential Coefficient is simply the present restatement of the reached beyond, or limiting value of infinite approximation, as implied in the fundamental theorem of pure Analysis (see Chapter VII), with particular regard to the now to be embodied supersession of the standpoint of the Quantitative Relation as such. In so far, therefore, as the Differential Coefficient is usually given the form of the direct ratio dy/dx, it is self-understood —from our standpoint—that the Differentials dy and dx have not the nature of the sides of a Direct Ratio, as particular values, but that the very standpoint of the Quantitative Relation is in this respect to be treated as superseded.

In order to get a geometrical illustration of the meaning to be associated with the symbol dy/dx, let us consider that, when a straight line, cutting a curve in two points, is conceived as revolving about one of them, the length between the two intersecting points, or, in a word, the chord becomes more and more nearly equal to the arc. If the fixed point is identified with the origin of the co-ordinate axes, the tangent representing the ratio of the ordinate of the shifting point to its correspondingly varying abscissa diminishes and finally vanishes; when, that is, the revolving straight line just touches the curve, and in that case indicates the latter's slope at the point of coincidence of the two intersecting points, in exemplification of the geometrical

meaning of the primarily only trigonometrical tangent. Now, in so far as the geometrical tangent represents, with respect to the trigonometrical tangent, or the ratio between the ordinate and the abscissa of the shifting point, the reached beyond of infinite approximation, it embodies the conception of the Differential Coefficient and hence is symbolized also by dy/dx, along with its own equation, framed independently of its character, as the beyond of infinite approximation. The fact that dy/dx comes thus to be equated with *tan* a (when, that is, the angle made by the geometrical tangent with the x-axis is a), does not, however, mean that the differentials dy and dx stand for the co-ordinates y and x of a particular point lying on the geometrical tangent.

That the standpoint of the Quantitative Relation is to be treated in connection with the symbol dy/dx as superseded, is mathematically put on record in the very formula, generalising the just adopted mode in the determination of the thus symbolized value. When $f(x)$ represents the ordinate y of the shifting point on the curve as a function of the abscissa x, whilst δx refers to the difference between the abscissæ of two positions of the shifting point close to one another, and hence stands for a positive magnitude, however small, the formula becomes—

$$\frac{dy}{dx} = \frac{\text{Lt}}{\delta x = 0} \frac{f(x \pm \delta x) - f(x)}{\delta x}.$$

In so far as $f(x \pm \delta x) - f(x)$ obviously refers to the difference between the ordinates of the two positions of the shifting point on the curve, the expression reduces to δy, and the ratio $\delta y/\delta x$ represents the tangent of the angle made by the cord with the x-axis. The fact that, when this cord is conceived as rotating round one of the two intersecting points, it finally reaches the position corresponding to the geometrical tangent drawn to the curve at the fixed point, is put on record by the prefix $\frac{\text{Lt}}{\delta x = 0}$. Accordingly, whilst, on the one side, representing the limiting value of $\delta y/\delta x$, when δx is made indefinitely smaller and smaller, without yet reaching zero, the symbolical ratio dy/dx is, on the other side, meant to put on record the fact, that the finally vanished chord is also to be viewed

from the standpoint of its established unity with the curve, and hence not as simply shrunk to a mere point, in which all distinguishedness disappears, but to a point having the express significance of an element common both to the curve and to the straight line.

In so far, by the way, as dy/dx stands for $y/x = tan\ a$ when y and x are the rectangular co-ordinates of any point on the geometrical tangent applied to the curve $y = f(x)$ at the point $P(x_1, y_1)$, the co-ordinates of this point must equally satisfy the equation of the tangent. That is to say, along with $y_1 = f(x_1)$ we have now also $y_1 = x_1\ tan\ a$. Now, seeing that the geometrical tangent at the same time contains an infinite number of points which do not lie on the curve in question, the condition that it may pass through the point $P(x_1, y_1)$ is found by subtracting the equation $y_1 = x_1\ tan\ a$ from the equation $y = x\ tan\ a$, the resulting equation for the geometrical tangent being

$$y - y_1 = (x - x_1)\ tan\ a,$$

or rather, seeing that $tan\ a = dy_1/dx_1$,

$$y - y_1 = \frac{dy_1}{dx_1}(x - x_1),$$

in which case we are symbolizing, not only the identification of the origin with the point $P(x_1, y_1)$, but also the substitution of the geometrical tangent for the chord.

In this way we have before us at once also an elementary illustration of the great advantage secured by the invention of the differential Calculus in the treatment of problems whose solution before then caused no end of perplexities. As is seen, in order to find the equation of the tangent to a curve, all we have to do is to substitute, in the equation representing the tangent as passing through the point it has in common with the curve, for the tangent of the angle a which the straight line in question makes with the x-axis, the first differential coefficient of the ordinate of the curve. For instance, if it is the equation of the tangent to a parabola that is required, we are in a position to determine at once the value of $tan\ a$, by identifying it with the expression resulting for dy_1/dx_1 from the general equation for parabola $y^2 = 4ax$. That is to say, $tan\ a = 2a/y$, and the required equation of the tangent at the point $P(x_1, y_1)$, consequently,

$$y\,y_1 = 2a(x + x_1).$$

Or, seeing that a *normal* line is a perpendicular to the tangent at a given point on the curve, drawn to the x-axis; by substituting for *tan α* the negative value of the reciprocal of the first differential coefficient, we get the general equation of the normal line:—

$$y - y_1 = - \frac{dx_1}{dy_1}(x - x_1),$$

and hence for the parabola $2a(y - y_1) = y_1(x_1 - x)$.

Yet what is nowadays mathematically a mere platitude, was still to Descartes a problem of such magnitude that he felt justified to express himself as to the merits of his own solution thereof as follows: *j'ose dire que c'est ceci le problème le plus utile et le plus général, non seulement que je sache, mais même que j'aie jamais desiré de savoir en géometrie.* In order to vindicate the claim made on behalf of the simplicity of the differential Calculus, we have only to contrast the just indicated method of determining the equation of the tangent with Descartes' own tangential method. In Hegel's words (the already quoted expression of Descartes being a part of the present quotation from the "Wissenschaft der Logik," Berlin, 1841, Vol. I, p. 336-7): "He bases his solution on the analytic equation of the rectangled triangle formed by the ordinate of the point on the curve, through which the postulated straight line is to be drawn; then by this line itself, the normal, and finally, by the portion of the axis cut off by the ordinate and the normal, the subnormal. Now, from the known equation of a curve, the value of either the ordinate or the abscissa is substituted in that equation of the triangle, the result being an equation of the second degree (and Descartes shows how even curves, whose equations contain higher powers, reduce to this), in which there is found only *one* of the variables, in the square and the first degree, i.e. a quadratic equation and a so-called impure one. But he makes next the reflection that, if the point on the curve is taken in the sense of an intersecting point between the curve and the circle, the latter will intersect the curve additionally in another point, when we shall get for the thus arising unequal x two equations with the same constants and of the same form, or else only *one* equation with unequal values of x. But the equation becomes only one for the

one triangle, in which the hypotenuse is perpendicular to the curve or represents the normal; this case being conceived in such wise, that the two points of intersection between the curve and the circle are made to coincide, so that the circle is just touching the curve. Therewith, however, the circumstance of the unequal roots of x and y of the quadratic falls away. Seeing then that in a quadratic of two equal roots the coefficient of the term containing the unknown in its first degree is the double of the only root, we get finally an equation yielding the sought result. The procedure adopted is to be regarded as a genial artifice of a genuinely analytic head, in comparison with which the quite assertively assumed proportionality of the subtangent and ordinate with the presumably infinitely small increments of the abscissa and ordinate takes a back seat."

The concluding sentence of this quotation does not, of course, mean that Hegel disdained the advantage of a solution of Descartes' problem at the level of the differential Calculus, but that such a solution had at first only the appearance of a trick; until, that is, it began to dawn on mathematical consciousness that the contravention of ordinary rules of reckoning, implied in connection with the process of differentiation, does not affect the correctness of the result. As Hegel says, the first discoverers of the connection between the equation of a curve and the tangent applied to it "record their findings only in a quite empirical manner, without being able to account for the implied operation, with the consequence that this operation remained quite external. I content myself so far with Barrow's record—of him who was Newton's teacher. In his *Lect. Opt. et. Geom.*, where he deals with the problems of Higher Geometry according to the method of indivisibles (which method does not yet show forth the distinctive feature of the Calculus), he also puts his procedure of determining the tangent on record, 'because his friends begged him to do so' (lect. X). Those who wish to get proper conception of how this procedure is formulated in the sense of a purely external rule, must read it expounded in his own words. The exposition is typical of the style, in which the *regula de tri*, or, better still, the rule called 'casting out the niner' of the arts of reckoning, used to be presented in arithmetical textbooks. He draws those tiny lines which received subsequently, in

the characteristic triangle of a curve, the name of Increments, and then simply prescribes, in the guise of a mere rule, to reject as superfluous terms brought in by the expansion of the equations (as powers of those increments or as products: *etenim isti termini nihilum valebunt*), as well as those containing magnitudes to be found in the original equation (in anticipation of the subsequent deduction of the original equation from the one formed with the increments), and finally to substitute for the increment of the ordinate the ordinate itself, for that of the abscissa the subtangent. The procedure could hardly be stated, if one may be permitted to say so, in a more pedantic manner. That substitution concerns the assumption of that proportionality of the increments of the ordinate and the abscissa with the ordinate and the subtangent, on which rests the determinaton of the tangent in the ordinary differential method; in Barrow's rule this assumption appears in its quite naïve nakedness.

"A simple manner of determining the subtangent was found. The artifices of Roberval and Fermat are of a similar kind; the method of finding the greatest and smallest values which Roberval made his point of departure, rests on the same basis and procedure. It was the mathematical craze of those times to find so-called *methods*, i.e. rules of the stated kind and at the same time to make a mystery of them; which was not only easy, but in a sense even necessary, for that very reason for which it was easy—seeing, that is, that the discoverers discovered only an external rule of empirical origin, not a method, i.e. nothing derived from acknowledged principles. Leibnitz took over such so-called methods from his contemporaries, and so did Newton, who received them immediately from his teacher. By generalizing their form and applicableness, they opened up new paths to sciences, but so they felt at the same time the need of raising the adopted procedure above its primary import as a merely external rule and hence endeavoured to procure for it the still lacking justification.

"If we analyse the method more narrowly, we find the true procedure to be as follows. Firstly, the involved determinations (of course, of the variables) of an equation are reduced to their first functions. Thereby, however, the value of the equated terms gets changed and the equation is replaced by

a relation between the first function of the one variable to that of the other. Instead of $px = y^2$, there is $p/2y$, or instead of $2ax - x^2 = y^2$, there is $(a - x)/y$, i.e. that relation which subsequently came to be designated dx/dy. Now, whilst the equation represents a curve, the relation derived from it (at first according to a mere rule) is a linear expression involving certain lines: $p/2y$ or $(a - x)/y$ are themselves relations between straight lines of the curve, co-ordinates and parameters; but with this nothing is as yet known. The interest centres in finding that the derived relation applies to other lines connected with the curve; in finding its equality with another relation. And so there is, secondly, the question as to which are the straight lines, determined by the nature of the curve, standing in such a relation. But this is, precisely, what is known beforehand: that, namely, the relation in question is that of the ordinate to the subtangent.

"This had been found by the ancients in a clever geometrical manner. The more recent discoverers discovered merely the empirical procedure, how to adjust the equation of a curve in such wise that there results that ratio which is known in advance to be equal to a relation containing the line (here the subtangent) to be determined. But the planning and methodical carrying out of that adjustment of the equation—in Differentiation—led to the invention of imaginery increments to co-ordinates, as well as an imaginary *characteristic* triangle formed by those increments and an equally imaginary increment to the tangent, with a view to investing the proportionality of the ratio, resulting from the differentiation of the given equation, with the ratio formed by the ordinate and subtangent with the character of a fresh discovery, in supersession of the appearance of its having been accepted empirically on the strength of its old-established currency. Nevertheless, the adduced form of rules generally and unmistakably points to this old-established currency as the sole occasion and respective justification of the assumption of that characteristic triangle as well as of that proportionality.

"Lagrange openly rejected this simulation and struck the genuinely scientific course. His method brought into relief the point at issue by separating the two transitions, implied in the solution of the problem, and considering, as well as

justifying, each of them on its own account. The one part of this solution—if, for the closer indication of its course, we confine ourselves to the elementary problem of finding the subtangent—the theoretical or general part, i.e. the finding of the first function from the given equation of a curve, is settled by itself, the result being a linear relation, hence of straight lines belonging to the system determined by the curve. The other part of the solution concerns the finding of the lines in that system standing in the resulting relation. This is effected in a direct manner (Théorie des Fonct. Anal., Part II, Chap. II), i.e. without the characteristic triangle and thus also without assuming infinitely small arcs, ordinates and abscissæ, and letting the two latter stand for the sides of that relation in the sense of dy and dx, while at the same time substituting for them the ordinate and subtangent themselves. A line is properly determined only in so far as it forms the side of a triangle, this being equally the determination of a point. (Such, by the way, is the fundamental position of the Analytic Geometry with its co-ordinates, just as in Mechanics the same standpoint has its immediate embodiment in the Parallelogram of Forces, the great anxiety as to the proof of this latter principle being really uncalled for.) The subtangent is thus taken as the side of a triangle, whose further sides are the ordinate and the corresponding tangent. This latter has for its equation $p = aq$ ($+ b$ needs not be added because it is not necessary for the determination of a straight line; it is usually added only for the sake of popular universality). The determination of the ratio p/q falls into a, the coefficient of q, which, whilst being the respective first function of the equation, may be viewed also only as the essential determination of the straight line applied to the curve. But the first function of the equation of the curve stands equally for a straight line. Seeing, then, that the co-ordinate p of the first straight line and the co-ordinate y of the curve are assumed to be identical (the point at which that line which is tangent touches the curve being equally the starting-point of the straight line determined by the first function of the curve), the problem is to demonstrate that this second straight line coincides with the first. Now, the notorious increment turns up equally in Lagrange's demonstration of the coincidence, by virtue of which the straight line resulting from

the differentiation of the equation of a curve proves itself to be a tangent. But the way the increment is introduced for the just intimated purpose, and the development of the function in its terms must be carefully distinguished from its previous employment in connection with a differential equation or the characteristic triangle. Its employment is now quite in place, because it falls into the sphere of geometry, the geometrical determination of a tangent implying that between it and the curve, with which it has a point in common, there can pass no other straight line. This determination reduces the quality of a tangent or not-tangent to quantitative difference, that line being tangent which implies greater smallness with respect to the characteristic determination. This seemingly only relative smallness contains absolutely nothing empiricial, but is established qualitatively through the nature of the formula, when the difference of the moment, on which the magnitude to be compared depends, is a difference of powers. In that this difference becomes that of i and i^2, and i which is, after all, a positive magnitude, is to be conceived as a fraction, i^2 is *in and for itself* smaller than i, so that the very conception of an arbitrary magnitude in connection with i becomes superfluous and in fact out of place. Just for this reason the demonstration of the greater smallness has nothing to do with the infinitely small, which need not be dragged in here at all." [1]

"Were Hegel's mathematics correct?" asks Professor McTaggart in his "Commentary on Hegel's Logic," p. 43. "Was he right about the mathematics of his own time, and, if so, would he be right about the mathematics of the present day? To answer these questions requires a knowledge of mathematics which I am very far from possessing. Mr. Bertrand Russell—one of the few philosophers who are also mathematicians— says: 'In Hegel's day, the procedure of mathematicians was full of errors, which Hegel did not condemn as errors but welcomed as antinomies; the mathematicians, more patient than the philosophers, have removed the errors by careful detailed work on every doubtful point. A criticism of mathematics based on Hegel can, therefore, no longer be regarded as applicable to the existing state of the subject.'"

[1] "Wissenschaft der Logik," pp. 330-335.

But that Mr. Russell's comment is simply prompted by an excess of mathematical *amour-propre* becomes at once obvious when we inspect the argument justifying the identification of the function derived from the equation of a curve with the tangent of the angle made by the slope of any part of the curve in question with the x-axis, in a modern textbook of Higher Mathematics. J. W. Mellor, D.Sc., who is likely to be abreast of the most recent developments in the science of Mathematics, adduces the following version of this argument in the second edition of his textbook for students of Chemistry and Physics, published in 1905, p. 101-2:

"We sometimes define a tangent to a curve as a straight line which touches the curve at two coincident points. If in Fig. 24, P and Q are two points on a curve such that EP = NR = y; RQ = dy; OM = x; MN = PR = dx; the straight line PQ = ds. Otherwise the diagram explains itself.

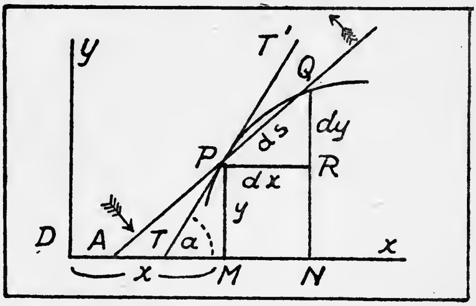


FIG. 24.

Now let the line APQ revolve about the point P. We have already shown, on page 15, that the chord PQ becomes more and more nearly equal to the arc PQ as Q approaches P; when Q coincides with P, the angle MTP = angle RPQ = a; dx, dy and ds are the sides of an infinitesimally small triangle with an angle at P equal to a: consequently

$$\frac{dy}{dx} = \tan a."$$

If we understand Hegel's version of Lagrange's method properly, the argument ought to be as follows:

If the straight line represented by the first function of the given equation is not a tangent, it cuts the curve in some other point, and the ordinate of any point on the implied chord will differ from the ordinate of the point on the curve corresponding to the same abscissa. For the abscissa

$x + \Delta x$ we shall have the ordinate $y + \Delta y$ for a point on the chord and $y + \delta y$ for the point on the curve. On passing to a new system of coordinate axes, having for its origin the point which is already given as common to the straight line and the curve, the difference between $y + \Delta y$ and $y + \delta y$ reduces, of course, simply to the difference between Δy and δy, which is a difference of powers, seeing that Δy is determined from a linear equation, and δy from a quadratic. But so we are negating the very possibility of another intersecting point between the straight line and the curve. Just because outside the point which the straight line, represented by the first function of the quadratic, *must* have in common with the curve represented by the quadratic, the ordinates Δy and δy *cannot possibly* become equal, the straight line in question is a tangent.

Of course, in order to realize the full force of the *cannot possibly*, we must put ourselves to the trouble of performing an act of pure thought. To try to read off the impossibility of another intersecting point between the straight line represented by the first function of the equation of a curve with this curve from a diagram, in which the straight line figures as a chord, naturally entangles one only in a hopeless vicious circle. In order to be conclusive, the proof must be viewed as an analytical version of the dialectical background of the Relation between the Original and the Derived Function, and hence must steer clear of the mist of the infinitely small which so conveniently, but for all that quite thoughtlessly and falsely, serves to hide the necessary contravention of acknowledged mathematical principles in the method of differentiation. It is the true meaning of quantitative Infinitude, i.e. that we thus reflect upon the Quality represented by a Quantity, that must be brought into relief instead. That the Derived Function is *qualitatively* different from the Original Function, is a proposition resting wholly on *à priori* ground; this is, precisely, why, apart from its origin in philosophical or pure thought, this proposition remains of empirical origin, entering mathematical consciousness in the guise of a statement of fact which is finally mathematized in the method of differentiation. Seeing, therefore, that this method is in its very nature only a mathematical device, having, apart from its usefulness in application to certain problems, no further significance, an attempt to prove its applicableness in the solution of a

particular problem can only amount to showing that the result of differentiation is of the same qualitative meaning as the result to be secured by the solution of the problem in question. Accordingly, seeing that the differentiation of a quadratic yields a linear equation, it is plain that the method of differentiation (which, by the way, may be equally devised from the relation between the Square and the Sum of its two roots in embodiment of the dialectically grasped Relation between the Original and the Derived Function and hence, to begin with, independently of its present geometrical application) may possibly simplify the determination of a tangent to a curve, because a tangent is equally a straight line. Any uncertainty is, of course, at once banished, if the tangent is determined independently by Descartes' tangential method. But if we insist on establishing the connection of the derived straight line with the tangent applied to the curve also directly from within the method of differentiation, then we must satisfy ourselves that this method admits, indeed, of being construed in this application into a circumstantial record of the simple definition of a tangent, as of a straight line touching a curve at two coincident points.

B. *Integration.*

In so far as the Relation between the Original and the Derived Function stands for the concluding phase of the dialectical whole of Quantity, when we find ourselves compelled to supersede our initial insistence on treating Quantity in the sense of Magnitude and hence cannot but acknowledge that the from the very first anticipated fundamental unity of Quality and Quantity is not simply due to an external bringing of them together, but to a dialectically necessary re-arising of the one in the other; we see that this Relation restates the vicious circle between them in explicit correspondence to the alternating determination between the infinitized Finite and the finitized Infinite. We cannot treat the two Functions isolatedly apart from one another, and hence, in so far as systematic completeness of our exposition requires that we should, all the same, distinguish between them, we must contrast them within their practically already established unity, in which respect we,

of course, reflect upon the double transition from the one into the other. Accordingly, after having dealt with the transition from the Original into the Derived Function, in a word, with *Differentiation*, we proceed to deal with the opposite sense of this transition, or with *Integration.*

"Differentiation and Integration are reciprocal operations in the same sense that multiplication is the inverse of division, addition of subtraction. Thus

$$a \times b \div b = a;\ a + b - b = a;\ \sqrt{a^2} = a;$$
$$d\int a \,.\, dx = a \,.\, dx;\ \int dx = x;$$

$3x^2dx$ is the differential of x^3, so is x^3 the integral of $3x^2dx$, The differentiation of an integral, or the integration of a differential always gives the original function. The signs of differentiation and of integration mutually cancel each other. The integral $\int f(x)dx$ is sometimes called an *anti-differential.* Integration reverses the operation of differentiation and restores the differentiated function to its original value, but with certain limitations to be indicated later on." [1]

Obviously, then, the operation of integration depends on the results of differentiation as stored up in memory or in a table containing sets of corresponding functions. Unless we know a function whose differentiation gives the function we should like to integrate, we are helpless, because the different artifices employed in integration are equally only of the nature of a device. Mr. Mellor remarks in a footnote that Clairaut, after having expressed his ideas of the moon's motion in the form of a set of complicated differential equations, left them in this incomplete stage with the invitation, "Now integrate them who can". Of course, if the mathematical expression of a particular fact leads to equations which cannot be integrated, the hypothesis suggesting itself in explanation of this fact must either be verified in some other way, or else be relegated to the great repository of unverified assumptions.

In so far as Integration, no less than Differentiation, has its real meaning only in application to a certain circle of problems involving transition of two Qualities into one another, and the equation resulting from the differentiation of, say, an equation of the second degree, representing a

[1] Mellor's "Higher Mathematics," p. 189.

line of the second order, i.e. a curve, eliminates the term unaffected by the variable, the purely formal term by term integration of the resulting equation of the first order is not sufficient to restore the differentiated function to its original value. There is an infinite number of expressions, differing only in the value of the constant term which, when differentiated, produce the same differential. Therefore, in stating the result of any integration, any possible constant term is provided for by the addition of an undetermined constant, called the *constant of integration* and usually represented by the letter C. That the geometrical significance of this constant is analogous to the constant added to the tangent form of the equation of the straight line, in reflection upon the possible ordinate of the intersection point of the line with the y-axis, is obvious. In so far, however, as the determination of the constant of integration is feasible only in cases when the nature of the problem under treatment permits of a determination of the value of the original function for particular values of the variable, there arises also a need to *eliminate* rather than to *evaluate* the constant C. The elimination is easily secured by confining the integration between definite limits which, of course, must not be confounded with the "limiting value" of a function. By finding the values of x corresponding to two different values of the independent variable (indicated by way of a superscript and subscript to the symbol "$\int$"), and substituting the two sets of results in the given equation, the constant can then be made to disappear by subtraction. In that case, we have before us a *Finite Definite Integral*, whose purely analytical definition is as fol l —м

"*Let $f(x)$ be a single valued, finite and continuous function of x in the interval $a \leqq x \leqq b$.*

"*Then the sum*

$$(x_1 - a)f(a) + (x_2 - x_1)f(x_1) \ . \ . \ . \ + (b - x_{n-1})f(x_{n-1}),$$

where the interval (ab) is divided up into the n intervals

$$(ax_1), (x_1x_2) \ . \ . \ . \ (x_{n-1}b),$$

has a limit as the number of intervals increases indefinitely, the intervals (x_rx_{r+1}) tending towards zero, following any law whatever, provided that their sum remains equal to $(b - a)$.

"This limit is written $\int_a^b f(x)dx$,

and it is called the Definite Integral of $f(x)$ in the interval (ab).

"In the usual geometrical proof of this theorem it is assumed that the function to be considered may be represented by an ordinary curve. The limit of this sum is then equal to the area enclosed between the curve, the axis of x, and the bounding ordinates. Assuming the existence of this limit, it is easy to prove, with the help of the First Theorem of Mean Value that the integral

$$\int_a^x f(x)dx$$

is a function of x whose differential coefficient is $f(x)$.

"In this way it is shown that

$$\int_a^b f(x)dx = \mathrm{F}(b) - \mathrm{F}(a), \text{ where } \mathrm{F}(x) = \int f(x)dx.$$

"The assumption that the function with which we deal may be represented by a curve is the one indispensable assumption of all mathematics that is not arithmetical, but in this case it is worth while to examine the question also from the arithmetical point of view, since the existence of the limit of the sum may be proved. Also the geometric proof restricts the theorem to continuous functions whose graphs may be drawn, whereas there are continuous functions with an infinite number of maxima and minima in a finite interval, and for these functions we could not draw a curve. Other such peculiar forms of continuous functions have also been discovered. It seems also advisable to have a proof of the existence of this limit other than the geometrical one, since the definition of an area involves the existence of the limit."[1]

In order to round off our attitude towards the standpoint of Pure Analysis, we proceed to examine this proof of what may be termed the fundamental theorem of Integration (*ib.* §§ 34, 35).

"Let us assume that the law of subdivision is such that we pass from subdivision to the following by cutting up its intervals into still smaller parts.

"Let $$x_{r+1} - x = \delta_{r+1},$$

and let M_{r+1} and m_{r+1} be the maximum and minimum values of $f(x)$ in this interval.

[1] Fourier's Series, § 33.

"Then the two sums $M_1\delta_1 + M_2\delta_2 \ . \ . \ . \ + M_n\delta_n$,
$m_1\delta_1 + m_2\delta_2 \ . \ . \ . \ + m_n\delta_n$

have each a limit; since the first continually decreases, and the second increases; and they are bounded, the first on the right, the second on the left; and the first is always greater than the second."

We are interrupting our quotation in order to draw attention to the fact that unless we associate with the terms of the two sums the conception of *area*, the trend of the proof remains meaningless.

"We now prove that the limits are the same. Since $f(x)$ is continuous, being given the positive quantity ϵ, as small as we please, there exists a positive quantity η_x such that

$$| f(x') - f(x) | < \epsilon \text{ for } | x' - x | < \eta_x.$$

"Let η be the smallest value of η_x for points in the given interval. Hence if we carry our subdivision so far that all the δ's are less than η, we shall have $(M_r - m_r) < \epsilon$, and the difference of the two sums will be less than

$$\epsilon(\delta_1 + \delta_2 \ . \ . \ . \ + \delta_n)$$

or

$$\epsilon(b - a).$$

"The difference of the two limits may thus be made smaller than any given number, and therefore the limits coincide. Let μ be their common value.

Then, since $$M_{r+1} \geqq f(x_r) \geqq m_{r+1},$$

$$M_{r+1}\delta_{r+1} \geqq f(x_2)\delta_{r+1} \geqq m_{r+1}\,\delta_{r+1}.$$

Therefore $$\text{Lt}\ \Sigma f(x_r)\delta_{r+1}$$

exists and is equal to μ."

As is seen, the proof is nothing more than a plain reflection upon the process of finding the area enclosed between the curve represented by $f(x)$, the axis of x and the bounding ordinates, the area being in this case conceived as subdivided or cut up into an indefinite number of small strips —called surface elements—perpendicular to the x-axis. M_{r+1} and m_{r+1} of the proof obviously refer to the bounding ordinates of each strip, but so that m_{r+1}, which at the same time is identical with M^{r+2}, is conceived as measured off on M_{r+1}, the area of each strip being implied as determined either by excess or by defect, according as it is symbolized either by $M_{r+1}\delta_{r+1}$ or by $m_{r+1}\delta_{r+1}$ respectively. The error

on either sides decreasing *pari passu* with the increasing narrowness of the strip, the area of this latter is ultimately symbolized by the ordinate itself in the explicit sense of an element of the total area A to be determined, i.e. by the expression

$$\mathrm{Lt}_{\delta x = 0} \frac{\delta A}{dx} = \frac{dA}{\delta x} = y$$

$$\therefore \quad dA = y dx.$$

$$\therefore \quad A = \int y \,.\, dx + C$$

$$\therefore \quad A = \int_a^b f(x) dx.$$

In so far, however, as the purely analytical proof aims at obliterating the elementary geometrical meaning of the Finite definite Integral, it insists on regarding the two sums, representing the total area A either by excess or by defect, in the sense of two aggregates determining an irrational number. That is to say, the proof is simply a more circumstantial restatement of the definition of the irrational number and for that reason equally includes that very thing which it aims at excluding, i.e. the geometrical meaning. But we have not yet completed our quotation:

"We have now to show that any other mode of subdivision will give the same result.

"Consider the system of intervals (ax_1), $(x_1 x_2) \ldots (x_{n-1} b)$, comprised in the law of subdivision which has given the limit μ. We shall suppose that all these subdivisions are less than η, as above.

"Take now another method of subdivision, and this a system of intervals (ay_1), $(y_1 y_2) \ldots (y_{n-1} b)$, in which the division has been carried so far that the greatest of the intervals of y are smaller than the smallest of the intervals of x. Between two consecutive values of x will be found at least one value of y. Writing in one line this set of points

$$a y_1 y_2 \ldots y_r, x_1 y_{r+1} \ldots y_s, x_2 y_{s+1} \ldots b,$$

let S_x be the sum relative to the x divisions and S_y to the y divisions.

"Then we can write S_y as follows:—

$$(y_1 - a)f(a) + (y_2 - y_1)f(y_1) \ldots + (x_1 - y_r)f(y_r)$$
$$+ (y_{r+1} - x_1)f(y_r) + \ldots + (x_2 - y_s)f(y_s) + \ldots$$

"In the first line $f(y_1), f(y_2) \ldots f(y_r)$ differ from $f(a)$ by less than ϵ, since $(x_1 - a) < \eta$, therefore we may put this line in the form

$$f(a)(x_1 - a) + R_1, \text{ where } | R_1 | < \epsilon(x_1 - a).$$

Similarly for the second line we may put

$$f(x_1)(x_2 - x_1) + R_2, \text{ where } | R_2 | < \epsilon(x_2 - x_1),$$

and so on.

"Thus $$| S_x - S_y | < \epsilon(b - a)$$

"But, as we have seen, S_x tends towards the limit μ; thus it follows that setting out from a certain time in our y subdivision, S_y differs from μ by as little as we please. That is to say, S_y has μ for its limit, as we wished to establish."

Again, the reference to the further subdivision of the intervals of x in the sense of *another* method of subdivision and the very need of this further subdivision remain a mystery until it occurs to us to interpret it geometrically. Namely, the intervals of x present themselves in that case as based on as small a difference between the corresponding ordinates as we please, so that if the curve represented by $f(x)$ slopes from left to right upwards only at a slow rate, the intervals of x may be of an appreciable size and hence invite further subdivision. That the increase of their number means then further reduction of the margin of error implied in the previous approximation to the total area is obvious. The proof concludes as follows:

"Instead of the sum

$$(x_1 - a)f(a) + (x_2 - x_1)f(x_1) \ldots + (b - x_{n-1})f(x_{n-1})$$

we may take

$$(x_1 - a)f(a + \theta_1\delta_1) + (x_2 - x_1)f(x_1 + \theta_2\delta_2) \ldots + (b - x_{n-1})f(x_{n-1} + \theta_n\delta_n)$$

where the δ's are taken arbitrarily between zero and unity. The limit of this sum will be the same as of the former. For, the intervals being supposed less than η, this sum differs from the first by a quantity less than $\epsilon(b - a)$, since

$$| f(a + \theta_1\delta_1) - f(a) | < \epsilon,$$
$$| f(x_1 + \theta_2\delta_2) - f(x_1) | < \epsilon, \text{ etc.}$$

"Therefore $\Sigma y_r\delta_r$, $\Sigma y\delta_r$, and $\Sigma y^{r+1}\delta_r$

have all the same limit, y_r, y_{r+1} being the values of $f(x)$ for x_r and x_{r+1}, and y being any intermediate value.

"When these sums have a limit the function is said to be integrable, and we have thus proved, independently of any assumption of the possibility of the graphical representation of the function, that all continuous functions are integrable."

We have said that, in submitting this proof to examination, we wished to round off our attitude towards the standpoint occupied by Pure Analysis. In this way we referred not only to an anticipation of a further confirmation of our previous inference that the standpoint in question comes under the head of the vicious circle between Quality and Quantity, but we also wished to bring out the fact that Pure Analysis does not explicitly concern itself with the method of differentiation or integration. This method is equally matter of the same vicious circle, but under the head of the Relation between the Original and the Derived Function. That is to say, the vicious circle is no longer to be reflected upon in its immediate connotation, as embodied in the fundamental theorem of Pure Analysis, when the transition of the infinite quantitative progress into its Limit, i.e. into its true meaning as Quality, although already resting on a dialectical foundation, is ushered on the scene only as a matter of immediate fact: at the concluding level of the dialetical whole of Quantity, the transition in question comes up for treatment in explicit reference to its concretion owing to the implied supersession of the intervening standpoint of the Quantitative Relation as such, when, of course, we are referring to the embodiment of the double transition between the Original and the Derived Function in Differentiation and Integration. In so far, then, as Pure Analysis at the same time equally presupposes familiarity with the Calculus, whilst, conformably with the standpoint which it occupies within the dialectical whole of Quantity, it yet is not concerned with the possibility of a mediation in connection with the transition of the infinite quantitative progress into its Limit, it follows that the method of differentiation or integration is *de facto*, if not in so many words, reflected upon in the sense of a mere device.

"A necessary and sufficient condition for the existence of a limit to the sequence u_1, u_2, u_3 . . . is that a finite positive

integer v exists such that $|u_{n+p} - u_n|$ becomes as small as we please for $n \geqq v$ and any positive integral value of p." Seeing that the existence of Limit is implied in the very conception of infinite approximation, there need be no difficulty whatever in connection with the transition from $\delta y/\delta x$ into dy/dx. Certainly, the simplest way to solve the difficulty in this connection is to deny its *raison d'être*. But, then, this has been done from the very first. Why did not the first discoverers of the Calculus put themselves to the trouble of justifying their methods? Because there did not seem to be any need of rigorous justifications; the method was a convenient device! In justifying it simply on utilitarian grounds, they anticipated at once Carnot's reflections on the metaphysics of the Calculus as well as the inference to be drawn from the fundamental theorem of Pure Analysis. But how is it then that mathematical *amour-propre* resents an open acknowledgment of this its own tacit admission? Obviously, because the very convenience or utility of the device in question proves its qualitative import and hence violently clashes with the possibly implied suggestion that it is to be degraded to the rank of a mere trick. It is precisely the further mathematization of its qualitative import that we had in our mind in stating that, at the concluding level of the dialectrical whole of Quantity, the transition from the infinite quantitative progress into its Limit comes up for treatment in explicit reference to its concretion, owing to the implied supersession of the intervening standpoint of the Quantitative Relation as such.

To recapitulate. The formula for the determination of the Derived Function may be viewed simply as a record of the dialectical inwardness of the infinite quantitative progress and hence equally of the fundamental theorem of Pure Analysis. But so the Derived Function is immediately correlated only with the standpoint of the Quantitative Relation as such. That is to say, its qualitativity remains still submerged in its quantitative aspect; viewed purely analytically, its determination is another restatement of the definition of the Irrational Number. In truth, however, the Relation between the Original and the Derived Function is meant to throw into relief the immediately submerged element of Quality. Differentiation or Integration concerns

the final stage in the full establishing of the return from Quantity into Quality. Consequently, the Derived Function stands for a Quantum whereby a qualitative Something or Other is what it is. Now, in so far as the real meaning of the Derived, and *eo ipso* of the Original, Function is to be identified with a spatial magnitude, it is *à priori* plain that if the Original Function refers to a curve, the Derived Function cannot but refer to an element of this curve, to a point which it has in common with a straight line and hence to its gradient; whilst if the Original Function stands for the area enclosed by the curve, the x-axis and two bounding ordinates, the Derived Function cannot but refer to the variable ordinate, but in the sense of an element of the area. In drawing these inferences, we are simply explicating the qualitative character of the contrast between the Original and the Derived Function within their fundamental unity, in simultaneous reflection upon the dialectic implied in the idea of Space. Yet even though these inferences prove themselves correct, we still continue to wonder why they should be so. The surviving perplexity is to be traced to the fact that, in relying upon the *à priori* aspect of our inferences, we immediately are inclined to perform the operation of differentiation or integration purely mechanically without any reflection upon its significance as a middle term of the qualitative relationship between the Original and the Derived Function. Viewed broadly, we have here an instance of the way in which the fundamentally negative unity of Thought and Being forces itself on our attention. The *à priori* determination of the real meaning of the two Functions is the aspect of Thought as still lacking its embodiment by way of a geometrical demonstration. Or, if we associate the aspect of Being with the mechanical performance of the operation effecting the transition from the one Function into the other, then the aspect of Thought refers to a grasp of that qualitative import of this operation, whereby it ceases to be a mere device. We may also simply say that, after having immediately emphasized the qualitative aspect of the Relation between the Original and the Derived Function, we are at once also reminded of the fact that this aspect still remains also bound up with its quantitative background And it is the bringing to the front of this quantitative

background, in mathematization of the qualitative import of the method of differentiation or integration, that present itself to us as the characteristic business of the Calculus, in distinction from the standpoint occupied by Pure Analysis. In Hegel's words:

"The differentiation of an equation of several variables yields the derived function or differential co-efficient, not as an equation, but in the form of a ratio; to find in the moments of the given subject-matter another ratio equal to that one which is the derived function, is the problem. By contrast, the object of the integral calculus is the relation between the original and the derived function in so far as this latter is given to begin with; so that the problem concerns then the meaning to be assigned to the expression resulting for the original function. Or rather, in that this meaning is announced in advance, in the statement of the problem, the integral calculus aims at demonstrating that the resulting original function *has* that meaning and which is the moment of the given subject-matter that the given derived function must then represent.

"Now, the ordinary method makes the matter easy for itself by means of the conception of the difference in the sense of the infinitely small. Thus for the quadrature of curves it assumes an infinitely small rectangle, a product of the ordinate into the element, i.e. an infinitely small particle of the abscissa, in the place of the trapezium, having for one of its sides the infinitely small arc opposite to that infinitely small particle of the abscissa; the product is then integrated in the sense that the integral represents the sum of the infinitely many trapezoidal-shaped strips or the area to be determined. Again, from an infinitely small element of an arc and the corresponding ordinate and abscissa, the ordinary method forms a rectangled triangle, in which the square of that arc-element is assumed to be equal to the sum of the squares on the two other infinitely small elements, and the length of the given portion of the curve is found by integrating the resulting equation for an arc-element.

"This procedure rests on the discovery that the area enclosed by curves, or the length of an arc, etc., stands to the function given by the equation of a curve in the relation of the so-called original function to its derivative. The

integral calculus aims precisely at establishing which part of a mathematical object represents the original function to a given derived function, whose meaning is embodied in another part of the same object. But now it was discovered that, when the function representing the ordinate of a curve stands for the derived function, the relatively original function gives the quantitative expression for the area of the curve cut off by this ordinate; or when it is a certain tangential determination that is identified with the derived function, the original function indicates the length of the arc belonging to the tangential determination in question, etc. In so far, then, as the method, employing the infinitely small, and mechanically operating on the same, simply avails itself of this discovery, it saves itself the trouble of an explicit vindication of what it so far presupposes as a matter of fact. In this respect, analytical genius has the merit of having found out from otherwise established results that certain and which sides of a mathematical object stand in the relation of the original to the derived function.

"Of these two functions, it is, then, the derived one that is given to begin with in the integral calculus, the original function having to be found through integration. But the derived function is not immediately given, and neither is it immediately at once plain which part or determination of the mathematical object under treatment is to be correlated with the derived function, so that the original function might stand for that other part or determination whose magnitude is to be determined. In at once conceiving certain parts of the object as infinitely small, or in the form of derived functions, determinable from the originally given equation of the object under treatment through differentiation (thus in connection with the rectification of curves we have infinitely small abscissæ and ordinates), the ordinary method identifies the derived function with such parts of the object as admit of an elementary connection with the part to be determined (in the given example, with the arc which is then equally conceived as infinitely small), owing to which elementary connection, i.e. one that is established by elementary mathematics, the required magnitude is easily found from the known magnitude of the derived function Thus, in connection with the rectification of curves, the

indicated three infinitely small elements are connected in the equation of the rectangled triangle; or seeing that area is represented arithmetically as a product of lines, quadrature of curves construes the ordinate and an element of the abscissa into factors of a product. Transition from such so-called elements of an area, of an arc, etc., to the magnitude of the total area, arc, etc., passes then only for an ascent from the infinite expression to the finite, to the *sum* of the infinitely many elements of which the postulated magnitude is conceived as consisting.

"It is, therefore, rather superficial to say that the integral calculus is merely the reverse of the differential calculus: the *real* interest of the integral calculus concerns almost exclusively the demonstration of the applied meaning of the relation between the original and the derived function.

"Neither in this part of the Calculus did Lagrange content himself with those direct assumptions in smooth disposal of the difficulties of its problems. It will contribute to the elucidation of the nature of the subject-matter equally to indicate the details of his procedure in a few examples. The professed object of his procedure is precisely to supply the required proof [or mathematization] of the fact that the relation between the original and the derived function obtains between particular aspects of a mathematical whole. Of course, this fact cannot be proved in a direct manner because of the nature of the relation itself in this sphere; the mathematical object connects curves with straight lines, linear dimensions and their functions with plane or surface dimensions and their functions, etc., so that the relation implies *qualitative* distinctions. For this reason, the required determination must be sought in the sense of a middle term between the greater and less. Recourse must be had again to the form of an increment with *plus* and *minus*, and the vigorous: *Dèvelopons* is again to the front. But, as has already been pointed out, Increment has here only an arithmetical, finite meaning. From the development of the condition that the magnitude to be determined is greater than the one easily determinable limit and smaller than the other, it is then deduced, e.g. that the function of the ordinate is the first function of the function of an area.

"In that Lagrange starts from the Archimedean principle, his demonstration of the rectification of curves is interesting

on account of this very endeavour to include Archimedean method within the own Inwardness of the standpoint occupied by the more recent analysis, thus enabling us to peep, as it were, into the inner element and the true meaning of a procedure otherwise carried on mechanically. The mode of procedure is necessarily analogous to the one just indicated. Archimedean principle (that the arc of a curve is greater than its chord and smaller than the sum of the two tangents drawn through the limiting points of the arc and contained between them and their point of intersection) gives no direct equation, but simply postulates progress from what is too great to what is too small in endless alternation; its translation, however, into the modern analytic form leads to an expression which is *per se* a simple fundamental equation. Whereas the formalism of the infinitely small ushers in at once the equation $ds^2 = dx^2 + dy^2$, Lagrangean exposition starts from the intimated basis and demonstrates that the length of an arc is the original function to a derived function whose characteristic term is itself a function derived from the function of the ordinate." [1]

C. Expansion of Functions and Transition into the objective Display of the Fundamental Unity of Quality and Quantity.

All that remains now to be done, in order to mathematize the fundamental unity of Quality and Quantity, in marked correspondence to the notion of the true Being as ushered on the scene in inwardization of the alternating determination of the finitized Infinite and the infinitized Finite, is to exemplify the double transition between the Original and the Derived Function in the sense of a self-discernment on the part of one and the same Function.

In this respect, therefore, we refer to a particular function, no longer simply with respect to its differentiation alone or its integration alone, but with respect to its treatment in both these senses at once; yet upon the proviso that the immediate import of these operations ceases to affect the meaning of the function under treatment. For, just because the true Being alters only in the sense of Alterableness: if the function now in question is to become a mathematical

[1] "Wissenshaft der Logik," Berlin, 1841, pp. 344-348.

symbol of the true Being, an Alteration of its meaning in the course of its treatment must, with respect to the final result, remain only matter of Alterableness, i.e. a matter of number.

Accordingly, the treatment of a particular function concerns us now simply in reference to an approximate evaluation of its full value. Of course, in marked reflection upon the element of Ideality, bound up with the standpoint of the true Being, it is at the same time to be taken for granted that empirical methods of evaluation are now out of the question, the implied devices of approximate calculations having to be viewed in the sense of an explicit correspondence, in the sphere of Quantity or Mathematics to *à priori* derivation, in the sphere of pure Thought, of what otherwise comes to notice only *à posteriori* or as a matter of fact.

In order to have some concrete image before the mind, let us say that our present task concerns the evaluation of the ratio between the perimeter of a circle and its diameter, symbolized by π. That in this respect we cannot have recourse to the formula used for the rectification of curves, i.e. to the formula

$$l = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \cdot dx$$

becomes plain on applying it to the evaluation of the perimeter of a circle. For, seeing that π has equally the significance of the rectified circumference, it may suggest itself at first sight to seek its evaluation with the help of the general formula for the rectification of curves. In that case, given the equation of the circle

$$x^2 + y^2 = r^2,$$

we have $$dy/dx = -x/y$$

so that $$\sqrt{1 + (dy/dx)^2} = r/\sqrt{r^2 - x^2}.$$

Therefore $l = r\int \frac{dx}{\sqrt{r^2 - x^2}} + C$, or in the form of a definite integral, when it is convenient to integrate between the limits r and O, so as to make l equivalent to the length of the arc of the first quadrant, we get

$$l = r\int_0^r \frac{dx}{\sqrt{r^2 - x^2}}.$$

On consulting now Tables of Integrals, we find that $1/\sqrt{r^2 - x^2}$ is the differential coefficient of the inverse trigonometrical function $\sin^{-1}\frac{x}{r}$, this being the symbol for the variable angle made by the radius r with the x-axis, i.e. for the angle whose sine is x/r, in so far as the magnitude of this angle is referred, not to the degree-minute-second system of units, but to the circular system having for its unit the length of the arc equivalent to the radius, or in a word, the radian. Accordingly, the length of the arc of the first quadrant is given by the equation:—

$$l = r\left(\sin^{-1} 1 - \sin^{-1}\frac{0}{r}\right)$$
$$= r \sin^{-1} 1 = \tfrac{1}{2}\pi r.$$
$$\therefore \text{ Whole perimeter} = 2\pi r.$$

As is seen, if we insist on symbolizing the perimeter by π, the result of an attempt to evaluate it with the help of the formula used for the rectification of curves simply brings to the front the from the very first obvious fact that π represents the length of the perimeter only upon the proviso that the diameter counts as a unit of measure. In so far, therefore, as the evaluation of π remains thus still dependent on empiricial measurement, and the idealization of this kind of evaluation falls equally within the province of Mathematics, we cannot but postulate that the sought mode of evaluation comes under the head of that treatment of a function which mathematizes the fundamental unity of Quality and Quantity in explicit correspondence to the establishment of the notion of the true Being in conclusion of the dialectical whole of Quality. And that in that case the function under treatment refers to an inverse trigonometrical function, immediately follows from the fact that the evaluation of π concerns it as an amount of radians.

Accordingly, we have before us the function, say, $\theta = \tan^{-1}x$, with a view to the determination of the treatment, to which it must be subjected, in order to acquire a form suitable for a direct evaluation of π. But we have already inferred that evaluation must now have the character of infinite approximation. Therefore, θ or $\tan^{-1}x$ stands for the Sum of an Infinite Series. In fact, we are aware that the treatment now in question refers to the

so-called *Expansion* of Functions into an Infinite Series. Nevertheless, since it is incumbent upon us to usher this conception upon the scene as a result of our own thought, and hence with a view to an explication of its dialectical Inwardness, we must make a point of determining the character of the Infinite Series, resulting from the Expansion of a function, independently of our familiarity with the theory of Infinite Series. In that case all we have to fall back upon is the fact that the Sum of an Infinite Series is now before us in the explicit sense of a negative unity of Differentiation and Integration. This means that every term of the Infinite Series formally shares the same character. The true Being communes in its otherwiseness purely with its own self, its otherwiseness being only a matter of that Alterableness by virtue of which it brings itself before itself in order to reflect upon its own self and thus acquire the character of Self-knowledge. The Infinite Series of Expansion is to be grasped with emphasis on the qualitative homogeneity of its terms; quite conformably with the fact that this Series is meant to represent the culminating point of the dialectical whole of Quantity. In so far, then, as the simplest numerical symbol of the qualitativity of a magnitude is its Involution, and the immediate restriction on the range of a Power is now removed, by virtue of the implied possibility of limitlessly successive differentiation or integration, the terms of the Infinite Series must be Powers of the same indefinite degree; or rather, in reflection upon the dualism of meaning belonging to the Inwardness of each of them, they have the nature of homogeneous Products formed by the powers of *two* variables. Inasmuch as the series of ascending powers of one of the variables can only concern the implied possibility of limitlessly successive integration, and the series of descending powers of the other variable, by contrast, the equally implied possibility of limitlessly successive differentiation, it becomes plain that the sought Infinite Series of Expansion is found by substituting for the two basis a and b in the *binomial series* two variables x and y, the resulting series

$$x + nx^{n-1}\frac{y}{1!} + n(n-1)x^{n-2}\frac{y^2}{2!} + n(n-1)(n-2)x^{n-3}\frac{y^3}{3!} + \ldots$$

or $f(x) + f'(x)\frac{y}{1!} + f''(x)\frac{y^2}{2!} + f'''(x)\frac{y^3}{3!} + \ \ldots$

being known under the name of *Taylor's series*, published in 1715.

In so far as it seems, at first sight, that the only way to expand a particular function of a *single* variable is to convert it, to begin with, into a sum (or difference) of *two* variables, it soon suggests itself on further reflection that the implied transformation of the single variable can be obviated by equating the other variable in Taylor's series to zero; namely, that variable which is implied in this series as successively differentiated. In this way, Taylor's series acquires the form known as *Maclaurin's series:*

$$u = f(y) = f(o) + f'(o)\frac{y}{1} + f''(o)\frac{y^2}{1 . 2} + f'''(o)\frac{y^3}{1 . 2 . 3} + \ . \ . \ . \ ,$$

where $f(o)$, $f'(o)$ . . . stands for $f(y)$ and its successive derivatives after the substitution of o for x. That is to say, in order to determine the value represented by them, we must first of all find the value corresponding to the successive differential coefficients of $f(y)$ and then equate y to zero.

As is seen, the treatment, to which we must subject the inverse trigonometrical function $\theta = \tan^{-1}x$, in order to give it a form suitable for the evaluation of θ in radians, consists, *firstly*, in the determination of the successive differential coefficients:

$$\begin{aligned} d\theta/dx &= (1 + x^2)^{-1} = 1 - x^2 + x^4 - x^6 + \ . \ . \ . \ , \\ d^2\theta/dx^2 &= -\ 2x + 4x^3 - 6x^5 + \ . \ . \ . \ , \\ d^3\theta/dx^3 &= -\ 2 + 3.4x^2 - 5.6x^4 + \ . \ . \ . \ , \\ d^4\theta/dx^4 &= 2.3.4x - 4.5.6x^3 + \ . \ . \ . \ , \\ d^5\theta/dx^5 &= 2.3.4 - 3.4.5.6x^2 + \ . \ . \ . \\ . \ . \ . \ . \ . \quad & \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ ; \end{aligned}$$

secondly, in equating x to zero, when

$$f(o) = o, f'(o) = 1, f''(o) = 0, f'''(o) = -\ 2\,!, f''''(o) = 0, f'''''(o) - 4\,! \ . \ . \ . \ ;$$

thirdly, in substituting the resulting values in Maclaurin's series:

$$\begin{aligned} \tan^{-1}x &= 0 + \frac{x}{1\,1} + 0 . \frac{x^2}{2\,!} + (-\ 2\,!)\frac{x^3}{3\,!} + 0 . \frac{x^4}{4\,!} + 4\,!\frac{x^5}{5\,!} + \ . \ . \ . \\ &= x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \ . \ . \ . \end{aligned}$$

$$\therefore \theta = \tan\theta - \frac{1}{3}\tan^3\theta + \frac{1}{5}\tan^5\theta - \frac{1}{7}\tan^7\theta + \ldots$$

Seeing, then, that $\tan 45° = 1$, it follows that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \frac{1}{13} - \ldots$$

and finally $\pi = 3{\cdot}1415926 \ldots$

By proceeding as before, we find that

$$\sin x = \frac{x}{1} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots,$$

or

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots;$$

or, with the help of Taylor's theorem:

$$\log(1+y) = y - \tfrac{1}{2}y^2 + \tfrac{1}{3}y^3 - \tfrac{1}{4}y^4 + \ldots$$
$$\log(1-y) = -(y + \tfrac{1}{2}y^2 + \tfrac{1}{3}y^3 + \tfrac{1}{4}y^4 + \ldots)$$
$$\therefore \log\frac{1+y}{1-y} = 2\left(y + \frac{y^3}{3} + \frac{y^5}{5} + \frac{y^7}{7} + \ldots\right).$$

As this series is slowly convergent when $y < 1$, on making

$$y = \frac{1}{2n+1} \text{ when } n > 1,$$

we get the rapidly converging series:

$$\log(n+1) = \log n + 2\left(\frac{1}{2n+1} + \frac{1}{3(2n+1)^3} + \ldots\right),$$

enabling us to compute the numerical value of $\log(n + 1)$ when the value of $\log n$ is known, and of course for $n = 1$, $\log n = 0$.

"The reader will, perhaps, have been impressed with the frequency with which experimental results are referred to a series formula of the type

$$y = A + Bx + Cx^2 + Dx^3 + \ldots,$$

in physical or chemical textbooks. For instance, I have counted over thirty examples in the first volume of Mendeléeff's "The Principles of Chemistry," and more than this number in Preston's "Theory of Heat". The formula has no theoretical significance whatever. It has none of the properties of a natural law. All it postulates is that the phenomena in question proceed continuously. In the ab-

sence of any knowledge as to the proper setting of the 'law' connecting two variables, this formula may be used to express the relation between the two phenomena to any required degree of approximation. It is only to be looked upon as an arbitrary device which is used for calculating corresponding values of the two variables when direct measurements have not been obtained."[1]

In so far as it is the purpose of Maclaurin's theorem to determine values of A, B, C, . . . which will make the Power Series used as approximation formula correct, the theorem may be equally deduced directly from the possibility of differentiating the Power Series in its interval of convergence as often as we desire. In that case, of course, we reflect upon the Power Series as given to begin with in the sense of an explicit record of what is from the very first implied in the conception of the Sum of an Infinite Series when the terms of this latter are conceived as no longer simply constant, but as finite, single valued and continuous functions of x.

" *When we speak of the Sum of the Infinite Series*

$$u_1(x) + u_2(x) + u_3(x) + \ . \ . \ .$$

it is to be understood:

"(1) *That we settle for what value of x we wish the sum of the series;*

"(2) *that we insert this value of x in the different terms of the series;*

"(3) *that we then find the sum $S_n(x)$ of the first n terms of the series; and*

"(4) *that we find the value of the limit of this sum as n increases indefinitely, keeping x all the time at the value settled upon.*

" With this understanding, the series

$$u_1(x) + u_2(x) + u_3(x) + \ . \ . \ .$$

is said to be convergent for the value x and to have $f(x)$ for its sum, when, this value of x having first been inserted in the different terms of the series, and the positive quantity ϵ having been chosen as small as we please, there shall exist a finite positive integer v, such that

$$| f(x) - S_n(x) | < \epsilon \text{ for } n \geqq v.$$

[1] Mellor's "Higher Mathematics," p. 273.

"Also in terms of the second criterion for convergence and with the same notation as above we must have

$$| S_{n+p}(x) - S_n(x) | < \epsilon \text{ for } n \geqq v, \quad p = 1, 2, 3 \ldots$$

"We shall find it convenient as before to write $R_n x$ for

$$f(x) - s_n(x),$$

and to call this the residue after n terms and also $pR_n(x)$ for

$$S_{n+p}(x) = S_n(x),$$

and to call this a partial residue."[1]

Now, the sum of the Infinite Series obviously involves double Limits:

"If the sum of the series is denoted by $f(x)$, then

$$f(x) = \underset{n=\infty}{Lt}(S_n(x)),$$

and the limit towards which $f(x)$ strives as x tends to the value x_0 is given by

$$\underset{x=x_0}{Lt}(f(x)) = \underset{x=x_0}{Lt}[\underset{n=\infty}{Lt}(S_n(x))] \quad . \quad . \quad . \quad (1)$$

"This is the ordinate of the point towards which we move as we proceed along the curve

$$y = f(x)$$

the abscissa getting nearer and nearer to x_0, but not quite reaching that value.

"The sum of the series for $x = x_0$ is given by

$$\underset{n=\infty}{Lt}[S_n(x_0)],$$

and since the sum of n continuous functions is itself a continuous function,

$$S_n(x_0) = \underset{x=x_0}{Lt}[S_n(x)].$$

"Therefore the sum of the series for $x = x_0$ may be written

$$\underset{n=\infty}{Lt}[\underset{x=x_0}{Lt}(S_n(x))] \quad . \quad . \quad . \quad (2)$$

These two expressions (1) and (2) need not be identical. They are so only when $f(x)$ is continuous at $x = x_0$, in which case

$$\underset{x=x_0}{Lt}(f(x)) = \text{the value of } f(x) \text{ for } x = x_0 = f(x_0).\text{"}$$[2]

[1] Fourier's Series, § 22.

[2] Ibid., § 24.

In so far, then, as $f(x)$ is itself either an original or a derived function, it does not follow without further examination that the Infinite Series represented by $f(x)$ admits of the term by term Differentiation or Integration. The possibility of a lack of identity between the two expressions (1) and (2) means equally that the result of the term by term Differentiation or Integration may differ from the result secured by the direct Differentiation or Integration of the sum itself. Namely, there enters the question of the independence of the order in which we proceed to the limits, when two limiting values are to be taken. When we differentiate or integrate the sum $f(x)$, we emphasize the limit involved in finding the sum of the series as given to begin with; whilst, in reference to the term by term differentiation or integration, the limit involved in finding the sum of the series is reflected upon only subsequently to the performance of the limiting operation upon its terms. But it is easily seen that the term by term Integration presupposes convergence for all values of x in the interval of integration, i.e. the so-called *Uniform* Convergence as against its aspect of infinite slowness in those cases when the number of terms needed for a particular approximation transcends every assignable limit, and when therefore the two criterea for Uniform Convergence

$$|\, R_n(x) \,| < \epsilon \text{ for } n \geqq v \text{ or } |\, pR_n(x) \,| < \epsilon \text{ for } n \geqq v \quad p = 1, 2 \ldots$$

fail to apply.

"*If the series of continuous functions*

$$u_1(x) + u_2(x) + \ldots$$

is uniformly convergent in the interval $a \leqq x \leqq b$ *and* $f(x)$ *is its sum, then*

$$\int_{x_0}^{x_1} f(x)dx = \int_{x_0}^{x_1} u_1(x)dx + \int_{x_0}^{x_1} u_2(x)dx + \ldots,$$

where x_0, x, *are any two numbers in the closed interval* $a \leqq x \leqq b$.

"Since the convergence is uniform, we may put

$$f(x) = S_n(x) + R_n(x),$$

where $\quad |\, R_n(x) \,| < \epsilon$, for $n \geqq v$,

for all values of x in the interval.

"Therefore $\int_{x_0}^{x_1} f(x)dx = \int_{x_0}^{x_1} S_n(x)dx + \int_{x_0}^{x_1} R_n(x)dx$

and $$\left| \int_{x_0}^{x_1} f(x)dx - \int_{x_0}^{x_1} S_n(x)dx \right| < \epsilon \mid x_1 - x_0 \mid ,$$

since $| R_n(x) | < \epsilon$ in the whole interval.

"Now, on the left-hand side, since n is finite, we may replace

$$\int_{x_0}^{x_1} S_n(x)dx$$

by $$\int_{x_0}^{x_1} u_1(x)dx + \ldots + \int_{x_0}^{x_1} u_n(x)dx.$$

It follows that

$$\left| \int_{x_0}^{x_1} f(x)dx - \int_{x_0}^{x_1} u_1(x)dx - \ldots - \int_{x_0}^{x_1} u_n(x)dx \right| < \epsilon \mid x_1 - x_0 \mid$$

for $n \geqq v$, provided that x_0, x, are any two numbers in the closed interval.

"Therefore the series of integrals

$$\int_{x_0}^{x_1} u_1(x)dx + \int_{x_0}^{x_1} u_2(x)dx + \ldots$$

converges and its sum is

$$\int_{x_0}^{x_1} f(x)dx.\text{"}$$ [1]

The term by term Differentiation equally concerns the Infinite Series only as immediately transformed into a simple finite sum. But whereas in connection with the term by term Integration the transformation invests the result with the property of Uniform Convergence, the result of term by term Differentiation loses the form of a definite Finite Integral and hence *possibly* also the property of Uniform Convergence, in which case, of course, the operation becomes inadmissible, because the resulting series of differential coefficients cannot be integrated term by term. Hence, the theorem :

"*If each term of the convergent series*

$$u_1(x) + u_2(x) + \ldots$$

[1] Fourier's Series, § 28.

has a differential coefficient which is continuous throughout the interval $a \leqq x \leqq b$, *and if the series of differential coefficients*

$$u'_1(x) + u'_2(x) + \ldots$$

converges uniformly through the interval, then $f(x)$ *has a differential coefficient at every point of the interval, and*

$$f'(x) = u'_1(x) + u'_2(x) + \ldots$$

Let $$\phi(x) = u'_1(x) + u'_2(x) + \ldots$$

Then since this series converges uniformly, we know that $\phi(x)$ is a continuous function of x and that we can integrate the series term by term in the interval.

"Therefore

$$\int_{x_0}^{x_1} \phi(x)dx = \int_{x_0}^{x_1} u'_1(x)dx + \int_{x_0}^{x_1} u'_2(x)dx + \ldots$$

$$= [u_1(x_1) - u_1(x_0)] + [u_2(x_1) - u_2(x_0)] + \ldots$$

and since $$u_1(x_1) + u_2(x_1) + \ldots,$$
$$u_1(x_0) + u_2(x_0) + \ldots$$

are convergent series whose sums are $f(x_1)$ and $f(x_0)$, we may write this result in the form

$$\int_{x_0}^{x_1} \phi(x)dx = f(x_1) - f(x_0).$$

Put $$x_0 = x$$
and $$x_1 = x + \delta x$$
Since $\phi(x)$ is continuous

$$\phi(\xi)dx = \int_x^{x+\delta x} \phi(x)dx,$$

where ϕ is some value of x between x and $x + \delta x$.

"Therefore $$\phi(\xi) = \frac{f(x + \delta x) - f(x)}{\delta x}$$

and $$\underset{\delta x = o}{\mathrm{L}t}\,\phi(\xi) = \underset{\delta x = o}{\mathrm{L}t}\,\frac{f(x + \delta x) - f(x)}{\delta x};$$

or in other words, $\phi(x) = f'(x)$." [1]

In so far as the Power Series admits both term by term integration and differentiation, its suitability for the representation of functions in Infinite Series leaps to the eye; for the successive approximation curves approach then towards coincidence with the curve of the function to be

[1] Fourier's Series, § 29.

represented, not only with regard to the ordinate, but also with regard to slope, curvature, etc.

Just as Maclaurin's theorem is a special case of Taylor's, so the latter may next be said to be a special case of *Lagrange's theorem*, and this latter, in turn, a special case of *Laplace's theorem*. Then there is also *Fourier's theorem*, determining the law for the expansion of any arbitrary function, not simply in a series of ascending powers of the independent variable, but in terms of sines or cosines of its multiples, the resulting series becoming, by virtue of the periodicity of its terms, an artificial way of representing the propagation or progression of any physical quality by a series of waves or vibrations; a mode of dealing with motion said to be more advantageous than any other form of mathematical reasoning.

That the law determining the expansion of functions in Infinite Series is not a law of Nature, follows after all from the fact that the fundamental unity of Quality and Quantity is thus reflected upon in anticipation of its own objective display which comes to concern us properly only in effected transition into the standpoint to be occupied by the next dialectical whole, in renewed restatement of all that precedes. We have quite satisfied ourselves that even though we at first assume that Quantity is not Quality, we are ultimately bound to admit that the two are truly only moments of one and the same fundamental unity of both. The dialectical proof of this their unity meets in advance the possible objection that the qualitative Quantum remains an ordinary Quantum. Although the dialectic of the Quantitative Relation may be said to concern only a symbolization of the dialectic of the qualitative distinction belonging to the Inwardness of the notion of Limit, this symbolization ends in superseding the standpoint of Magnitude as such, so that in completion of the original transition from Quality into Quantity, there is finally before us also the reverse transition from Quantity into Quality. By virtue of this double transition both Quantity and Quality prove themselves to be only transitory aspects of one and the same fundamental unity of both, which, in simultaneous reflection upon its simplest objective significance as a unity of Space and Time, we propose to make the object of our further thinking under the name of *Measurableness*.

PART III.

MEASURABLENESS.

THE NOTION OF MEASURE.

In so far as Measure currently connotes a Quantum such that it indicates the extent or dimensions of a thing, it must be understood that things concern us still only in the sense of something measurable; i.e. only to the extent, to which they illustrate the unity of Quality and Quantity. We must keep in mind that, whenever we refer to an existing fact, we necessarily imply much more than can be discussed at the reached level of our progress in knowing, and that the more thus implied is, so far, simply taken for granted. Seeing that Counting is taught to children primarily in its applied sense, a reference to things forces itself on attention already at the stage of Magnitude; but just as things are reflected upon at this stage only in the sense of something countable, so now that the Countable is to be envisaged in the more concrete sense of a Quantum whereby something material is what it is, things have significance for us only in so far as they are bound up with the objective display, or demonstrableness, of the fundamental unity of Quality and Quantity. What they are in themselves or essentially, does not concern us—yet.

Therefore, the transition from Quantity into our present standpoint must not be interpreted in the sense of a transformation of a Quantum into a tangible thing. We are simply meant to put past once for all the impression that the Qualitativity of a Quantum rests in the nature of a particular operation performed upon it. So far, we would remain fixed in the ordinary conception of Quantity, as what negates Quality; and the present transition into the objective display of the fundamental unity of Quality and Quantity would continue to pass for undeduced, illegitimate, and what not. There is no need to transform Quantity into a thing, because its completed dialectical whole brings

home to us that the need for such a transformation does not arise, for the simple reason that Quantity is *in truth* only an aspect of its fundamental unity with Quality. It was, precisely, only in order to vindicate this our initial assumption that we attempted to think Quantity in negation of Quality—with the result, as we know, that Quantity at once lost all meaning, or else had to be applied to things by converting *them* into something countable. The perplexity in connection with the present transition is of the same kind, as if one wondered how Time as such comes to be connected with a particular event in Space and Time. There is no Time as such, nor Space as such; what there is, to begin with, is a display of their fundamental unity in Matter and Motion.

Quite in accordance with the now established Ideality of the distinction between Quality and Quantity, our ordinary consciousness leaves it, as a rule, out of the question. To do away with the quantitative aspect of something measurable obviously amounts to doing away with its qualitative aspect. And, of course, neither can we succeed in eliminating its Qualitativity and still have something left over for counting, or rather measuring. But that distinction there all the same *is* between the two aspects, is easily demonstrable. For, suppose that one disdains to take it into consideration, on the ground of its complete Ideality: what happens?

A countryman, whilst going to market with his produce on a donkey, saw some fine willow twigs in the hedge, and, thinking it would not overload the donkey, he added a bundle to his burden. He shortly added another and again another small bundle, with the running comment that "a little more won't hurt," until, forsooth, the donkey collapsed. The countryman simply overlooked the fact that "a little more or less" is also of qualitative significance in connection with something measurable.

Similarly, if in answer to the question: Does a heap of sand cease to be a heap if one grain is taken away?—one answers unhesitatingly No, one overlooks the fact that even a single grain concerns the qualitative aspect of the given heap. If one grain more or less does not matter, the heap may be diminished grain by grain, until nothing is left.

Or, take the puzzle of Achilles and the tortoise. Achilles goes ten times as fast as the tortoise and the latter has ten feet start. When Achilles has gone ten feet, the tortoise is one foot in front of him; when Achilles has gone one foot further the tortoise is 0·1 ft. in front; when Achilles has gone 0·1 ft. further the tortoise is 0·01 ft. in front; and so on without end, with the consequence, it would seem, that Achilles will never overtake the tortoise, whilst yet we know for a fact that the tortoise will be left behind in no time. The puzzle is obviously based on the oversight of the qualitative aspect of the distance separating Achilles and the tortoise. The argument that the tortoise must forever keep a little, however little, in front of Achilles, simply refuses to take into consideration that the distance separating them is just as much a function of time and coverable by Achilles' speed of movement in a *finite* time. As divorced from this its real meaning, this distances comes to be reflected upon only with a view to illustrating the purely analytical conception of the Sum of an Infinite Series, the implied quantitative progress *ad infinitum* being due to the ensued relapse into the standpoint of a mere Quest of the negated moment of Quality.

Therefore, although Quality and Quantity, or in explicit reference to their simplest objective meaning, Space and Time, are now fused into one fundamental unity, Measurableness or Matter, this unity is not an inert unity. We cannot expect anything else, because the original Being necessarily contains Alterableness in its very Inwardness. The completion of the dialectical whole of Quantity has again brought us back to our beginning, but so that instead of the pure Being we have now Quality, and instead of Nothing, Quantity. Or, if the double transition of Quality and Quantity into one another is envisaged in present restatement of Origin and Decease, then the notion of Measure obviously stands for the present restatement of Presence as such, thus acquiring the significance of the original background, or Substrate of Alterableness. In fact, it is only with a view to the ensuing discussion of the import of the *measurable* Alterableness that we have been at pains to show that the material Something or Other must be credited with a distinguishable double aspect, even though the distinction is from the very first just as much to be

negated, in reflection upon its fundamental Ideality. Inasmuch, now, as Quality comes up for treatment before Quantity, we propose to call the immediate object of our further thinking *measurable Quality;* or seeing that we shall thus at the same time refer to the standpoint occupied in Physics, we may also say that measurable Alterableness will concern us immediately, in reference to its objective meaning, in the alternative sense of *Physical Quantity*, in which way we are at once putting also on record the present interchangeability of Quality and Quantity.

Section I.

THE MEASURABLE QUALITY OR PHYSICAL QUANTITY.

CHAPTER I.

THE DISPLAY OF THE DIRECT RELATION IN A SYSTEM OF UNITS.

SEEING that Measure is the Quantum through which a material Something or Other is what it is, the postulated Alterableness of such a something immediately concerns only its physical or quantitative aspect. The Alterableness is a matter of increasing or decreasing, but no longer in simple exemplification of the definition of Magnitude as such. Quantity is now before us only as an aspect of its fundamental unity with Quality. Consequently, the increasing or decreasing in connection with Physical Quantity must be envisaged in the sense of a display of the nature of the *qualitative* Quantum. And as the qualitative Quantum makes its nature explicit in the sense of a Quantitative Relation, measurable Alterableness of Physical Quantity is to be treated in present embodiment of the forms of the Quantitative Relation, and hence, first of all, in display of the Direct Relation.

In this respect, it is obvious that we reflect upon the objective meaning of the fundamental unity of Quality and Quantity, i.e. upon Matter as, to begin with, self-differentiated into an endless number of particular amounts of Masses, in concrete restatement of the original result of Self-differentiation. Instead of the present Ones having separately no *raison d'être* or Qualitativity, we have now physical Quantities or Bodies, the distinction between which remains immediately purely quantitative. That Quality is now no longer to be treated as conspicuous by its absence, follows

from the fact that, at the level of our present progress in knowing, the One Being refers to the Substrate of the objective display of the fundamental unity of Quality and Quantity. Just because idealization of the immediate meaninglessness of Quantity means its return into Quality, and the resulting unity of both acquires with respect to its objective display the sense of Matter, the original lack of Qualitativity in connection with the Ones of Self-differentiation means now that their distinctiveness, as separately existing physical Quantities, concerns us immediately only in the sense of Alterableness, the from the very first just as much anticipated Qualitativity of the distinction between them having to remain implicit until a reflection upon it becomes dialectically necessary.

Accordingly, we are immediately concerned with a measurable Quality in the sense of an amount of a given unity, called the *Standard.* That, in this respect, we do not refer to a principle of natural measures in the sense of a *natural* or *à priori* derivable Standard of things, is obvious. A standard measure may well have the nature of an original measure, i.e. of a measure whose fixation is not simply a matter of a capricious resolve on our part, as is the case in the C.G.S. (centimetre-gramme-second) system of units, but of a measure placed at our disposal by Nature herself, as is partly the case in the British (foot-grain-second) system of units. The dialectical element entering into the determination of naturally fixed Standards of measure, and the distinction itself between arbitrary and natural standards is, however, in connection with the present display of the Direct Relation, of no consequence, because, in this respect, a standard is postulated solely for the purpose of external comparison of the magnitude of things and its fixation, consequently, is meant to rest primarily on general agreement.

Since all physical quantities can be expressed in terms of Space, Time and Mass, the standard units of these three quantities are called *fundamental units ;* and any complete system based on these units is termed an *absolute system of units*. That is to say, such a system concerns only the so-called *Dimensions* of physical quantities.

"A 'physical quantity' may be defined as a measurement of some physical property: a length, a mass, or a time is a fundamental physical quantity, and most other physical

quantities are related to these. The 'dimensions of a physical quantity' denote an expression which shows how the quantity in question is related to the three fundamental units of length L, mass M, and time T.

"For instance, the area of a rectangle is found by multiplying its length by its breadth, i.e. by multiplying together two magnitudes each of which represents a certain number of units of length; the area of any other figure involves the mnltiplication together of two lengths and hence the dimensions of an area are given by $L \times L$ or L^2. Similar reasoning shows that the dimensions of a volume are given by L^3. To find the density of a substance, we must determine the volume of a weighed mass of that substance, and then divide the mass by the volume; hence the dimensions of a density are given by $M \div L^3$, or ML^{-3}."[1]

The expressions denoting dimensions of physical quantities are also called *dimensional equations.* Thus the equation for a velocity comes to be transformed from its general form $v = s/t$ into $[V] = [LT^{-1}]$, when it is understood that [V], [L] and [T] represents v, s, and t of the general equation as expressly reduced to unity. In this way, the dimensional equations serve as a record of the definitions of the so-called *derived* units in terms of the fundamental units.

Derived units should refer to anything and everything measurable, because at the present stage of our progress in knowing, represented by the standpoint of Physics, the whole Universe has interest for us only as an objective display of the fundamental unity of Quality and Quantity, and hence as Measurableness. As is seen, the main industry of experimental science to *explain* the whole Universe in terms of Space, Time and Mass, is a dialectically necessary stage in human ascent towards Self-knowledge. Even though in advance aware of the essential futility of such an *explanation*, we yet find at its back the nature of the true Being. For, the true Being is this: to create a distinction within its own Inwardness and hence to presuppose its own self in the guise of an Other to its own self, in order to reflect upon this its own Otherwiseness and, by means of this kind of self-communion, vindicate its own nature as the negative unity of Thought and Being; as the Creator of the Universe.

[1] "General Physics for Students," Edser, p. 6.

Just as, in order to vindicate the from the very first anticipated fundamental unity of Quality and Quantity, we had to begin by denying it and doing our best to affirm this negative attitude, not to give it up until compelled to do so by sheer dialectical necessity in the course of our progress in knowing; so now, too, quite comformably with our resolve to take nothing for granted, we must make common cause with that attitude towards the Universe which goes by the name of Materialism. The fact that we are committing ourselves to this standpoint out of sincerest love for truth may be recommended to the attention of those who are accustomed to view in it a sin against the Holy Ghost.

CHAPTER II.

THE DISPLAY OF THE INVERTED RELATION IN MECHANICAL MOVEMENT.

In display of the Inverted Relation, the measurable Quality or physical Quantity is to be grasped as a fixed product of two variable factors. But that the determination of the present meaning of these two factors must not yet call for an internal or chemical analysis of the nature of the physical Quantity, is obvious from our insight into the trend of the dialectical whole now under discussion. For the time being, we are concerned only with *physical* change, i.e. a change not involving a qualitative alteration in the nature of the measurable.

Although the moments of Quality and Quantity, of which the measurable is to be viewed as an immediate unity, refer objectively to its spatial and temporal existence, and Space and Time, consequently, are to be viewed as immediately belonging to its own Inwardness, the measurable Quality or physical Quantity nevertheless immediately presents itself as also indifferent to this its inwardness. To begin with, we have before us only a particular Quantum of Mass *in* Space and Time, from the standpoint of the current conception that its Presence or Absence makes no difference to the nature of Space and Time; as if, namely, these latter had no immanent connection with what is found to fill them. This current conception of Space and Time, as independently presupposed substrata of concrete existence, represents, of course, another instance of the proclivity of the loose thinking to hypostatize the two sides belonging to the own Inwardness of the true Being; but it is quite in place in connection with the attitude to be adopted towards the Physical Quantity at the outset of its dialectical whole in embodiment of the true Being. When, however, we advance to its grasp in display of the Inverted Relation, the initial insistence on its inert fixedness in Space and

Time is out of place. Instead of Indifference, there is then before us its Relation to Space and Time. And since we are thus immediately negating its immediate characterization as an *inert* body, the Relation now in question refers to it as a *moving* body, but upon the proviso that its Movement still lacks spontaneity; seeing that spontaneous or absolutely free movement can become an object of our consideration only in the concluding section of the dialectical whole of Measurableness, which, our insight into the systematically circumstantial character of our progress in knowing enables us to foresee, will deal with the now established unity of Quality and Quantity as concreted into the unity of *measurable* Quality and Quantity, in objectively adequate display of the nature of the true Being.

In so far, then, as instead of the simply inert Matter we have now before us Matter *and* Movement, our familiarity with facts makes plain to us that the display of the Inverted Relation undoubtedly refers to the effect produced by an inert body set in motion. For, under the same conditions, this effect is found to have the sense of a fixed product of Weight and Velocity; and that Weight and Velocity do represent the meaning acquired by Matter and Movement in express correlation with the two variable factors on the side of the ratio, may be vindicated by expanding the Relation between Matter and Movement into a dialectical whole, relevantly interpolated in the main thread of our discussion with a view to securing a more circumstantial treatment of our present subject-matter.

Therefore to begin with we reflect upon the possibility of envisaging Matter and Movement in complete independence of one another. But that such a possibility amounts to its own impossibility, is already plain from the preceding. We cannot conceive of Movement apart from Space and Time; and since the same applies to Matter, we contrast them only on the ground of their fundamental unity, and hence at once in the sense of Reality and Negation. But since in that case, too, we are simply anticipating the possibility of an abstractly distinguishable display of their fundamental unity, the Relation between them properly steps upon the scene only in the sense of Something or Other, when we find ourselves occuping the standpoint voiced in Newton's first Law of Motion:

"Every body continues in its state of rest or of uniform motion in a straight line, except so far as it may be compelled by impressed forces to change that state." [1]

It is plain that, so far, Matter and Movement remain only undistinguishably distinguished. Conformably with the immediate, still quite abstract import of the Relation between them, the distinction remains a matter of simple emphasis on either the one or the other aspect of their fundamental unity. On the one side, therefore, we get the idea of Matter as expressly negative of Movement, or, in a word, as *inert;* whilst on the other side, we have the idea of Movement as expressly negative of Matter, and hence of a Movement such that it is unlike any movement displayed in Nature.

"This law cannot be proved directly, for it is impossible to arrange that a body shall be acted upon by no forces; to do this we should have to experiment on a body at an infinite distance, not only from the earth, moon, sun and stars, but also from ourselves, so that none of these bodies could exert forces on it." [2]

At first sight, it might suggest itself to associate the utterly unimpeded *perpetuum mobile* with the absolutely free movement, as exemplified in connection with celestial bodies. But the fundamental unity of Matter and Movement is before us still only in the sense of a direct or inert unity, and hence as devoid of the principle of spontaneity. For this reason, the abstraction of Movement now in question cannot even be associated with conditionally free movement, as exemplified in the descent of a falling body; no, nor even with ordinary mechanical movement, for reasons implied in the quoted passage. Obviously, the postulated eternity, whether on the side of Matter or that of Movement, is only a way of stating that, so far, either is considered in pure self-reference and thereby converted into a non-existent abstraction. In other words, Newton's first law of motion is no law of nature. To continue the quotation:—

"It is more logical to consider Newton's first law as giving a definition of force—that which modifies, or tends to modify the state of rest or uniform motion of a body; in this case a body which continues at rest or in uniform motion is, by

[1] "General Physics," Edser, p. 12. [2] Ibid., p. 12.

definition, either acted upon by no force, or if forces act upon it they must neutralize each other's effects."

In this way, Matter and Movement cease to be contrasted purely abstractly, but so that the Relation between them is advanced to the stage of Something *and* an Other. Either is *in itself* identical with the other and only for that reason becomes a meaningless abstraction, if credited with a Being *sui generis* in absolute exclusion of the other. "Everyone" let us quote again from our text-book. p. 1, "has a more or less definite idea of the meaning to be attached to the word 'matter,' it is not easy, however, to frame a rigid definition of matter which will prove useful in our subsequent investigations. *The property which distinguishes matter from any other substance is that it has weight;* in other words, when matter is near to the earth (where alone we can *directly* investigate its properties), it is pulled downward towards the ground. The word *weight* is applied to the pull which the earth exerts on matter near its surface." As is seen, in attempting to exclude from Matter the possibility of Movement, or in other words, in crediting it with absolute Inertness, we do away with its very In-itself. Even if at rest, a body is heavy: and its Heaviness is nothing but a direct exemplification of its notion, as the unity of Space and Time in objective display of the fundamentally negative unity of Quality and Quantity. And inasmuch as Movement is only another aspect of the same fundamental unity, its exemplication is bound up with Matter.

Whether we speak of a state of rest or of movement, we refer to the Relation between Matter and Movement. A body set in motion must come to a stop, not merely out of consideration for the various obstacles impeding its motions such as friction, impact on other bodies, resistance of the air or water, but in reflection upon the very notion of mechanical movement. Just as the state of rest is qualified by a reference to Movement, so this latter is qualified by a reference to the state of rest. The two states of a body are, indeed, distinguishable only in the sense of the In-itself and Constitution: either is in itself what the other is, whilst yet at the same time remaining only externally related to it. Everything may be at rest or in a state of motion, but only so that the one state admits of taking the place of the

other. And in so far as this their exchangeability presupposes their fundamental unity in the sense of Limit, the magnitudes representing the body in reference to its two states are just as much to be viewed in the sense of two variable factors of the same fixed product. In that case, the magnitude representing the body in a state of rest obviously refers to the Weight which it has by virtue of its fundamental property; and that the magnitude of the other factor can only refer to the Velocity with which the body moves, is obvious.

In so far as the Effect or Work produced by a body set in motion is a display of the Force originating mechanical Movement, and the conception of Force arises through hypostatization of material Heaviness, the general equation for Work is $w = fs$, unit work being done when unit force acts through unit distance, on the understanding that unit force produces or creates unit acceleration in unit mass and that a body has unit acceleration when its velocity changes by unity in unit time.

That forces should be measured by their capacities to set matter in motion, is put on record in *Newton's second law of Motion:*

"*Change of motion is proportional to the impressed force, and takes place in the direction of the straight line in which the force acts.*

"In applying this law, it must be understood that the conditions under which the forces act must be similar: that is, they must set equal masses of matter in motion, and must act for equal times; in this case the forces are defined as being proportional to the changes of motion (more strictly the changes of velocity) produced. Thus, if a number of different forces successively act on the same mass of matter, which in each case is at rest to start with, then the velocities produced after equal times serve to measure the magnitude of the forces." [1]

Because the velocity v is thus reflected upon as directly proportional to the impressed force f and to the time t during which the force acts; and when a given force acts for a given time, the velocity produced is at the same time also inversely proportional to the mass m of matter set in motion, the expression for the fundamental quantitative

[1] "General Physics," Edser, p. 12.

laws of dynamics is $v \propto \frac{ft}{m}$; an expression reducing to equality when its terms are each equal to unity. In that case, it is implied that if unit force acts for unit time and sets unit mass of matter in motion, then unit velocity will be generated.

In the C.G.S. system, the unit force is called the *Dyne*, i.e. a force producing a velocity of one centimetre per second when acting for one second on one gram of matter, whilst in the British or foot-pound-second system we have the *Poundal.*

In so far as the product $m.v$ is also called the *momentum*, the force acting on a body is represented by the rate at which its momentum is increasing, whilst the force per unit mass stands for the increase of velocity per unit time, i.e. for acceleration.

CHAPTER III.

THE DISPLAY OF THE INVOLVED INVERTED RELATION IN THE LAW OF DESCENT OF A FALLING BODY.

THAT Newton's second law of motion constitutes in the dialectical whole of the measurable Quality or physical Quantity the transition from the Display of the Inverted Relation into that of the Involved Inverted Relation, comes home to us when we set ourselves the following problem:

"*A body of mass m, initially at rest, is acted upon by a constant force f; how far will the body move in t seconds?*

"The velocity at any instant is proportional to the time during which the force has acted; thus the velocity increases uniformly from zero at the start, to the final value $(ft)/m$, and the average velocity of the body during these t seconds is $(ft)/2m$. If s represents the distance through which the body travels in the t seconds,

$$s = average\ velocity \times time = \frac{ft}{2m} \times t = \frac{1}{2}\frac{f}{m}t^2."$$ [1]

In so far as this result represents equally the space traversed by a freely falling body, when, that is, the expression $f/2m$ stands for the space traversed in the first second, it suggests itself at first sight to identify Newton's second law of motion with the law of descent of a freely falling body. But what has been said about his first law, must be repeated even now. Neither of these laws concerns naturally existing motion and, therefore, has really no claim to be called a *law of motion*. This does not, of course, dispute the logically necessary import of their content.

In so far as Matter and Movement are necessarily correlated, first of all, only in the sense of Something or Other, and hence only in anticipation of a real relationship between them, it is in place that the record of such an

[1] "General Physics," Edser, p. 20.

abstract relation should bring home to us that, unless we postulate a middle term, Matter and Movement remain empty abstractions. Whether Newton worded his so-called first law of motion with this intent, or whether he himself believed in the eternity of Matter, or the *perpetuum mobile* implied in connection with the abstraction of Movement, is of no consequence, because one does not necessarily fathom all one says, any more than one always manages to say exactly all one knows. For us, it is sufficient to find a textbook on general Physics declaring that "it is more logical to consider Newton's first law as giving a definition of force—that which modifies or tends to modify, the state of rest or uniform motion of a body". And we are equally in full agreement with the further remark that "Newton's second law of motion indicates the manner in which forces are to be measured". It so happens—in fact, how could it be otherwise?—that we are thus in a position to measure the space traversed by a freely falling body, but that a method of measuring forces cannot be converted into a principle of the law of descent of a falling body is obvious—at any rate, to us, whatever is the verdict of physicists blinded by *amour propre.*

We are assigning to Newton's explication of Galileo's method of measuring forces the significance of a transition from the standpoint of the mechanically originated and sustained Movement with uniform velocity into the standpoint of the conditionally already free or spontaneously manifested Movement, because in the dialectical whole of Matter and Movement (which, it is now easy to see, amounts to a dialectically pertinent recapitulation of the dialectical whole of Quality under the head of the measurable Quality, even though, in reflection upon the original meaning of this Quality, in record of the true meaning of the qualitative Quantum, its dialectical whole presents itself at first sight in the light of a higher restatement of the forms of the Quantitative Relation and of its concluding transition into the next higher standpoint which is now to be identified with the Measurable Quantity or chemical Quality), the method in question has for its background the dialectic of Limit in transition into the standpoint of Finitude.

In so far as Limit at once includes and excludes Something and an Other, it has the sense of Something or

Other only as a middle term at once joining and disjoining two qualitatively distinct Somethings. We are thus returning to the original meaning of Something or Other, whilst at the same time putting on record our fuller grasp of this meaning. Thus, if the original meaning refers simply to a heavy body in a state of rest, the fuller grasp of this meaning at the stage of Limit refers to its Heaviness in the explicit sense of a physical Quantity, as Weight; and it is then also understood that the way to determine the Weight of a body is to invest it with the sense of a product of two variable factors, as is the case in connection with the lever. That is to say, instead of the Effect produced by a body set in motion, we have thus only an equilibrium of forces, because this is the way in which the Inwardness of the notion of Limit comes to the front in connection with a body in a state of rest. A body is at rest either because it is acted upon by no force or else because the forces acting upon it are together equivalent to no force, i.e. are in equilibrium. But the original meaning of Something or Other just as much refers to Movement, and in so far as the fuller grasp of this meaning in connection with a heavy body concerns its uniform Velocity in alternative exemplification of the fixed product of two variable factors, it is obvious that the unit weight whose distance from the fulcrum of the lever measures the Weight of the body suspended on the other side of the fulcrum at a unit distance, refers them to a unit velocity, and that the other factor represents the number of times the chosen unit velocity must be increased in order to become equivalent to the Velocity to be measured. That, however, the implied amount must be qualified in reference to the element of time, follows from the qualitative contrast between the two exemplifications of the notion of Limit. It is not a particular distance that is to be measured, but a given Velocity of mechanical movement; in so far, then, as the Velocity is to be traced to the action of a force on a primarily resting body, and the unit velocity, consequently, to the action of a unit force during a unit time, the magnitude of the Velocity to be measured acquires the sense of a sum of unit forces acting together during a unit time, or else of a sum of the effects of the shocks which the body set in motion keeps receiving every unit time by the unit force,

in which case the unit velocity comes to be reflected upon in the sense of Acceleration. But, then, the unit velocity acquires this significance only on the understanding that a body once set in motion becomes a *perpetuum mobile*, under the reservations set forth by the so-called first law of motion. It follows, therefore, that Acceleration refers to no fact in Nature. That is to say, there is no natural movement whose velocity increases in the manner indicated by Newton's second law of motion, and neither is it, therefore, possible to demonstrate this law. The standpoint occupied by it is still the standpoint of the first law, which, it is admitted by physicists themselves, cannot be proved directly.

But, then, what about Attwood's machine? "Two equal masses of matter A and B are attached, one at each end of a long cord of fine silk. This cord passes round the grooved edge of a wheel, the axle of which is supported so that rotation can take place with as little friction as possible. The two masses A and B being equal, gravity exerts equal forces on them; and as one cannot move downwards without pulling the other upwards, the effect of the pull of gravity on one is neutralized by the equal pull of gravity on the other. In consonance with Newton's first law of motion, the two masses, if originally at rest, remain in that condition; when they are set in motion by any force, they continue to move with uniform velocity after that force ceases to act." [1]

True, in this way it is proved that if a given force sets a given mass of matter in motion, the velocity increases uniformily with the time; but it is to be noticed that the given force, setting a given mass of matter in motion, is in this case no longer external to the body in motion, but is simply identified with its own weight. That is to say, the experiment does not exemplify the standpoint of the mechanically originated and sustained movement, but simply vindicates our own anticipation that, in so far as Newton's second law of motion is really concerned only with a method of measuring forces by their capacities to set bodies in motion, its exemplification brings to the front the complementary aspects of the fundamental unity of Matter and Movement when the dialectical exposition of this unity is advanced to

[1] "General Physics," Edser, p. 13.

the stage at which Limit comes up for treatment with respect to its transition into the standpoint of Finitude. As we know, the two complementary aspects concern an exemplification of the notion of Limit in connection with either of the two moments belonging to its own Inwardness, i.e. on the one side in connection with the Weight, and on the other side in connection with the uniform Velocity of a heavy body. As qualitatively distinguished moments in one fundamental unity, the two exemplifications at once implicate and negate one another. That is to say, by virtue of its own Weight, a body is capable of the kind of movement postulated in the alternative exemplification of its own Inwardness, but in that case there comes to view the simultaneously implied element of Negation in the ensued conversion of mechanical movement into the conditionally free movement; and that the mechanically moved body just as much implicates Weight only negatively, is obvious from the relation of its own weight with the velocity of its movement coming to the front in the fact that when a given force acts for a given time, the velocity produced is inversely proportional to the mass of matter set in motion. Attwood's machine displays the from the very first presupposed unity of Matter and Movement in negation of the equilibrium of forces defining the state of rest at the level of Limit and in this way just as much negates the alternative exemplification of the Limit in connection with the mechanically originated and sustained movement. In so far as the state of rest is negated in negation of an external source of movement, the standpoint of Limit is advanced to the stage of Finitude, at which the fundamental Ideality of the distinction between Matter and Movement steps upon the scene in the sense of a freely falling body.

"The transition of Ideality into Reality has its explicit demonstration in the well-known mechanical appearances that, namely, Ideality may be put in the place of Reality and vice versa; and it is only due to pictorial and analytical thoughtlessness, if the Identity of both is not at once inferred from their interchangeability. For instance, in connection with the lever, Distance takes the place of Mass and vice versa, and a Quantum in idealistic reference produces the same effect as the corresponding moment of Reality. In connection with the magnitude of movement, too, Mass may

be replaced by Velocity, representing simply a quantitative relation of Space and Time; or else the result is the same when Mass is increased and Velocity proportionately decreased. A brick does not kill a man just because it is a brick, but brings such a result about only by virtue of the Velocity acquired by it; that is to say, man is killed by Space and Time. It is only because the conception of Force is invested by the intellect with the sense of a first principle, that one does not attempt to fathom its own meaning. Even so, however, it is granted that the Effect of Force is something real, appealing to sense, and that Force is constituted by that very thing which comes to view in its Expression and that this its realistic Expression acquires its Force through the relation between its idealistic moments, Space and Time. But in keeping with the thoughtless character of intellectual reflection, the so-called Forces are viewed as *implanted* in Matter, that is to say, as what is given to begin with externally to Matter, with the consequence that the very identity of Space and Time which is at the back of the conception of Force and which in truth constitutes the Essence of Matter, is, after all, assumed to be something alien to it; something accidentally introduced from outside."[1]

It is plain, then, why the analytical explanation of the conditionally free movement—*conditionally* free, because its demonstration is possible only under the condition that a body is first of all raised to the height from which it is falling—fails to satisfy us. In advancing the dialectical whole of Matter and Movement to the stage of Finitude, we are concerned with an explication of the fundamental property of Matter in supersession of the immediately arisen appearance of Asunderness between its two aspects. The analytical explanation, however, continues to maintain the immediate attitude to Matter in connection with the very demonstration of the ultimate untenability of such an attitude. A body is said to be in a state of rest or uniform velocity except in so far as it is acted upon by a force. But, then, what force acts upon a body falling from a height? Such a body is neither in a state of rest nor does it move with uniform velocity; hence, it ought to be acted upon by a force, when yet we fail to discover any trace of it! But

[1] Hegel's "Encyklopädie," § 261.

what about the force of gravity? Of course, we have heard of Newton's famous discovery of this force, only we fail to discover it for ourselves; in fact, it does not really occur to us to insist on its discovery, because *in truth* there is no such force *to act upon a body.* If the falling movement of a body is due to the action of a force upon it, the body is stripped of its fundamental property, because, in reflection upon the simultaneous identification of gravity with the effect due to material heaviness, it is the own Weight of the body that is then acting upon it. That, however, a body cannot be separated from its own Weight for the purpose of discovering, or rather verifying Newton's discovery of, the force of gravity, is *to us* obvious. What is wrong with the simple statement that a body falls because it is heavy? What need is there for discovering what is not there, in order, presumably to explain, but really to explain away, what there is? We have here an instance of the thoughtlessness characteristic of analytical explanations of simple facts. In so far as such explanations are not a record of speculative or pure thought, they simply break up a given whole into empty abstractions, taking the place of first principles, and thus degenerate into fanciful metaphysics, at variance at once with actuality and philosophy. At the back of the discovered force of gravity, there is simply the thoughtless prejudice that every movement must be due to the action of an external force; a prejudice embodied in Newton's first law of motion.

"The laws of the relatively free Movement concern the Magnitude of the Time elapsed and of the Space traversed; they are immortal discoveries redounding to the greatest honour of the analysis pursued by intellect. The next step concerns their proof independently of empiricism; and this proof has been equally supplied by mathematical Mechanics, so that even the science basing itself on facts is not satisfied with their simple demonstration. The *à priori* proof in question rests on the fact that the Velocity of the falling movement is uniformly accelerated, but the moments of the mathematical formula are at the same time converted into *physical* Forces: into an accelerating force, imparting one and the same impulse in each unit of Time, and into a force of Inertia, perpetuating the velocity acquired—determinations utterly devoid of empirical sanction and just as much

repudiated by the Notion. The determination of Magnitude, containing in this respect a relation of Powers, is reduced to the form of a sum of two independent elements, and the qualitative, notionally determined element is in this way killed. It is presumably from the law as thus proved that it follows 'that in the uniformly accelerated movement velocities are proportional to times'. As a matter of fact, however, this statement is nothing than the quite simple definition of the uniformly accelerated movement itself. The simply *uniform* movement implies proportionality between the spaces traversed and the times: the *accelerated* movement is that one in which velocity increases in every successive unit time; hence the *uniformly accelerated* movement stands for a movement in which velocities are proportional to times elapsed. Therefore V/t, i.e. s/t^2. This is the simple, true proof. V stands for velocity as such, as still undetermined, and hence just as much for the abstract, i.e. purely uniform velocity. The difficulty in connection with the proof in question lies in this, that V stands, to begin with, for the indeterminate velocity as such, but in the mathematical formula presents itself as s/t, i.e. acquires the sense of the purely uniform velocity. In so far, then, as the proof hangs together with the mathematical exposition, it seeks to justify in an indirect manner the necessity of treating V in the sense of s/t, with a view to the transition into s/t^2. The statement that velocity is proportional to time, concerns primarily velocity as such; and it is only because the velocity is superfluously mathematized into s/t, that the proof introduces the force of Inertia, in order to explain the presence of the purely uniform velocity. But as velocity is from the very first implied as proportional to time, it is uniform only as uniformly accelerated, as s/t^2, so that the concern with s/t is here out of place.

"As against the abstract uniform velocity of the dead, externally determined mechanism, the law of descent of a falling body is a *free* law of Nature, i.e. a law implying an element hanging together with the Notion of the body. . . . This connection obviously lies in this, that, as it is the Notion that here arrives at its determination, the notional determinations of Time and Space become free against one another; that is to say, they are quantitatively correlated according to their notional determinations. Seeing now

that Time is the moment of Negation, of Being-for-self, the principle of Oneness, its magnitude (any empirical number) is to be taken, in the relation to Space, as the Unit or as Denominator. Space on the contrary is the Out-of-itself-ness, and that of no other magnitude than the one belonging to Time; for the velocity of this free movement means precisely that Space and Time are not contrasted externally or contingently, but that they blend in one determination. But as against the form assigned to Time, as Unity, the form to be assigned to Space, in express elimination of any other determination, becomes the Square—Magnitude, as what steps out of itself, raises itself into the second dimension, hence spreads out, but only by virtue of its own character, which alone sets the limit to its extension and thereby converts its becoming otherwise into pure self-reference.

"This is the proof of the law of descent of a falling body, as derived from the Notion of the thing. The Relation of Powers is essentially a qualitative relation and the only relation belonging to the Notion." [1]

[1] "Enc.," § 267.

CHAPTER IV.

TRANSITION INTO THE MEASURABLE QUANTITY:

A. In Restatement of the Original Transition into the Measurable Quality.

AFTER having completed our treatment of the measurable Quality as a display of the forms of the Quantitative Relation as such, it would seem that its concluding form, the just derived mathematical expression of the law of descent of a falling body, should become the object of our further discussion in higher restatement of the explicitly involved Inverted Relation. But it is to be realized that the present restatement of the Relation between the Original and the Derived Function does not concern its formal import, but its dialectical significance, as the conclusive vindication of the fact that, even though the Involved Inverted Relation appears to rest only on Quantity, it nevertheless just as much already is pregnant with a wholly qualitative meaning in supersession of the standpoint of the Quantitative Relation as such, and hence is representative of the fundamental unity of Quality and Quantity in transition into Measurableness. That is to say, we are now concerned only with the vindication that, even though the law of descent of a falling body presents itself at first sight as what involves only the quantitative aspect of the measurable Quality, when, that is, this latter is emphasized in the sense of physical Quantity, it nevertheless does also already implicate its qualitative aspect in transition into the standpoint of the chemical Quality; and that for the purpose of this vindication, the formal possibility of treating the formula of the law in question as an original function is irrelevant, is obvious, since in this respect we would only bring home to ourselves that the already discussed analytical interpretation of the conditionally free movement does, indeed, hang

together with mathematical exposition rather than with the actual nature of the measurable. In so far as Hegel's exposition of the Being-for-self in Measure ("Wissenshaft der Logik," I., p. 402-406) entirely coincides with the treatment which we would accord to the transition now under discussion, we shall avail ourselves of it—in further evidence, as it were, that the much complained of obscurity of Hegel's discourse is mainly due to a failure, on the part of his students, to discover the key to its real import in the circumstantially systematic nature of the dialectic movement of pure thought.

"1. In the just considered form of the specified Measure, the quantitative element of both sides is determined qualitatively; the sides are in Involved Inverted Relation and hence moments of one Measure-determinateness of qualitative nature. At the same time, however, the two sides are so far established only as immediate, simply different Qualities which are not themselves related in the same manner as their quantitative determinateness; namely, they cannot be said to have no sense, nor presence, outside the Involved Inverted Relation between their quantitative determinatenesses. The qualitative element figures, therefore, in the relation as what specifies, not its own self, but the quantitative determinateness; only as in this latter is it (the qualitative element) established, whilst remaining on its own immediate Quality as such which, beside the fact that magnitude is established in difference from it and beside its reference to its other, has yet an independently subsisting presence. Thus Space and Time are both valid apart from that specification, which their quantitative determinateness implies in the conditionally or absolutely free movement, as Space in general or Time in general, Space subsisting for itself as what endures apart and without Time, and this latter too as for itself flowing independently of Space.

"This immediacy of the qualitative element, as against its specific correlatedness as Measure, is, however, just as much bound up with a quantitative immediacy and hence also with the indifference of this its quantitative aspect against its specific correlatedness; the immediate Quality has also the sense of an only immediate Quantum. So comes it that the specific Measure has also a side, on which it

remains subject to external Alterableness in the sense of arithmetical progression, undisturbed by its specific nature, and into which falls the external and therefore only empirical determinateness of magnitude. Quality and Quantum, as thus prominent also outside the specific Measure, are at the same time correlated with this latter; the immediacy is a moment of such as belong themselves to Measure. Therefore, the immediate Qualities, belonging also to Measure, are equally in quantitative relation, which, as outside the specified Involved Inverted Relation, is itself only a Direct Relation, in embodiment of an immediate Measure. This inference and its import is to be indicated closer.

"2. Even though the immediately determined Quantum as such becomes a moment of Measure by virtue of an *à priori* element at its back, with respect to the specific Measure it has the character of something given to begin with. As thus established, the immediacy negates the qualitative determination of Measure, whose sides have already been found to have equally the character of independent Qualities. Such negation and the returning to the immediate character of Quantity lies in the qualitatively determined Relation, because the relation of distinct ones generally implies their correlatedness as *one* determinateness, which latter, in distinction from the form of relation, is here, in the element of Quantity, a Quantum. As negative of the distinct qualitatively determined sides, this exponent is a Being-for-self or pure Self-determinateness. But it has this significance only in itself, being, as regards its presence, a simple, immediate Quantum, which is Quotient or Exponent of the direct ratio between the sides of the Measure, but generally refers to the empirically determined Unit connected with the quantitative aspect of Measure. In the movement of falling bodies, traversed spaces are proportional to the square of ellapsed times, $s = at^2$. This is the specifically determined, the Involved Inverted Relation of Space and Time. The other, the direct Relation, should concern Space and Time as mutually indifferent Qualities; the space traversed in the first unit time. The same coefficient a remains in all the following units of time; the unity for the amount determined by the specifying Measure is an ordinary Quantum. But this unity at the same time

passes for the exponent of that direct ratio which represents the fancied bad, i.e. formal, not notionally sanctioned Velocity. Such a velocity does not exist here, any more than the one mentioned before which is supposed to accrue to the falling body at the end of a unit of time. The velocity under discussion refers to the space traversed in the *first* unit of time; but this so-called unit of time is itself only an assumed unit, having as such atomic point no presence. The beginning of movement (the smallness alleged for it could make no difference) is at once a magnitude and one specified by the law of descent of a falling body. The empirical Quantum *a* is ascribed to the force of gravity, on the understanding that this force has no connection with the actual specification, i.e. with the Involved Inverted Relation characteristic of the Inwardness of Measure. The immediate moment, that in the falling movement the space traversed in the first second amounts to some fifteen spatial units called feet, is an *immediate* Measure, just as the standard size of the members of the human body, the distances or diameters of the planets, etc. The determination of such a standard goes beyond the simple qualitative Inwardness of a specified Measure, here of the law of descent of a falling body; at any rate, concrete sciences have so far failed to enlighten us as regards the *raison d'être* of numbers representing the immediate and therefore only empirically ascertained element of a measure.[1] Here we are concerned only with the notionally determined fact that the empirical coefficient in the law of descent of a falling body stands for the Being-for-self as embodied in a Measure, and that too only in so far as this aspect of the latter lacks explicitness and hence concerns its simple

[1] Hegel is obviously assuming that in so far, for instance, as the mean planetary distances from the Sun, or the planetary diameters, are systematically correlated, it is thinkable that this their correlation may take the shape of a mathematical formula whose discovery is the business of empirical science. But that such a mathematization would not take the place of an *à priori* derivation, is obvious; and we are surprised that neither Hegel nor any of his followers tackled the problem of the connectedness of immediate Measures with the Notion. That measurable features of the Solar System do admit of an *à priori* derivation, will be proved by our explication of the fundamental unity of the *measurable* Quality and Quantity; and that we may then equally claim to have determined *à priori* the space traversed by a falling body in the first unit time or second, follows from the calculableness of this space from the magnitude of the Moon's orbit and her sidereal revolution.

immediacy in contrast to the developed aspect of its Being-for-self in the specific relationship between its sides, their Involved Inverted Relation. According to this second aspect of the Being-for-self, Heaviness of the falling body acquires the significance of a natural force whose expression depends on the nature of Space and Time. So comes it that the specific relationship between the sides concerns the law of descent of a falling body, whilst the simple direct Relation refers only to the relationship of Time and Space in the formal, externally produced and determined Velocity.

"(3) Measure has thus acquired the sense of a specified quantitative relation, whose qualitative aspect implies an ordinary, external Quantum, but not in its purely immediate connotation, as a Quantum as such, but essentially as a determining moment of the Relation as such, and hence in the sense of an Exponent, namely, of the objectively fixed Exponent of the already mentioned Direct Ratio between the two Qualities whose quantitative relation is specifically determined only in reference to this Ratio. This Direct Ratio is anticipated and assumed as given in connection with the law of descent of a falling body: but, as said, it does not yet exist in this movement. The fact, however, that the two aspects of Measure are themselves Measures, each immanently specified in immediate or external distinction against the other within the same fundamental unity of both, means that Measure is now before us as further determined, as *realized*. It contains the Relation, in which magnitudes are determined and differently specified through the nature of Qualities in such wise that its determinateness, as quite immanent and self-subsistent, at the same time presents itself in the sense of the Being-for-self of an ordinary Quantum, i.e. of the Exponent of a direct Ratio. The self-determination of the [Involved Inverted] Relation is thus negated, because the Exponent of the direct Ratio, by virtue of which it has its independence, is an Other to it; but then, this its Other is an immediate Measure of immanent Qualitativity only in so far as it, in turn, implicates the simultaneously negated otherwiseness. This negative unity is *real Being-for-self*, the category of a Something as unity of Qualities in measurable relation; complete Self-subsistence. The two aspects which have presented themselves in the sense of two distinct Relations,

acquire now immediately also the significance of two distinct Presences; but, in more precise reflection upon the present import of their fundamental unity, as a self-subsistent whole in restatement of the One Being, we must at once postulate its Self-differentiation into many distinct Self-subsistences whose qualitative nature and subsisting (Materiality) is bound up with their measureable aspect.

"Measurableness comes thus to imply a connection of Measures constituting the Quality of distinctly independent somethings, so-called Things. The previously discussed Measure Relations concern abstract Qualities, such as Space and Time; those to be discussed presently are exemplified in Specific Gravity, next in Chemical Properties, i.e. in features characteristic of *material* existence. Space and Time are equally implicated in connection with such measurableness, but their relation ceases to depend simply on their own nature, because it becomes subordinate to further determinations. Among the determining moments, e.g. in Sound, there is the *time* in which a number of vibrations occurs, and the *spatial* element of length and thickness of the vibrating body; but the magnitudes of those ideal moments are determined externally. That is to say, Space and Time present themselves as no longer in an Involved Inverted, but only in the ordinary Direct Relation, harmony being reduced to a quite external simplicity of numbers whose relations are most easily graspable, yielding on that account a satisfaction falling entirely within the element of sensation because of the absence of representation, fanciful image, thought and such filling of Spirit. In that the sides which now constitute Measure Relation are themselves Measures, but at the same time material somethings, their Measures are, to begin with, immediate Measures and with respect to their character as Relations, Direct Relations. It is the inter-relation of such Relations that is now to be considered in its development."

B. *In Completion of the Dialectical Whole of Matter and Movement.*

1. Before definitely turning our back on the standpoint of the measurable Quality, it is in place that we should

equally reconsider the trend of the just effected transition into the standpoint of the measurable Quantity with a view to rounding-off the dialectical whole of Matter and Movement which we have left behind as advanced, in connection with the conditionally free movement, only to the stage of Finitude. In this respect, then, we are immediately concerned with the infinitization of the falling, or simply heavy body in transition into the attitude to be adopted towards it, in order to have it before us in explicit correspondence to the direct unity of the Infinite and Finite.

But, then, as this unity is ushered on the scene in higher restatement of the notion of Limit, it suggests itself at once to correlate its present embodiment with *Newton's third law of motion*, according to which to every action there is always an equal and opposite reaction. For, in this way, we are restating the equilibrium of forces, mentioned in connection with the standpoint of limit, in simultaneous hypostatization of the two determining moments of the conditionally free movement in exemplification of the notion of Finitude.

Namely, in correspondence to the immediate Transitoriness, we are reflecting upon the heavy body as a pure Being-for-other. The fact, however, that the heavy body is thus envisaged simply from the standpoint of Newton's first law of motion, reminds us that Transitoriness is from the very first implied as in fundamental unity with Transcendence, and consequently refers to the heavy body as remaining subject to mechanical movement in simultaneous vindication of the notional import of the fundamental property of Matter, i.e. of its Heaviness, which is to be correlated with the aspect of Transcendentalness, the dual nature of the In-itself on the part of the Finite, as what keeps transcending its own self, having its correspondence in the fact that a heavy body too is only a self-centred Negation, its falling movement, or the pressure which it exercises when at rest, being precisely a demonstration of its Finitude. In so far, then, as the two aspects of the Finite come to be restated from the standpoint of Infinitude, we reflect upon an immediate embodiment of the notion of the true Being in connection with a heavy body; namely, that an action upon it converts itself into its own action—that to every action there is always an equal and opposite reaction,

or that the mutual actions of any two bodies are always equal and oppositely directed.

The law of *conservation of momentum* is, of course, another restatement of the just derived Newton's third law of motion, in reflection upon the measurableness of force by rate of change of momentum. If two bodies A and B are moving in any manner, subject to the condition that no force exerted by any other body acts on either of them, then any change in the motion of one of these bodies must be due to a force exerted by the other body. In that case, the momentum (i.e. the product *m.v.*) of A is increasing at a finite rate in one direction only on the understanding that the momentum of B is increasing at an equal rate in the opposite direction, so that if we prefix a positive sign to the rate of increase of A's momentum, we must prefix a negative sign to the rate of increase of B's momentum, the addition of these two quantities giving zero as a result.

2. But as we are, so far, reflecting upon the two aspects of the Infinite, only from the standpoint of their direct or immediate unity, we must next bring to view also the moment of mediation implied in their convertibility into one another. The two bodies under discussion just as much act *against* their action upon one another. Each is implied in the sense of an aspect of their fundamental connectedness, i.e. in explicit correspondence to the infinitized Finite or the finitized Infinite. Consequently, we cannot stop at the simple equilibrium of the rate of increase of their momenta, but must also concern ourselves with the element of opposition implied in their action upon one another. In that case, of course, we refer to the *work* performed when a body moves in any direction against an opposing force, the work done being measured by the product of the opposing force and the displacement of the body. With respect to this its capacity of performing work, a body is said to possess *Energy.*

As is seen, in explicit correspondence to the reciprocal determination of the Infinite and Finite, we reflect upon the idealistic distinction between Potential and Kinetic Energy, in higher restatement of the two aspects specified in connection with the conditionally free movement in exemplification of the standpoint of Finitude. In demonstration of the notional import of Heaviness, a stone has a tendency to fall towards the centre of the earth ; but before

it can fall, it must have been lifted to a certain height. Inasmuch, now, as the work implied in this negation of its fundamental property is to be viewed as preserved or *stored-up* in its own Inwardness, the Energy due to this inwardization of external action is called *Potential Energy*, whilst the capacity for doing work, acquired by the stone in the course of its subsequent falling movement, is termed *Kinetic Energy*—the energy due to actual motion as against the other one due to a position of advantage with respect to any force.

3. The fact that no form of Energy can be generated without an *equivalent* loss in the same, or some other, form of Energy, leads to the inference that the total amount of Energy in the universe is a constant quantity which can never be increased or diminished, although its form may be changed—a generalization termed the *Law of Conservation of Energy*, in higher restatement of the afore-mentioned Law of Conservation of Momentum. In order, however, to avoid going beyond the scope of the dialectical whole of Matter and Movement under the head of the measurable Quality, we must immediately concern ourselves only with the inference, to be drawn from the mutual convertibility of the Potential and Kinetic Energy, in explicit reference to a falling body, when the physical constitution of this body is still left out of the question. Since in that case the further transformation of Kinetic Energy, e.g. into Heat, is to be excluded, the falling body must no longer be conceived as liable to striking the ground, i.e. as an instance of the conditionally free movement. In short, the law of conservation of energy concerns us immediately only with respect to its embodiment in the Absolutely Free Movement as demonstrated in planetary motion.

When this movement is analytically treated as a case of the Circular Motion (in the image of a weight attached to string and mechanically whirled round), there is postulated, to begin with, an original impulse, so that the immediately resulting motion, according to Newton's first law, represents the true Being as reduced to its simplest finite expression; which further serves only as an abstract substratum for an exemplification of the two forces entering into Presence in connection with the curbing of the original straight path into a circular one. It so happens, however, that the orbital

movement is not circular, but elliptical in shape. Consequently, if we attempt to interpret it in terms of the centripetal and centrifugal force (as potential and kinetic energy come to be restated in this connection), we must grasp their sum as a sum of two factors in inverted relation : by as much as the one of them increases or decreases, by so much does the other decrease or increase, whilst neither must be supposed to decrease to zero or else to increase to the maximum extent of their abstract range of alterableness. Nevertheless, whilst remaining an unattainable beyond of infinite approximation, the sum of the two forces at the same time belongs to the very Inwardness of either of them ; this follows from the very nature of the mediation between them as aspects of one self-decerning unity. They are in equilibrium only in so far as this their equilibrium rests on its own negation ; or, the distinction between them being purely idealistic, their fundamental unity presents itself in the form of an all-round contradiction. So comes it that in final restatement of the equilibrium implied in connection with Newton's third law, in simultaneous inwardization of the idealistic distinction between Potential and Kinetic Energy, we arrive at the conception of the Absolutely Free Movement.

4. In so far, however, as this conception is here ushered on the scene only incidentally to the alternative version of the transition into measurable Quantity, in formal recapitulation of the original transition from Quality into Quantity, we are concerned with the stated embodiment of the true Being as immediately reduced to a condition of Oneness. In this respect, we may be said to refer to the scientific conception of Ether—also called the *luminiferous* ether, because originally assumed for the purpose of explaining the phenomena of Light.

" Any discussion of ether leads out upon the highroad to incredulity. A thing must be defined by its properties and the properties of the ether are for the most part negative ; so negative, indeed, are they, that when one says boldly that we cannot see ether, hear it, taste it, smell it, exhaust it, weigh it, or measure it, one feels timid that sane-minded people will meet these negative qualities of our ether by a decided negation of belief in its existence. But the fact of the matter is that if this thing 'ether' is not

visible to the eye of sense, it is visible to the eye of the mind, which is much less liable to err. . . . Once convinced that light consists of waves, the mind insists that these waves shall inhere in something. The ocean waves are made of water,—sound waves of air,—light waves of, we must say, —*something*. This *something* cannot be air or water or any form of matter as we know it, for throughout that great reach of 93,000,000 of miles between the sun and us there exists but empty space. Filled this empty space is, however, and to the brim. There is no such thing as emptiness. From corner to corner of the universe, wherever a star shines or light darts, their broods this vast circumambient medium —the ether. Not only through interstellar spaces, but through the world also, in all its manifold complexity, through our own bodies ; all lie not only encompassed by it but soaking in it as a sponge lies soaked in water." [1]

That Emptiness is really another word for the Ideality of the true Being, when this latter becomes the object of further thinking in the sense of Oneness, became plain to us already in transition to the dialectical whole of Quantity, Consequently, we cannot object to the scientific postulate of an all-pervading ether in explicit correspondence to the transcendental Omnipresence of the One Being. Materiality should in this respect remain purely *in statu nascendi :* a mere entering into Presence, arguing its immediate vanishing. Its purely idealistic character is forced on scientific attention in the fact that " gravitational attraction is independent of the medium in which the attracting bodies are placed, and is unaffected by interposing matter of any kind between them ". [2] The tendency to endow ether with the properties of ordinary matter, in attempted explanation of action at distance, simply shifts the problem without solving it. As regards physical science, such an action must needs remain only a problem, because the standpoint of the externally originated movement is incapable of explaining direct demonstrations of the nature of the one true Being. "Gravitational attraction is propagated with a velocity so great that it may be considered to be infinite. To understand this statement, let it be supposed that a body A is attracted by another body B. While the bodies are station-

[1] " The New Knowledge," R. K. Duncan, pp. 3, 4, 5.
[2] " General Physics," Edser, p. 211.

ary, an equal attraction is exerted on B by A. If A suddenly moves towards B, the attraction on A increases; but if gravitational force were propagated with a finite velocity, an increased attraction would not be felt at once by B. If such an action as this occurred, it would affect the motions of the planets, and no effects which could be explained as due to this cause, have been observed."[1]

Therefore, whilst identifying Ether with the One Being, we reflect upon it more precisely as the objective correspondence of the Emptiness *filling* the One. In so far, then, as the One Being is equally implied as abstractly self-discerned, and this self-discernment is next to be grasped in the sense of Self-differentiation, we get in correspondence to the many present Ones simply Points, because the self-differentiated One is then equally contrasted with Space, of which Ether is a higher restatement.

If we now remind ourselves that we have assumed Matter as self-differentiated to begin with, and that we next concerned ourselves with heavy bodies only as centres of gravity actively negative of their out-of-one-another-ness, the dialectical whole of Matter and Movement ending by ushering on the scene the common centre of universal gravitation, implied in the concluding idea of Absolutely Free Movement; the present embodiment of the self-differentiating One strikes us at first sight as a higher restatement of the Self-differentiation postulated at the outset of the dialectical whole of the measurable Quality. But in thus returning to the beginning, we have it before us as further qualified. We are meant to bring home to ourselves that what we at first simply called heavy body, is to be equally credited with self-centredness in supersession of its immediate tendency to fall. That is to say, we have it now before us in the sense of a concrete Being-for-self; in the sense of a material something explicitly constituted in pure self-reference and hence endowed with distinct qualitativity of its own.

As is seen, the dialectic of the One Being concerns us only in the sense of a purely abstract middle term between the standpoint of the measurable Quality and that of the measurable Quantity. Its distinct hypostatization can be sought only in connection with the *Stellar System;* but

[1] "General Physics," Edser, p. 211.

since in that case we look in vain for a formative principle specifying this system from within its own self, its closer consideration is of no speculative interest and hence primarily concerns the standpoint of observational Astronomy. However great its appeal to emotion; to self-conscious Reason the host of stars is, as Hegel says, "no more admirable than an eruption on our skin, or a swarm of flies. The fact that it is boundlessly spread out, means nothing to reason; so far, reference is only made to mere externality, emptiness, purely abstract infinitude. Reason transcends all that and hence declines to take share in an admiration of pure negativity." [1]

[1] "Enc.," § 268, Zusatz.

SECTION II.

THE MEASURABLE QUANTITY OR CHEMICAL QUALITY.

CHAPTER I.

THE DISPLAY OF THE DIRECT RELATION IN CHEMICAL MIXTURE.

IN effected transition into the standpoint of the measurable Quantity, we have before us the self-differentiated One Being in the shape of material somethings, whose materiality is no longer simply considered with respect to the demonstrableness of its fundamental property, but with respect to its chemical nature. By virtue of the now inwardized dialectic of the One Being, the centre of gravity of a particular mass of Matter is equally a representative of the common centre of universal gravitation, with the consequence that the state of rest ceases to be traceable solely to an external circumstance, but to the own self-poise of a material something. Inasmuch, however, as this its self-poise must, nevertheless, be grasped in union with its fundamental property, and this property or Heaviness is, consequently, reflected upon as further specified in explicit reference to the own material Being-for-self, we are immediately concerned with the Specific Gravity of a Mixture of two substances of different specific gravities.

Now, as is well-known, such a Mixture has a specific gravity other than the Sum of the two specific gravities of the mixed substances, even though their Weight is simply summed-up. Nothing else can be expected. Weight represents the Quantum through which a material something is what it is. It refers to the fundamental property of Matter, apart from which there is no possibility of material Being-for-self. This is, precisely, why the stellar system

presents itself to us only as a playground of empty abstractions. In so far, then, as Specific Gravity refers to the series of comparative numbers representing the Weight of a constant Volume of different substances, and weight does not itself admit of alteration; the aspect of Alterableness necessarily to be postulated for the chemical Quality, in reference to its further discussion, can come to the front only in connection with the idealistic moment of the direct ratio embodied in specific gravity. So comes it that a Mixture of chemical substances sums up only their Weight, but not Volumes.

But a mixture of two substances is only one among indefinitely many ones. Accordingly, a substance or chemical element (the series of specific gravities obviously admitting also of being identified with the series of *atomic* weights, in which respect of course we must keep in mind that "the chemist uses the term 'atom' much as he uses that of *element*, not as expressing an absolute fact, but as a convenient term to express what is observed to be the case according to our present knowledge"[1]) has its immediate meaning in an endless series of Exponents which it, taken as unity, forms with the rest of specifically heavy substances. Nevertheless, in order to get at the truth of this meaning, we require only a finite series of Exponents. In that case, however, the series of Exponents ceases to refer to the comparative numbers resulting for the specific gravities of simple mixtures of substances or elements, but must be interpreted conformably with the implied shifting of the ground from the standpoint of mere Alterableness to that of Alteration. The quantitative infinitude associated with the immediate meaning of a chemical element is, as always, only a transitional form ushering upon the scene the next higher qualitative aspect of the subject-matter under discussion. In reflection upon the correlation of the unfoldment of the measurable Quantity with the dialectical whole of Quantity, it will be seen that we have now arrived at the stage, at which the endless flux of Counting is inwardized in the Sum of an Infinite Series, in transition into the standpoint of the geometrical Magnitude.

[1] Kemshead, "Inorganic Chemistry," p. 34.

CHAPTER II.

THE DISPLAY OF THE INVERTED RELATION IN CHEMICAL AFFINITY.

INSTEAD of a simple series of indefinitely many substances of different specific gravity, we postulate now this series as, to begin with, divided into two chemically distinct groups; and we are concerned with the series of Exponents resulting when a member of either group enters into chemical Combination with all the members of the opposite group. We see at once that the chemical distinction between the members of the two opposite groups is postulated with a view to the formation of a compound body; in correspondence to the concluding unity of the extensive and intensive Magnitude, in transition into the standpoint of the Quantitative Relation. Immediately, however, the finite series of Exponents now in question obviously concerns only the amounts required for the saturation of individual members on either side. That is to say, we do not at once refer to the Elective, but only to the simple chemical Affinity.

Now, the fact is that, along with counting as Unity, with respect to the Amounts recorded by the series of Exponents representing the Combinations now under discussion, any and every member of either group is at the same time one of many on its own side. Therefore, in so far as the Neutralities resulting from the Combination of any and every member on either side with all the members on the opposite side in the same fixed order yield a different series of Exponents, the difference between the resulting series of Exponents must represent the series of comparative numbers which the members of the group on the side of Unity form on their own side.

There is nothing else to fall back upon for the determination of the difference in question. The resulting series

of Exponents must be comparable, because the members of either group are only quantitatively distinct instances of the same general Quality. Consequently, the resulting series of Exponents must be, with respect to a particular member on the side of Unity, the same amount of a unit-series of Exponents as the particular member on the side of Unity is of the chosen unit-member on this side. Obviously, we are ushering on the scene the *Law of Reciprocal Proportions*, stated as follows in Newth's "Inorganic Chemistry," p. 24:—

"The weights of different elements which combine separately with one and the same weight of another element are either the same as, or are simple multiples of, the weights of these different elements which combine with one another; or in other words, the relative proportions by weight in which the elements A, B, C, D, etc., combine with a constant weight of another element X, are the same for their combination with any other element Y."

In this respect we are simply recapitulating the distinction between the extensive and intensive Magnitude. Each of the members on the side of Unity represents the intensive Quantum in distinction from the resulting series of Exponents in embodiment of the extensive Quantum. Inasmuch, however, as the intensive Quantum is directly also extensive in distinction from the extensive Quantum, and similarly the extensive Quantum directly also intensive in distinction from the intensive Quantum, each member on the side of Unity is also an amount of a standard unit on its own side, whilst any and every series of Exponents carries with it equally the sense of unity in embodiment of the specific meaning of a particular member on the side of Unity. Consequently, the two groups are qualitatively distinct in such wise that their negation of one another is just as much negated.

The immediate import of this negation is that the Neutralities resulting from the combination of the members of the two opposite groups are mere Mixtures, connected with one another in purely arithmetical continuity. But just because the standpoint of chemical affinity is negated in the sense of being superseded, and hence is just as much included as excluded, the treatment of a Neutrality in the sense of a direct unity of two chemically distinct substances

entangles itself in a vicious circle, in present embodiment of the vicious circle between Quality and Quantity in transition into the standpoint of the Quantitative Relation. True, on the one side, chemical combination does present itself as resting solely on Quantum. This follows from the fact that the vicious circle in question is just as much of objective as of subjective significance. The stellar system shows that even empty abstractions are objectively hypostatized; in proof of our fundamental assumption or thesis that all that is dialectically necessary is just as much a matter of objective fact. The vicious circle between Quality and Quantity must be of some objective significance in the dialectical whole of the measurable Quantity.

Accordingly, we ourselves postulate that there must be chemical combinations which prove themselves to rest solely on Quantum. In that case, we are plainly referring to the second law of chemical combination, called the *Law of multiple Proportions:* "When the same two elements combine together to form more than one compound, the different weights of one of the elements which unite with a constant weight of the other, bear a simple ratio to one another; or in other words, when one element unites with another in two or more different proportions by weight, these proportions are simple multiples of a common factor."[1] "Thus, if A and B be the elements in question, if A be constant, B will be found to exist in a series of multiples, such as—

$$\begin{aligned} &A + B,\ A + 2B,\ A + 3B,\ A + 4B, \text{ etc., or} \\ &A + B,\ A + 3B,\ A + 5B, \text{ etc., or} \\ &2A + B,\ 2A + 2B,\ 2A + 3B, \text{ etc., or} \\ &2A + 3B,\ 2A + 5B,\ 2A + 7B, \text{ etc.} \end{aligned}$$

or in some similar series."[2]

But we must just as much postulate the possibility of chemical combinations proving the very opposite of the foregoing contention. As a matter of fact, a great many substances amongst organic bodies are composed of the same elements in the same relative proportions and yet exhibit physical and chemical properties perfectly distinct from one another. "For instance, oil of lemons, oil of turpentine, oil of rosemary, and many others, all contain the

[1] Newth's "Chemistry," p. 24.

[2] Kemshead, "Chemistry," p. 28.

same elements, united together in the same proportion; or the crystallized portion of *essence of roses* and common *coal* gas are chemically identical. To all such bodies the term *Isomeric* (from ισος, equal, and μερος, part) is applied."[1]

Accordingly, chemical combination is a mixture of chemically distinct substances, in connection with which quantitative element is to be reflected upon as just as much superseded. In this way, we are ushering upon the scene the standpoint of *Elective* Chemical Affinity in transition into the display of the Involved Inverted Relation.

[1] Kemshead, "Inorganic Chemistry," pp. 27, 28.

CHAPTER III.

THE DISPLAY OF THE INVOLVED INVERTED RELATION AND NODAL LINE OF MEASURE RELATIONS.

A. *The Compound Body.*

INASMUCH as we are treating the display of the forms of the Quantitative Relation by the measurable Quantity or chemical Quality in simultaneous recapitulation of the dialectical whole of Quantity, the display of the Involved Inverted Relation is to be immediately correlated with the qualitative Quantum in its immediate reference to the direct Relation.

As we know the qualitativity of the Quantum refers at this stage to its simple Fixedness, the reason why a particular Quantum should be treated as fixed remaining still arbitrary, even though already taking the shape of a direct ratio. For, the possible objective significance of this latter comes to view only in connection with something measurable and on that account remains immediately only an anticipation. Accordingly, too, chemical attraction, under whose influence elementary bodies arrange themselves into numberless compounds, presents itself to us only in anticipation of its full significance, whose proper discussion goes indeed altogether beyond the scope of Measurableness. For the present, we are concerned with it only in immediate hypostatization of the true Being in anticipation of the concrete demonstrableness of its nature in the explicit sense of Essence.

In so far, then, as the qualitative nature of the Exponent is to amount, to begin with, only to quantitative Fixedness, we obviously refer to the first law of chemical combination, called the *Law of Constant Proportions*, according to which "the same compound always contains the same elements combined together in the same proportion by weight; or expressed in other words, the weights of the constituent elements of every compound bear an unalterable ratio to each other, and to the weight of the compound formed".[1]

[1] Newth's "Inorganic Chemistry," p. 24.

But because the standpoint of the direct Relation comes now also already under the head of the displayed Involved Inverted Relation, we must *pari passu* treat the Exponent as a qualitatively established third to the sides of the direct Ratio. For, we are now also already presupposing the standpoint of the Inverted Relation as behind us, and hence must view the result of chemical attraction as taking the place of the Exponent of the Inverse Ratio, the third being in this respect only a Quantum determining how much smaller or greater we must make the Quantum of a particular member in either of the two opposite groups, if the product representing the resulting combinations is to remain a fixed number. Whereas the third stands thus for a series of combinations, now it stands only for one particular combination, on the understanding that the firmer holding together of the two combined substances, as against the previously implied openness to any other combination on the part of either, is due to their Elective Affinity and hence is of a distinctly qualitative kind.

As a matter of fact, the most striking and characteristic feature of chemical attraction is the entire change of properties with which its result is attended. Under its influence solids become liquified, liquids solidified, gases both, along with changes of colour, taste, smell, which by no possible reasoning could have been predicted.

B. *The Finitude of the Compound Body.*

The present emphasis on the qualitative character of chemical combination reminds us that the Compound Body is to be also viewed in the sense of an *implicit* embodiment of the fundamental unity of the measurable Quality and Quantity. The dialectical whole of the Measurable Quantity recapitulates the original trend of the dialectic of Quantity as a returning movement into its beginning. In so far, however, as the fundamental unity of Quality and Quantity comes to the front, to begin with, with an accent on Quantity, the Compound Body does not yet concern us in fully established return into the standpoint of the measurable Quality, but corresponds to the stage reached in the dialectical whole of this latter in the conditionally free movement. This demonstration of the finite character of the self-centredness on the part of a heavy body is to be

read into the chemical nature of a compound body; and in that case, we obviously refer to an explicit exemplification of the contradictory nature of its Inwardness, in so far as the possibility of other combinations on the part of its constituents is at once to be granted and negated.

Even though the compound body or a salt rests on Elective Affinity between two chemically distinct substances, and consequently immediately excludes the formation of other possible combinations on their part, the Being-for-self thus reached retains the significance of a hypostatized present One of Self-differentiation. The fundamental Oneness of all that is, is obviously reflected upon only negatively in connection with an exclusive measure relation. In this respect, therefore, stability remains pregnant with a striving to transcend its immediate character as a fixed Negation, with the consequence that compound bodies must be also viewed as subject to alterable Alteration. The fact that their qualitativity still also continues to rest on the quantitative element, argues its Transitoriness. In so far, however, as Alteration must then remain a matter of Alterableness, the exclusive Elective Affinity involves no further principle of specification, but exemplifies simply the separability of its constituents with a view to the formation of other neutralities of a similarly transitory exclusiveness. As is seen, in this respect we bring again to the front the third law of chemical combination (the law of reciprocal proportions), but the members of the two groups entering into combinations are now to be grasped as at the same time resulting from the analysis of compound bodies into their constituents.

"Whether equivalent proportions of sodium nitrate and potassium chloride, or of sodium chloride and potassium nitrate are mixed together in aqueous solution at constant temperature, each solution will, after the elapse of a certain time, contain these four salts distributed in the same proportions. Let m and n be positive integers, then

$$(m+n)NaNO_3 + (m+n)KCl = mNaCl + mKNO_3 + nNaNO_3 + nKCl;$$
$$(m+n)NaCl + (m+n)KNO_3 = mNaCl + mKNO_3 + nNaNO_3 + nKCl.$$

This is more concisely written—

$$NaCl + KNO_3 \rightleftarrows NaNO_3 + KCl.$$

"The phenomenon is explained by assuming that the products of the reaction interact to reform the original components simultaneously with the direct reaction. That is to say, two independent and antagonistic changes take place simultaneously in the same reacting system. When the speeds of the two opposing reactions are perfectly balanced, the system appears to be in a stationary state of equilibrium. This is another illustration of the principle of the coexistence of different reactions." [1]

The quoted explanation of chemical equilibria and the method adopted for the calculation of the rate of a particular transformation reminds us that the present recapitulation of the standpoint of the Inverted Relation does, indeed, at the same time come under the head of the displayed Involved Inverted Relation. In that case, as we know, the Inverse Ratio becomes a moment of the Derived Function, either side increasing, not as many times, but by as much, as the other side decreases, whilst the Exponent acquires the sense of their sum, and hence in the present application, of the sum of the changes taking place in the same reacting system. Thus:

"Consider a system containing two reacting substances A_1 and A_2 such that $A_1 \rightleftarrows A_2$

Let a_1 and a_2 be the respective concentrations of A_1 and A_2. Let x of A_1 be transformed in the time t, then by the law of mass action

$$\delta x/\delta t = k_1(a_1 - x).$$

Further, let x' of A_2 be transformed in the time t. The rate of transformation of A_2 to A_1 is then

$$\delta x'/\delta t = k_2(a_2 - x').$$

But for the mutual transformation of x of A_1 to A_2 and x' of A_2 to A_1, we must have for equilibrium

$$x = -x'; \text{ and } \delta x = -\delta x';$$
$$\therefore \delta x/\delta t = -k_2(a_2 + x).$$

The net or total velocity of the reaction is obviously the algebraic sum of these 'partial' velocities, or

$$dx/dt = k_1(a_1 - x) - k_2(a_2 + x)."$$ [2]

[1] Mellor's "Higher Mathematics," p. 223. [2] Ibid., p. 224.

Inasmuch as the law of mass action puts on record the fact that the amount of chemical change is directly proportional to the quantum of a reacting substance present in the system, it may be written also in the form of an exponential function, which is then the Original Function to the alternative expression for the same law in the form of the Derived Function. The relation between the two functions remains so far purely idealistic, because our recapitulation of the dialectical whole of Quantity is properly advanced only to the stage of the Inverted Relation.

A distinct demonstration of this stage is, by the way, also found in connection with the dissociation of the molecules of an acid or salt into two parts called *ions*. In this, case, when the concentration of the solution is very great, the amount of dissociation is very small, and vice versa. Inasmuch as complete dissociation remains the beyond of infinite approximation, the graphic curve illustrating the amount x of ionised molecules and the concentration c, the so-called dissociation isotherm, is assymptotic towards the two axes.

C. The Periodic System of Elements.

The distinct recapitulation of the Involved Inverted Relation itself brings us back again to the homogeneousness resulting from chemical combination, but with the difference that this homogeneousness is to be grasped more precisely in the sense of the fundamental Oneness or *Substratum*, of all that is measurable. For, the homogeneousness characteristic of a compound body is to be treated only as an anticipation of that ultimate, all-embracing homogeneousness, in the light of which the material universe reduces to the condition of Oneness.

Therefore, Alteration is finally to be grasped in explicit reference to one and the same Substrate. That it generally comes to notice only in its results, was pointed out already at the beginning of Chapter IV under the head of Quality. Owing, however, to our progress in knowing since then, our explanation of this fact is no longer simply limited to our immediate insight into the nature of Alteration, as a more concrete instance of the collapse of Becoming into Presence, on the understanding that this latter is the real First with which beginning is made. Now we no longer simply pick

up something or an other in simple illustration of the qualitative distinction to be postulated to begin with. Something and an Other are now before us also as instances of the Countable, or rather of the Measurable. In fact, we have now superseded even the immediate connotation of this latter, as that Quantum through which Something is what it is; so far, the Measurable remains qualitatively finite. In deepening our grasp of the homogeneousness characteristic of compound bodies, we have now also infinitized their immediate finitude. That is to say, we have finally imparted to the Measurable the significance of an aspect of what *in itself* truly transcends all measurable distinctions and hence is itself the *Immeasurable.*

We have been aware from the very first that the Measurable must display the nature of the qualitative Quantum. But that its Qualitativity and Quantitativity are only Aspects or Phases of a truly immeasurable or measureless Being, is properly brought home to us only in the present conclusion of the dialectic of the measurable Quantity. In so far as the fundamental Oneness or Substrate is to be viewed in concrete embodiment of the double transition between Quality and Quantity ushering on the scene the notion of Measure, it contains the distinction between Continuous and Discrete Magnitude; but instead of the mere flux of Counting, we have now an entering into Presence of one and the same Substrate in the guise of heterogeneous substances. The transition from a particular substance into another one necessarily implies Continuity; but along with being apparently only gradual, it is just as much also abrupt or of the nature of a leap. The new substance does not really arise from the old one, because this latter remains another qualitative Presence up to the very last. Inasmuch as the merely quantitative continuity of gradual alteration is no Limit in its own self, the new substance remains with respect to the old one the beyond of infinite approximation and hence arises truly from that inner specifying unity which has not yet entered into Presence. And since the new substance is subjected to the same progress, and this going over of Qualitativity and Quantitativity into one another is meant to explicate the conception of the Substrate as a self-discerning unity, it follows that this latter enters into Presence in a series whose members are qualitatively

distinct in quantitative continuity, thus forming a Nodal Line of Measure Relations in one and the same Substrate.

When we try to present to ourselves this line graphically, there suggests itself at once the logarithmic spiral—a curve made up of an infinite number of turns, the logarithm of the ratio of the distance of any two points from the pole being proportional to the angle between their radii vectores. If these latter, by the way, represent the number of vibrations of a sounding body in a given time, the corresponding angles measure the logarithms of the intervals between the tones produced by these vibrations. But, then, we are primarily interested in the Nodal Line of Measure Relations as exemplified in the Periodic System of Elements.

Just because Elements must be conceived as arisen directly from the inner specifying unity called the Substrate, and hence as being at once qualitatively heterogeneous and quantitatively continuous, it suggests itself to seek an analytical explanation of their connectedness in terms of a growing number of ultimate particles, of which atoms are to be conceived as consisting, on the understanding that the properties distinguishing various elements are bound up with the qualifying points emerging in the implied quantitative progress *ad infinitum*. As a matter of fact, this is the standpoint adopted in explanation of the empirically ascertained fact that the properties of Elements are a periodic function of their atomic weight. In the so-called *corpuscular* theory of Matter, the weight of atoms is viewed as due to the weight of a certain number of corpuscles, such as are thrown off by glowing metals, incandescent carbon, gas flames, or simply by metals on mere exposure to light, especially to the light obtained from an arc lamp or by burning magnesium, i.e. to the light rich in ultra-violet rays. But since the determination of the mass of a corpuscle is made to depend on the determination of its electrical charge, the corpuscular theory of Matter merges into the *electronic* theory, the mass of a corpuscle being ultimately viewed as due solely to the amount of ether dragged along, or bound by, the moving negative electrical unity. Now, since a negative charge of electricity has always associated with it an equal positive charge, and the carriers of positive electricity are found to be of the same magnitude as atoms of ordinary matter, an atom is finally

assumed to be a sphere of positive electrification enclosing a number of negatively electrified corpuscles, in a word, *electrons*, the negative electricity of these latter exactly balancing the positive electricity of the enclosing sphere. Inasmuch as this equilibrium necessitates also a definite arrangement of the involved number of constituent electrons, it follows that a regular, arithmetically continuous growth in this number goes together with an alternating increase and decrease in the stability of the resulting equilibria, and further that the same cycle of varying degrees of stability keeps repeating itself, i.e. there follows the Periodic System of Elements.

"What there is herewith present is (α) one and the same Substrate, established as the basis of its own distinctions and hence as perennial. The specification of Being in contrast to its general character as Quality begins already in Countableness, Something or other being great only as indifferent to its Determinateness. In Measurableness, the Something or Other is at the outset viewed in implicit embodiment of the two moments, which constitute the distinction within the general sphere of Being and of which the one is the Beyond of the other, with the consequence that the Measurable is from the very first a representative of the perennial Substrate. In explication of this fact, we find (β) that qualitative Self-subsistences, representing the self-differentiated One of Measure, rest on quantitative distinctions and hence are just as much purely continuous, whilst (γ) this Continuity of the qualitative element all through the quantitative progress, in so far as this progress also implies Negation of the qualitative element and therewith equally of the merely quantitative externality, has its explicit record in the infinite progress of the Nodal Line. The purely quantitative character of the endless progress goes under in the emergence of a measure relation, i.e. of a Quality, whilst in so far as this new Quality is itself only a quantitative Relation, the qualitative character of the transition is just as much negated. This going over of Qualitativity and Quantitativity into one another is taking place on the ground of their unity, and the sense of this process is only the showing or Establishing that at its bottom there is indeed such a Substrate."[1]

[1] "Wissenschaft der Logik," I., pp. 437-8.

CHAPTER IV.

TRANSITION INTO THE DISPLAY OF THE FUNDAMENTAL UNITY OF THE MEASURABLE QUALITY AND QUANTITY.

A. Absolute Indifference.

CONFORMABLY with the fact that the fundamental unity of Quality and Quantity remains, in outcome of the dialectic of the Quantitative Relation as such, still only established with an accent on Quantity, the Substrate of the Nodal Line of Measure Relations is before us only in the sense of *Absolute Indifference*:—in higher restatement of Quantity as such, in so far as the latter's immediate Indifference to Quality is now extended to all the features of Measurableness, since these features pass now for purely external phases lacking individual Being-for-self. Our further task concerns, therefore, the present restatement of the Relation between the original and the Derived Function, whose object, as we know, is to supersede the primarily only quantitative aspect of the Quantitative Relation, this aspect coming now to the front in the immanent unconnectedness of the Substrate with its present embodiment in Measurableness. In order to supersede this unconnectedness, the immediate nature of the Substrate, as Absolute Indifference, must be deepened into an embodiment of the true Being, seeing that, so far, it is before us only in correspondence to the direct unity of the Finite and Infinite; or as we may say now, to the direct unity of the measurable Quality and Quantity. Even though the two main aspects of Measurableness have been already brought together in an alternating unity of both, this alternating unity, i.e. the Nodal Line of Measure Relations, has not yet the free Notion, i.e. the true Being, for its specifying principle, because distinguishing concerns only an external phase of the Substrate. Measurable distinctions must be ultimately grasped in record of the own notional nature of the Substrate; but before this

is possible, we must first of all get beyond the immediate connotation of the Substrate as Absolute Indifference—a term putting on record its primary aloofness from its measurable embodiment.

Accordingly, the Substrate as such is immediately before us as Absolute Indifference in contradistinction against its entrance into Presence by way of a Nodal Line of Measure Relations. Since, however, we reflect upon Absolute Indifference as a result of the previous progress in knowing, we are really concerned with the reappearance of its dialectical part within its own Inwardness. Just because the at first sight utterly excluded circle of the determinatenesses falling within the sphere of facts is just as much also included in its own Inwardness, we cannot but postulate an inwardized aspect of these determinatenesses in connection with our intended explication of the implicit nature of the Substrate as such. But that Inwardization means here Mathematization is at once brought home to us by the fact that no empirical science is viewed as established on a firm basis unless its generalizations are expressed in terms of number.

After all, that the Substrate as such refers ultimately to our own mind is obvious. Inasmuch, however, as at the present stage of our progress in knowing, our concern is only with the way Measurableness acquires a *metaphysical* embodiment, or gets to *be* in the sphere of knowledge, we can refer to our mind only as correspondingly emptied of any higher content of its own. At any rate, its characteristic mission to *comprehend*, to display itself as the true Being, must still remain conspicuous by its absence, because of the simultaneous aloofness on the part of the Substrate from its own embodiment. In exemplification of this aloofness, the process of knowing can concern only the surface without any endeavour to penetrate the inner nature of the subject-matter. In short, mind comes here for consideration only as displayed in Mathematics. "It is quantity, a form of difference that does not touch the essential nature, which alone mathematics deals with. It abstracts from the fact that it is the notion which separates space into its dimensions, and determines the connections between them and in them. . . . The principle of quantity, of difference which is not determined by the notion, and the principle of equality, of abstract, lifeless unity, are incapable of

dealing with that sheer restlessness of life and its absolute and inherent process of differentiation. It is therefore only in an arrested, paralysed form, only in the form of the quantitative unit, that this essentially negative activity becomes the object-matter of this way of knowing, which, itself an external operation, degrades what is self-moving to the level of mere matter, in order thus to get an indifferent, external, lifeless content."[1]

In so far as the relation between the sphere of facts and its metaphysical counterpart in mathematical knowledge is analogous to the relation between Quality and Quantity, the dialectical whole of these two determinatenesses of Presence might also be viewed in the sense of an original element in our progress in knowing, Quality assuming with respect to its higher restatement the significance of Measurableness and Quantity that of Mathematics. Just as the dialectical consideration of Quality anticipates the standpoint of Quantity, so Measurableness is equally all along bound up with mathematical knowledge. But just as when Quantity definitely enters upon the scene, it becomes the object of further thinking only as, to begin with, stripped of all qualitative meaning, in the sense, that is, of Magnitude pure and simple, so now too mathematical knowledge becomes our object only as explicitly negative of its fundamental correlatedness with the rest of empirical sciences, i.e. only in the sense of pure mathematics. But that we shall adopt this treatment of mathematical knowledge only with a view to establishing its fundamental unity with the primarily negated Measurableness, is now also obvious from the trend of the dialectic whole of Quantity. As regards, by the way, the final objective meaning of the fundamental unity in question, we may at once anticipate that, having to be characterised at once by objective demonstrableness of Measurableness and at once also by subjective Transcendentalness, it must refer to a transcendental demonstrableness of Measurableness. The next higher correspondence to Measurableness, as the third to Quality and Quantity, refers to the Solar System, as the third to Measurableness and Mathematics.

Seeing, now, that Mathematics concerns us in meta-

[1] Hegel's "Phenomenology," Baillie's trans., I., pp. 42, 43.

physical embodiment of the dialectical past of the Substrate as such, when this latter is simultaneously viewed in higher restatement of Quantity, it follows that our explication of the system of Mathematics takes its point of departure from simple Counting. Inasmuch, however, as it is really the nature of the Substrate that is to be exemplified under this head, Counting or Number comes to concern our explication mainly in respect of the growing concretion of its primary elementariness in progressive correlation with the successive stages of the familiar dialectical ascent up to the level of the Substrate itself, the scope of this recapitulation acquiring *pari passu* also the significance of a transition from the standpoint of Arithmetic into that of Geometry, under whose head we shall have to proceed once again to recapitulate Substrate's Inwardness in transition into the next higher branch of Mathematics; and so on, until the scope of the undertaken explication proves itself exhausted and the Substrate finally enters upon the scene in fully established return into the standpoint of the, to begin with, excluded Measurableness, when it will be before us in explicit correspondence to the Relation between the Original and the Derived Function in ultimate transition into the display of the fundamental unity of the measurable Quality and Quantity in the Solar System.

As is seen, the systematization of the science of Mathematics presents itself to us in correspondence to the original supersession of the Quantitative Relation as such. Just as in this respect we proceeded to explicate the Inwardness of the Involved Inverted Relation in the sense of the explicitly involved Inverted Relation ushering on the scene the standpoint of the Derived Function; so now, too, we aim at a final restatement of the Substrate as such in the sense of Indifference as a Sum of two factors in inverse ratio. But instead of explicating the Inwardness of the Involved Inverted Relation, we are now under dialectical necessity to explicate the dialectical past of the Substrate as such; and this explication takes the shape of a metaphysical counterpart of the Substrate's entering into Presence by way of a Nodal Line of Measure Relations, in so far as their Substrate comes to stand for the mathematical consciousness entering into Presence in the system of Mathematics.

I. Arithmetic and Elementary Algebra.

The immediate embodiment of the Substrate under the head of the arithmetical Magnitude is obviously found in the ordinary arts of reckoning: (1) Addition or Subtraction, (2) Multiplication or Division, (3) Involution or Evolution.

That which distinguishes *Addition* from simple Annumeration is this, that we are summing up unities which already represent the result of the simple Counting, and hence have the significance of immediately quite indefinite and on that account generally unequal Numbers. That we at once also already reflect upon the nature of the Substrate, as a Nodal Line of Measure Relations, is made manifest in the very system of rational Numbers. For the numbers to be added together are of the type

$$a_n 10^n + a_{n-1} 10^{n-1} + \dots a_2 10^2 + a_1 10 + a_3,$$

when the coefficients of the powers of 10 range from 1 to 9, the radix 10 being clearly of digital origin.

Subtraction is, of course, simply the negative aspect of Addition. Accordingly, too, $a + b$ may be written also in the form $a - (-b)$, in record of the logical meaning of the affirmative as a negation of the negation.

The transition from Addition into the next higher positive art of reckoning must be sought in the dialectic relationship between the two moments of a Number. In this respect, we are at once also concerned with the embodiment of the dialectic of the Quantitative Relation. The two moments of the number are themselves also distinct numbers. Now, *Multiplication* obviously refers to the immediate connotation of the Direct Relation, in so far as the Numerator represents the result of the Multiplication of the Denominator by the Exponent; or also vice versa. Next, the standpoint of the Inverted Relation just as obviously concerns the resolution of a given number into two Factors, and hence brings to the front *Division*, i.e. the negative aspect of Multiplication as an addition of a particular amount of *equal* numbers. When, finally, the two factors of a given product are made equal, so that one and the same number is added to itself as many times as is indicated by its own amount, Multiplication becomes *Involution*, and Division correspondingly Evolution.

In so far, now, as the further display of the Quantitative Relation under the head of Measurableness can immediately be interpreted only in the sense of a further exemplification of the fundamental arts of reckoning in connection with the Counting of the number of times a number is to be multiplied by its own self, it is at once plain that the dialectical whole of the measurable Quality has its correspondence in the *theory of Indices.* And it is equally easily seen that in specific reflection upon the internal character of the alteration in connection with the measurable Quantity, any and every number must next be envisaged in the sense of a Power of a permanently fixed base.

That is to say, after considering numbers with respect to their Involution to any given Power, we are next meant to associate with them the character of a Power to begin with. In so far, now, as, in reflection upon the self-differentiated One of the measurable Quantity, we must at the same time postulate a permanently fixed base, the Index acquires the significance of a *Logarithm.* Thus, by representing numbers as different powers of 10, we get the system of Briggsian or common logarithms. But logarithms could be calculated to any other number employed as base. In this way, we get an analogy to the two groups of chemical Qualities in a series of different bases on the one side and the systems of corresponding logarithms on the other. But that it suffices to calculate the series of logarithms only to a particular base is obvious. For, given

$$n = \alpha^a = \beta^b,$$

we get by taking logarithms to the base α and remembering that $\log_\alpha \alpha = 1$:

$$a = b \log_\alpha \beta,$$
$$\therefore \log_\alpha n = \log_\beta n \log_\alpha \beta,$$

where $\log_\alpha \beta$, serving for the purpose of recalculating the given system of logarithms to the base β, is called the *modulus* of the system of logarithms to the base α.

As regards the present significance of the deepening of the simple chemical affinity into elective affinity, we are obviously meant to supersede the initial element of arbitrariness pure and simple in the choice of the base to a standard system of logarithms. In so far, now, as this supersession must at the same time equally inwardize the entanglement

arising between the standpoint of simple annumeration on the side of the bases, whose series is identifiable with the system of rational numbers from 2 upwards, and the standpoint of arithmetical continuum forcing itself on our attention in connection with the series of logarithms, seeing that integral numbers acquire in this series also the sense of a beyond of infinite approximation; the standard base must be grasped in the explicit sense of a qualitative Quantum, and hence have the form of an Irrational Number. Because, however, this number is to represent the permanent base in inwardization of the significance attaching to the rest of possible bases, as different powers of one fundamental number, and hence is itself *in itself* a power, its determination implies Involution.

That this should be so follows also from the fact that the recapitulation of the standpoint of the Direct Relation, in immediate inwardization of the contradiction between simple counting and arithmetical continuum, falls in the dialectical whole of the measurable Quantity under the head of the immediate display of the Involved Inverted Relation in the Compound Body. Also, seeing that the standpoint of the measurable Quality is, in the present recapitulation of the Substrate's Inwardness, correlated with an exemplification of the fundamental arts of reckoning in connection with the Counting of the number of times a number is to be multiplied by itself, so that the Index of the resulting Power becomes itself a Power, and a Power of an indefinitely increasing degree, just because the Index of the already involved Index always admits of further Involution: when Numbers become next the object of further consideration in the explicit sense of different Powers of a fundamental number, it is by contrast this latter that finally should be exemplified in the sense of an indefinitely increasing Power.

In fact, inasmuch as the postulated Standard Base is to be ushered on the scene in present exemplification of the return from the measurable Quantity into the measurable Quality, it must share the nature of the Involution typifying the standpoint of the measurable Quality. But since the Power in question is at the same time to enter into Presence in the sense of an Irrational Number, it is at once also obvious that this significance must equally apply to the

base of the Power representing the postulated standard base. In so far, now, as the Index and the Base thus in question belong to the Inwardness of the irrational number representing the calculable value of the postulated standard base, and hence are to be contrasted in the sense of the infinitely great and small, the irrational number standing for the base of the Infinite Power representing the standard base must be envisaged in the sense of an infinite progress towards the minimum value which can be associated with the base of a system of logarithms. The beyond of infinite approximation being in that case 1, we arrive finally at the inference that the Standard Base is represented by the expression $\left(1 + \frac{1}{n}\right)^n$, in so far as n stands for a limitlessly increasing number. As a matter of fact, the limiting value of this expression, symbolized by the letter e ($= 2{\cdot}718281828 \ldots$) forms the base of the Napierian or *natural* logarithms, preferred in higher mathematics to the Briggsian or common logarithms on account of the simplicity of expression secured by their employment for the differential coefficients of logarithmic or exponential functions.

In making the Binomial Theorem, implied in the just derived expression for the standard base, the object of further consideration, we are brought back to our initial attitude towards a number as a base of a power of indefinite degree, in so far as this power is at the same time correlated with the stage marked in the chemical analysis of a compound body. Owing to the thus implied return into the standpoint of the measurable Quality, the power acquires again the sense of an algebraic function, whose argument becomes a sum. That is to say, x^n becomes, to begin with, say, c^n and next

$$(a+b)^n = a^n + \frac{n}{1}a^{n-1}b + \frac{n(n-1)}{1 \, . \, 2}a^{n-2}b^2 + \ldots + \frac{n(n-1)}{1 \, . \, 2}a^2b^{n-2} + \frac{n}{1}ab^{n-1} + b^n.$$

As is seen, the simple counting of the number of times a particular number is to be multiplied by its own self acquires now the form of a sum of homogeneous products formed by the powers of the two numbers into which the original base

is resolved, in so far as these products have for their coefficients the number of *Combinations* of n things taken r at a time, the number r being indicated by the index of b and hence ranging from 1 to n. Inasmuch as we are thus referring to a specification of the number of arrangements that can be made by varying the order of some or all of the same number of things, we may say that it is more particularly *Permutations* of n things taken all together, i.e. the so-called "factoral n," symbolized by $\underline{|n}$ or $n!$, that represents Counting at the stage of the Substrate. That which is thus counted is no longer a given number of things, or else a counting arbitrarily broken off at a particular amount, but a counting of the arrangements into which a given number may be self-differentiated. The factoral n presents itself, therefore, to us as a symbol of the fundamental One as regards both its Self-differentiation and Self-integration, i.e. in the sense of a finitized One of Quantity, which becomes the object of the next recapitulation of Substrate's Inwardness under the head of Geometry.

II. *Geometry and Trigonometry.*

Inasmuch as Geometry is not a philosophical science, it must present itself as grounded on a set of axioms and postulates. An axiom was considered by Euclid as a proposition which is so self-evident that it needs no demonstration; and he adduces five such axioms dealing exclusively with equalities and inequalities of magnitudes. The more definite character of geometrical objects is further made to depend upon five postulates, i.e. upon propositions which, although no longer exactly self-evident, do not admit of a *geometrical* proof. In both respects, reference is made to those common notions which either immediately belong to the dialectical Inwardness of the notion of Quantity, as is, for instance, the case with the axiom of Continuity, lying at the base of Pure Analysis, or to an application of notions whose discussion is still ahead of us.

" In so far as Geometry deals with lines, angles, areas and other spatial properties, it suggests itself at first sight to call it the science of Space. It is, however, to be realized that "the 'spaciness' of space does not enter into our geometrical *reasoning* at all. It enters into the geometrical

intuitions of mathematicians in ways personal and peculiar to each individual. But what enter into the reasoning are merely certain properties of things in space, or of things forming space, which properties are completely abstract in the sense in which abstract was defined in Chapter I; these properties do not involve any peculiar space-apprehension or space-intuition or space-sensation. They are on exactly the same basis as the mathematical properties of number. Thus the space-intuition which is so essential an aid to the study of geometry is logically irrelevant: it does not enter into the premises when they are properly stated, nor into any step of the reasoning. It has the practical importance of an example, which is essential for the stimulation of our thoughts. Examples are equally necessary to stimulate our thoughts on number. When we think of 'two' and 'three' we see strokes in a row, or balls in a heap, or some other physical aggregation of particular things. The peculiarity of geometry is the fixity and overwhelming importance of the one particular example which occurs to our mind. . . . However, for all its overwhelming importance, it is but an example." [1]

Accordingly, too, we propose to deal with Geometry in the sense of a further application of the fundamental laws of reckoning. Thus, Addition or Subtraction refers now to the summation of unequal lengths. Multiplication refers equally either to the standpoint of the Direct or to that of the Inverted Relation, according as the product represents a line or an area, the factors being in the latter case the result of the geometrical significance of factorization. And finally, when the factors become equal, the area represents a square.

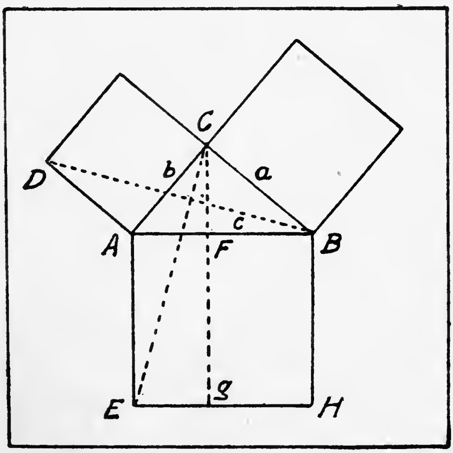
In so far as at the stage of the measurable Quality the Fixedness of magnitude is immediately capable of a concrete embodiment, the previous application of the arts of reckoning concerns us now in connection with the establishing of mensuration formulæ. The Display of the Direct Relation has its obvious correspondence in the treatment of the spatial magnitude as an amount of a given unit length, square or cube. Consistently with the original import of the display of the Inverted Relation, geometrical figures are next considered in reference to their transformation. For

[1] "An Introduction to Mathematics," A. N. Whitehead, Sc.D., F.R.S., pp. 242-3.

instance, a parallelogram admits of being transformed into a triangle, which in turn may be given any shape provided its apex lies on a parallel to its base. If, in transition into the standpoint of the Involved Inverted Relation, we seek the construction by means of which a parallelogram is transformed into a square, we find, to begin with, that

$$\begin{aligned} ab &= \left(\frac{a+b}{2} + \frac{a-b}{2}\right)\left(\frac{a+b}{2} - \frac{a-b}{2}\right), \\ &= \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2 \\ &= c^2. \end{aligned}$$

That is to say, the square representing the area of a parallelogram is equivalent to the difference between the squares on $\frac{1}{2}$ sum and $\frac{1}{2}$ difference of its base and height. It suggests itself, therefore, to construct the base of the resulting square by prolonging the side b up to its intersecting point with the circumference described by $\dfrac{a+b}{2}$. But the *necessity* of this construction depends on a purely geometrical demonstration of the implied Pythagorean theorem. In so far as this theorem concerns the transformation of a parallelogram into a square, its proof must be sought along the line of a mere figural transformation in display of the Inverted Relation. In fact, on inspecting the subjoined figure,



we see at a glance that

$$\tfrac{1}{2}b^2 = \triangle ABD = \triangle AEC = \tfrac{1}{2} \square AEGF.$$

Similarly $\tfrac{1}{2}a^2 = \tfrac{1}{2} \square BHGF$

$$\therefore a^2 + b^2 = c^2.$$

Inasmuch as $\overline{CF}^2 = \overline{AF} \times \overline{FB}$, it follows that C lies on the circumference of a circle having $\overline{AB}$ for its diameter. In this way, it is brought home to us that the circumference of a circle is also the *locus* of any right-angled triangle having the diameter for its hypotenuse. In making this fact the object of our further consideration, we may say to have effected transition into the standpoint of the measurable Quantity.

In correspondence to the specific gravity, we obviously refer to the direct ratio between arcs and chords subtending the same angle at the centre. In that case, the arc stands for the element of Being-for-self, because it is connected with the primary definition of the circle as a curve such that all points on it are equidistant from a given point. Consequently, in combining two ratios $\frac{\text{arc } \alpha}{\text{chord } \alpha}$ and $\frac{\text{arc } \beta.}{\text{chord } \beta}$ we get a ratio implying only the sum of the two arcs, not however of the two chords.

In display of the Inverted Relation, we get in formal correspondence to the two groups in chemical affinity, on the one side, a series of arcs of varying length, and on the other a series of radii of varying length. It will be seen that the angle subtended by an arc of a fixed length is inversely proportional to the length of the radius, and that the law of reciprocal proportions refers now to the fact that the series of angles subtended by an arc of a constant length for a fixed series of radii retains the same constant interrelation for any other arc; or vice versa. In exemplification of the double import of the concluding Negation of the Negation between the two contrasted groups we have the double system employed for the measurement of angles. In practical work, angles are usually measured in degrees, minutes and seconds, whilst in theoretical calculations they are referred to the circular or radian measure of an angle, i.e. the ratio arc/radius = 1. Obviously, if D stands for degrees, and α for radians, we have

$$D^\circ : 360^\circ = \alpha : 2\pi. \quad \therefore D^\circ = \frac{\alpha}{2\pi} 360^\circ; \text{ or } \alpha = \frac{D}{360} 2\pi.$$

In connection with Elective Affinity in transition into the display of the Involved Inverted Relation, the measurement of angles must bring to the front the deeper significance of the circumference of a circle as the locus of a right-angled triangle. In this way, we obviously refer to the trigonometrical ratios of an angle as functions of the sides of a right-angled triangle, when the hypotenuse is identified with the radius. In so far as the triangle has three sides, we get six trigonometrical ratios; namely, as many as is indicated by the "factoral 3". If the base, perpendicular and hypotenuse are symbolized by the letters a, b and c respectively, then the ratios a/c, b/c, and a/b are called respectively the *cosine*, the *sine*, and the *tangent* of the angle a made the hypotenuse with the base, whilst the corresponding reciprocals are the secant ($\sec a = \frac{1}{\cos a}$), the cosecant ($\operatorname{cosec} a = \frac{1}{\sin a}$) and the cotangent ($\cot a = \frac{1}{\tan a}$). Whereas the squares of any of these ratios are generally written $\sin^2 x$, $\cos^2 x$, etc., the symbols $\sin^{-1} x$, $\cos^{-1} x$, etc., are not used as equivalent to $(\sin x)^{-1}$, $(\cos x)^{-1}$, etc., meaning $\frac{1}{\sin x}$, $\frac{1}{\cos x}$, etc, but in reference to the so-called Inverse Trigonometrical Functions, in which case $\sin^{-1} x = y$ means that $\sin^{-1} x$ is an angle y whose sine is x. In so far as the angle is equivalent to y degrees, etc., the same meaning is usually symbolized by the expression $\sin y = x$. It follows, of course, that $\sin(\sin^{-1} x) = x$.

Pythagoras' theorem may now be exemplified in connection with the conversion of the trigonometrical ratios into one another. Seeing that $c^2 = a^2 + b^2$, on dividing through by c^2, we get $\cos^2 x + \sin^2 x = 1$, hence

$$\sin x = \sqrt{1 - \cos^2 x}, \text{ or } \cos x = \sqrt{1 - \sin^2 x}.$$

Or $$\operatorname{cosec} x = \sqrt{1 + \cot^2 x}, \text{ or } \sec x = \sqrt{1 + \tan^2 x}$$

$$\therefore \quad \sin x = \frac{\tan x}{\sqrt{1 + \tan^2 x}}, \text{ or } \cos x = \frac{1}{\sqrt{1 + \tan^2 x}}, \text{ etc.}$$

And since we may also at once read off from a pertinent diagram that

$$\sin(x \pm y) = \sin x \cos y \pm \cos x . \sin y$$

or $$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y,$$

we find that the establishing of a table of trigonometrical formulæ, involving expressions such as $\sin x \pm \sin y$ or $\tan x \pm \tan y$, is simply a matter of routine exercise in application of the fundamental arts of reckoning.

Our dialectical interest is aroused again in connection with the trigonometrical ratios of the *complement* of an angle, namely, of the angle $90° - x$ or $\frac{1}{2}\pi - x$. Obviously, $\sin(\frac{1}{2}\pi - x) = \cos x$, or $\cos(\frac{1}{2}\pi - x) = \sin x$. We are thus reflecting upon the right angle in the sense of a sum of two factors in inverted relation and hence in correspondence to chemical equilibria as Reversible Reactions, in illustration of the principle of the coexistence of different actions. In this connection, it is also forced on our attention that if the two complementary angles are viewed as formed by the rotation of the hypotenuse round its end-points, the rotation is in the one case clock-wise, in the other anti-clock-wise. This comes best to the front when the hypotenuse is treated as fixed in the sense of the diameter of a circle, though in that case we view the complementary angles as formed by the rotation of the sides round the end-points of the diameter. What interests us is the fact that when the two sides become equal, the primarily only implicit equilibrium between the two senses of rotation becomes explicit—but only as a phase entering periodically into Presence, and hence in correspondence to the Substrate as such with respect to its representation by way of a Nodal Line of Measure Relations. Accordingly, too, trigonometrical ratios are calculated on the basis of $\tan 45° = 1$, on the understanding, from the standpoint of the rotating hypotenuse, that ratios of angles greater than 45° refer either to the complement or to the supplement of an angle smaller than 45°, the *supplement* of an angle x referring to the angle $180° - x$ or $\pi - x$. Thus

$$\sin 120° = \sin(180° - 60°) = \sin 60° = \cos 30° = \sqrt{3}/2.$$

In this respect, however, it is also necessary to take notice of the change in the value of the ratio as it passes through the four quadrants. In so far as the radius may be viewed as rotating in two senses, the opposite direction to the hands of a clock is conventionally reckoned positive; but whether the *periodic motion* is positive or negative, there arise in either case also alternations in the direction of the two sides. Thus $\sin(\pi + x) = -\sin x$, and similarly

$\cos(\pi + x) = -\cos x$. That is to say, the base of the right-angled triangle, corresponding to any circumferential point when the hypotenuse is identified with the radius, is either positive or negative according as it extends to the right or to the left of the central point; whilst the perpendicular in turn is positive or negative according as it extends above or below the base. This suggests the idea that the simplest way to determine the position of a point on the circumference is to introduce a pair of co-ordinate axes, intersecting at the centre, and to specify the position of the point in terms of its distance from them. In that case, of course, we are ushering on the scene the standpoint of the Analytic Geometry, when the base comes to be called *Abscissa*, and the perpendicular *Ordinate.*

"The essence of co-ordinate geometry is the identification of the algebraic correlation with the geometrical locus. The point on a plane is represented in algebra by its two co-ordinates x and y, and the condition satisfied by any point on the locus is represented by the corresponding correlation between x and y. Finally, to correlations expressible in some general algebraic form such as $ax + by = c$, there correspond loci of some general type, whose geometrical conditions are all of the same form. We have thus arrived at a position where we can effect a complete interchange in ideas and results between the two sciences. It is impossible not to feel stirred at the thought of the emotions of men at certain historic moments of adventure and discovery—Columbus when he first saw the Western shore, Pizarro when he stared at the Pacific Ocean, Franklin when the electric spark came from the string of his kite, Galileo when he first turned his telescope to the heavens. Such moments are also granted to students in the abstract regions of thought, and high among them must be placed the morning when Descartes lay in bed and invented the method of co-ordinate geometry."[1]

In so far, then, as Co-ordinate or Analytic Geometry correlates the Algebra with Geometry, its explication concerns Substrate's Inwardness at the stage of the Quantitative Relation. But before definitely entering upon this stage, we ought to concern ourselves preliminarily with a mathematical correspondence to the vicious circle between

[1] "Introduction to Mathematics," Whitehead, pp. 121-2.

the two standpoints directly identified in Co-ordinate Geometry. We find that in that case we refer to the so-called Non-Euclidean Geometry.

III. *Non-Euclidean Geometry.*

Non-Euclidean Geometry takes exception to Euclid's fifth postulate, the so-called Parallel Postulate, running to the effect that if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than two right angles.

" The history of the theory of parallels is full of reproaches against the lack of self-evidence of this 'axiom'. Sir Henry Savile ('Praelectiones,' Oxford, 1621, p. 140) referred to it as one of the great blemishes in the beautiful body of geometry; D'Alembert ('Mélanges de Littérature,' Amsterdam, 1759, p. 180) called it 'l'écueil et le scandale des éléments de Géométrie'. . . . From the invention of printing onwards a host of parallel-postulate demonstrators existed, rivalled only by the 'circle-squarers,' the 'flat-earthers,' and the candidates for the Wolfskehl 'Fermat' prize. Great ingenuity was expended, but no advance was made towards a settlement of the question, for each successive demonstrator showed the falseness of his predecessor's reasoning, or pointed out an unnoticed assumption equivalent to the postulate which it was desired to prove." [1]

Now, inasmuch as a rigorous proof seemed a *sine qua non*, the repeated failure to demonstrate the theory of parallels led finally to its repudiation, first of all by Bolyai, Lobachevsky, and, rather hesitatingly, also by Gauss. " Much has been written on the relationship of these three discoverers, but it is now generally recognized that John Bolyai and Lobachevsky each arrived at their ideas independently of Gauss and of each other; and since they possessed the convictions and the courage to publish them which Gauss lacked, to them alone is due the honour of the discovery. . . . It is remarkable that it never occurred to Lobachevsky or Bolyai or their predecessors, Gauss, Schwei-

[1] "The Elements of Non-Euclidean Geometry," Dr. Sommerville, 1914 p. 3.

kart, Taurinus and Wachter, to admit the hypothesis that the sum of the angles of a triangle may be greater than two right angles. This involves the conception of a straight line as being unbounded but yet of finate length. Somewhere 'at the back of beyond' the two ends of the line meet and close it. We owe this conception first to Bernhard Riemann (1826-1866) in his Dissertation of 1854 (published only in 1866, after the author's death), but in his Spherical Geometry two straight lines intersect twice like two great circles on a sphere. The conception of a geometry in which the straight line is finite, and is without exception uniquely determined by two distinct points, is due to Felix Klein. Klein attached the now usual nomenclature to the three geometries; the geometry of Lobachevsky he called *Hyperbolic*, that of Reimann *Elliptic*, and that of Euclid *Parabolic*."[1]

When we now inspect the definition of parallel lines as postulated by Hyperbolic Geometry, we find that AA′ is said to be parallel to BB′ when (p. 30)—

"(1) AA′ and BB′ lie in the same plane,
(2) AA′ does not meet BB′, both being produced indefinitely, and
(3) every ray drawn through A within the angle BAA′ meets the ray BB′."

As is seen, but for the fact that at the same time Playfair's axiom is contradicted, it being assumed that "through any point O *two* parallels OL and OM can be drawn to a given line AB, so that OL || NA and OM || NB," the quoted definition just as well fits two parallel lines in Euclidean Geometry. There are postulated two distinct senses of parallelism, and the property of parallelism is at the same time also interpreted in Euclidean sense. That this confusion does not vanish by interpreting the term "parallel" in the sense of "asymptotic" is plain from pp. 10, 11: "The term 'parallel' (Greek παράλληλος = running alongside) originally connoted equidistance, but the term is used by Euclid rather in the sense of 'asymptotic' (Greek ἀ-σύμπτωτος = non-intersecting), and this term has come to be used in the limiting case of curves which tend to coincidence, or the limiting case between intersection and non-intersection. In non-Eucli-

[1] Ibid., pp. 24, 25.

dean geometry parallel straight lines are asymptotic in this sense, and equidistant straight lines in a plane do not exist. This is just one instance of two distinct ideas which are confused in Euclidean geometry, but are quite distinct in non-Euclidean."

Curiously enough, we would have thought that, in so far as Euclidean geometry associated equidistance with parallels and asymptotes with the limiting case of curves tending to coincidence, it is just the non-Euclidean geometry that confuses these terms. Anyhow, we can readily admit that a line need not intersect another line even though the distance between them continues to diminish indefinitely; in that case, however, the two lines cannot both be straight. What perplexes us is the fact that from the standpoint of hyperbolic geometry a straight line remains straight even when it becomes parallel to two other intersecting straight lines. Consequently, we cannot help asking, what rational need is satisfied by what at first sight appears to be sheer nonsense?

First of all, let us realize that the logic mathematics in general and non-Euclidean geometry in particular claim for their basis, is not the logic of pure thinking, but that of loose thinking. It may seem absurd to distinguish between different kinds of logic, seeing that we ourselves are committed to the standpoint that the nature of thought is unchangeable, and as a matter of fact are engaged on the task of proving that loose thinking is itself an embodiment of the stages cropping up in our own progress in knowing. But, then, the logic of loose thought, the so-called *formal* logic, is obviously simply meant to refer to the degree of self-consciousness reached by pure thought at the stage of its ordinary attitude towards objectivity, when this latter presents itself as something given to begin with, which we may well attempt to describe, compare, analyze and finally reduce to its substrate, but the essential nature of which, the Thing-in-itself, nevertheless still remains unknown and presumably unknowable. It is because Thought remains thus still unconscious of its own fundamental unity with Being, that we may call loose thinking also thoughtlessness and characterize its logic as the still thoughtless logic—in reflection upon the fact that it concerns itself with thought-forms purely analytically, tearing apart their elements, in order to hypostatize them in a series of so-called laws of thought, under

whose sway mental spontaneity acquires a forced and artificial character. We who are born thinkers find ourselves thus under the apparent necessity to *learn* how to think, whilst yet the more *learned* we become in this kind of thinking, the further we find ourselves removed from the true thinking, from the thinking of Truth. Nevertheless, this entanglement is not so much a censurable blunder, as part and parcel of the discipline without which we would never become fully self-conscious or pure thinkers. Our ordinary attitude towards objectivity, as to an immediacy given beforehand, is after all due to the own essential nature of the true Being, in so far as this latter immediately appears to exclude its own inwardness from within its own self, and, thus turning itself inside out, reduces on its own side to a mere Substratum of the very mediation belonging to its own inwardness. So comes it, therefore, that we seek the material for our thinking, first of all, outside our own self, assuming it only in the shape of something given and on that very account crediting it with an unknowable In-itself. For, progress in knowing can, so far, concern the dialectical inwardness of the true Being only with respect to its elements as displayed in the sphere of Presence—those very elements which we have already subjected to a circumstantial consideration. But because what we thus aimed at grasping as a product of our own thinking, the ordinary consciousness, or loose thinking, leaves at the stage of something given, its progress in knowing ultimates in this respect only in riveting its attention on its own presuppositions. Under the sway of the instinct of reason, we forever aim at taking nothing for granted, and hence no assumption, however immediately necessary, escapes an inquiry as to its passport. Accordingly, it is inevitable that mathematical consciousness too comes sooner or later to concern itself with the rationale of its own geometrical axioms and postulates. In that case, of course, unless mathematical consciousness *pari passu* supersedes its own scope and consciously reduces to a mere aspect (instead of fountain) of pure thought, such investigations cannot but end in a web of pure abstractions admitting of no confirmation by either common-sense or pure thought, and hence having the nature of metaphysical monstrosities originating in fancy pure and simple.

The relation between two straight lines in the same plane is embodied by the angle made by them. The variableness comes to our notice when one of them rotates either round their intersecting point or else round any other point lying on it whilst the other line remains fixed. In either case, we get an elementary illustration of the nature of the Measurable with respect to the double aspect of its alterableness. Since this latter is not merely matter of a change by degrees, but also of a change *per saltum*, there results the Nodal Line of Measure Relations, whose infinitude acquires in the present application the form of a cyclical reproduction of the same typical whole of phases. If the one line rotates round its intersecting point with the other line, the angle between them perennially vanishes after reaching its maximal value $\pi = 180°$. But when the rotating line has no fixed common point with the other line, the angle between them keeps disappearing whenever the rotating line passes through the position corresponding to its previous coincidence with the fixed line. It is this position that is hypostatized in the parallel-postulate. That this postulate does not admit of a geometric demonstration is at once obvious from the immediate absence of a relation between two parallels. When their parallelism is interpreted in the sense of a mere phase and hence in terms of the vanished relation, the parallel position of the rotating line remains a beyond of infinite approximation. Just because the vanishing of the relation in question comes then to notice only through the vanishing of this its vanishing (for, however far we trace the receding intersecting point between the rotating and the fixed line in one direction, it ultimately suddenly turns up returning from the other direction even though the extent of its shifting in either direction transcends any assignable magnitude), the parallel position of the two lines cannot be ushered on the scene, or demonstrated as a mere result of a gradual change. To insist on this attitude simply means to entangle oneself in the same kind of conundrum as is exemplified in the old fable of Achilles and the tortoise. In so far as the sudden reappearance of the intersecting point from the other direction may be viewed as an elementary geometric illustration of the change *per saltum* from one Quality into another one, a kind of change which we have from the very first

characterized as hidden from observation, the parallel position comes to represent the Substrate as such and quite appropriately passes for a postulate. After all, Gauss' hesitation in committing himself to an open repudiation of the theory of parallels need not be traced to a lack of courage to publish his convictions—or rather doubts.

Whilst, however, finding ourselves obliged to dismiss the hypothesis of hyperbolic geometry as an instance of metaphysical eccentricity of mathematical consciousness, we ought, it would seem at first sight, to give ready assent to the hypothesis of the other branch of non-Euclidean geometry, the so-called elliptic geometry. For, in that case, a straight line is to be treated as closed and hence as an exemplification of the dialectical inwardness of the idea of a Point, as an abstract negation of space which yet, at the same time, also is in space. As we know, owing to this its own incompatibility with its own Presence, a Point becomes an element of a Line, which, in immediate absence of any other determinant, as regards its Presence, except the negatively self-referent Point, is primarily endlessly straight, but, in further supersession of the simultaneously implied progress *ad infinitum*, becomes a circle. Nevertheless, we do not thus straightway identify a straight line with the circumference of a circle. Whilst defining either as an exemplification of the dialectical inwardness of a Point, we are aware that the negative self-reference of this latter in connection with a circle has a qualitatively distinct meaning from its immediate significance in connection with an infinitely produced straight line. In short, the unity of the straight line with the circle is with us a *negative* unity, and this is precisely where we differ from elliptic geometry, according to which the shortest distance between two points is *not* a straight line.

To argue that because a straight line admits of being dialectically identified with the circumference of a circle, the distinction between them is to be dropped, is quite in harmony with the so-called law of the excluded middle—either this or that, there is no third—but utterly contrary to the logic of pure thinking. According to this, our logic, that which is put past does not vanish without a trace, but is preserved as a lower aspect of the higher meaning, just because pure thought never loses sight of the fundamental unity of

Thought and Being, and of the fact that this unity is a living, a self-discerning one. The true Being is a fusion with its own self in the guise of presupposed immediacy, and even though this immediacy remains *per se* nothing pure and simple, from the standpoint of the self-discerning activity characteristic of the true Being, it retains an appearance of a Being *sui generis*. Accordingly, too, our attitude towards objectivity is not at once governed by pure thought, but remains under the sway of the dualistic standpoint of our ordinary consciousness. The assertion, on the part of elliptic geometry, that the shortest distance between two points is *not* a straight line, is simply another instance of the metaphysical confusion resulting when ordinary common-sense is flatly negated before the insight arising from pure thinking gets beyond its embryonic stage.

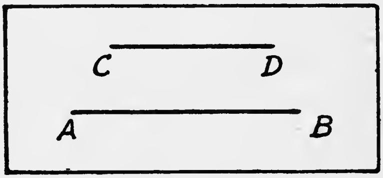
In so far as non-Euclidean geometry refuses simply to take for granted what does not admit of geometrical demonstration, and hence endeavours to supersede what at first sight quite legitimately may be treated as a mere prejudice, its industry commends itself to our philosophical consciousness. For, we are equally committed to taking nothing for granted. But as non-Euclidean geometry nevertheless is not a science of pure thought, and therefore cannot be expected to interpret the meaning of taking nothing for granted in our own way, it, as a matter of fact, simply substitutes for one set of assumptions another and still more questionable one. Whereas Euclidean axioms and postulates have at least the support of common-sense, not to speak of their philosophically tenable import, the assumptions advanced by non-Euclidean geometry remain probabilities arguable only in the name of the thoughtlessly presumed unknowableness of Truth. But, then, although ordinary common-sense is instinctively wedded to the standpoint of pure thought, of Absolute Idealism, it is immediately conscious of itself only as a quest for Truth; and, so long as this ultimate goal of all human industry presents itself only in the image of an unreachable beyond of infinite approximation, no conviction can be certain of its own finality, with the consequence that any absurdity may be rendered plausible.

" Nothing so triumphantly, one may almost say so insolently, ignoring of sense had ever been written before," is

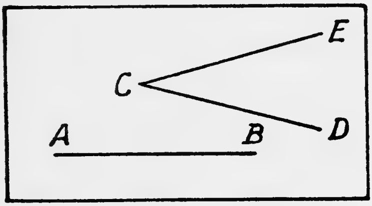
Mr. Hinton's enthusiastic comment on Bolyai's and Lobachevsky's fancies. "Men had struggled against the limitations of the body, fought them, despised them, conquered them. But no one had ever thought simply as if the body, the bodily eyes, the organs of vision, all this vast existence of space, had never existed. The age-long contest of the soul with the body, the struggle for mastery, had come to a culmination. Bolyai and Lobatchewsky simply thought as if the body was not. The struggle for dominion, the strife and combat of the soul were over; they had mastered and the Hungarian drew his line." [1]

Namely,

"Take a line AB and a point C. We say and see and know that through C can only be drawn one line parallel to AB.



"But Bolyai said: 'I will draw two'. Let CD be parallel to AB, that is, not meet AB however far produced, and let lines beyond CD also not meet AB; let there be a certain region between CD and CE, in which no line drawn meets AB. CE and CD produced backwards through C will give a similar region on the other side." [2]



On reading that "there can be no greater evidence of the indomitable nature of the human spirit, or of its manifest destiny to conquer all those limitations which bind it down within the sphere of sense than this grand assertion of Bolyai and Lobatchewsky" we find that with some people the main article of belief still reduces to *Credo quia adsurdum.*

But, then, whilst on the one side vindicating the immediate inclination on the part of ordinary consciousness to reject the hypotheses advanced by the two branches of non-Euclidean geometry, pure thought just as much grants them their *raison d'être* in the system of mathematics. As a recapitulation of the Substrate's Inwardness, i.e. of the dia-

[1] "The Fourth Dimension," Hinton, M.A., 1906, p. 57.
[2] Ibid., pp. 56-57.

lectic of the measurable Quality and Quantity, under each of the main stages in the dialectical whole of Quantity beginning with the standpoint of Counting or Arithmetical Magnitude, this system must equally have a branch corresponding to the Vicious Circle of Quality and Quantity. And it is now easily seen that this stage is indeed fittingly embodied in hyperbolic and elliptic geometry. The former insists on treating purely quantitatively what is essentially a purely qualitative relationship, whilst the latter, by contrast, insist on treating purely qualitatively what is essentially a purely quantitative relationship.

It does not occur to us to doubt the existence of finite lengths because the points, of which a length is analytically conceived as consisting, cannot be counted up; nevertheless, hyperbolic geometry does not hesitate to make the validity of the parallel-postulate depend on an assumption just as irrelevant to the meaning embodied by this postulate as is the analytical assumption that a line consists of points to its own meaning as continuous or extensive magnitude. Two parallels are not in the relation of Something *and* an Other, but in that of Something *or* Other. We may reflect upon them also as an instance of Self-differentiation in abstract contrast against Self-integration. They are one and the same line present as a Two, a Three, etc., *ad infinitum*, i.e. one and the same line when envisaged in the sense of an element of a plane which is yet *pari passu* analytically conceived as consisting of lines. Counting in this respect is eliminated. The relation between parallel lines is of the same kind as that between coincident lines—null and void or purely idealistic, because the One Being is unalterably one One. Just as the central point of a circle is the circumference in the form of intensive magnitude, so parallels belong to the inwardness of one and the same line. The whole space is *in itself* only a point. Hyperbolic geometry is not aware that the impossibility to demonstrate the parallel-postulate is of the same kind as the impossibility to demonstrate God's existence to those who are not yet self-conscious thinkers. But just as ordinary common-sense is content to grant God's existence even apart from any demonstration thereof (though, does not God exist for the very reason that We exist?), it equally readily grants the parallel-postulate, as if aware that the impossibility to de-

monstrate it amounts to a repudiation, not of the postulate itself, but of the analytical treatment irrelevantly accorded to it.

In so far as elliptic geometry quarrels, by contrast, with the every-day platitude that the shortest distance between two points is a straight line, on the ground that in truth there are no straight lines, the one-sidedness committed is obviously of exactly the opposite kind to the one characteristic of hyperbolic geometry. We have already explained that the dialectic attitude towards the straight line does not question its existence. Now, in so far as the distance between two points is a measure relation resting on Quantum, and the shortest distance, consequently, reduces this Quantum to its minimum, this minimum acquires the sense of a limiting value, in connection with which Otherwiseness is submerged in pure self-reference. Translated in terms of a line, this means that the shortest distance between two points is a purely self-referent or one-dimensional line, ie. a straight line. *In truth* there are no straight lines, but the same applies just as much to curved lines ; and, in fact, to the whole universe. In truth *God alone is !* All the same, God invests the aspects of His own Inwardness with an appearance of a Being *sui generis ;* and in that case there are closed lines only upon the proviso that there are also straight lines, planes, volumes, parallels, etc. In so far, then, as straight and closed lines are just as much qualitatively distinct, the shortest distance between two points can be straightway identified with a straight line, there being here only a *quantitative* difference in question. But as is seen, elliptic geometry chooses to repudiate this identification on *irrelevant qualitative* grounds.

In so far, by the way, as according to Kant : " My notion of Straightness contains nothing pertaining to magnitude, but only a Quality ; the notion of what is shortest is, therefore, wholly an addition and cannot be analytically inferred from the notion of a straight line ; it follows, then, that we must have here recourse to intuition which alone makes a synthesis possible," we may pertinently quote Hegel's criticism of Kant's position. " There is here no question of a notion of Straightness, but only of a straight line ; and this is already something spatial, an object of perception. The determination (or if one likes, the *notion*)

of a straight line is after all none other than that it is *absolutely* simple, i.e. that it, in its coming-out-of-itself (the so-called motion of a point), necessarily refers itself to itself, having no difference of any kind in its extension, no relation to any other point or line established outside its own self; it is an *absolutely simple direction within itself.* This simplicity is certainly its Quality, and should it seem difficult to define a straight line analytically, it would be only on account of its determination of simplicity or self-reference, and because every determining suggests to reflection, especially at first, plurality, a determining through something else; but *per se*, there is really nothing difficult in grasping this determination of extension as utter simplicity within itself, as what is not determined through something else. Euclid's definition contains nothing but this simplicity. Now, as regards the presumably synthetical transition from this Quality to the quantitative determination (of the shortest), it is altogether only analytical. As in space, the line is Quantity as such; the simplest, in terms of Quantity, is the *least*, and this, in terms of a line, is the *shortest.* Geometry may take up these determinations as corollary to its definitions; still, Archimedes, in his books on the sphere and cylinder (*v.* Hauber's transl. p. 4), was most consistent in constituting the said determination of a straight line into an axiom, in just as correct a sense as Euclid included the determination concerning parallel lines among axioms inasmuch as the development of this determination into a theorem equally would have required determinations which do not immediately belong to the element of space, but more abstractly qualitative ones of the same kind as the simplicity, equality of direction, and such like of before. These ancients gave even to their sciences a plastic character, strictly confining their exposition to the peculiarity of their subject-matter and therefore excluding all that would have been, in this respect, of heterogeneous nature."[1]

IV. *Co-ordinate Geometry.*

As we have already indicated, the interest of co-ordinate geometry lies in the fact that it relates together geometry, which started as the science of space, and algebra, which has

[1] "Wissenschaft der Logik," I, pp. 231-2.

its origin in the science of number. Whereas algebra concerns variable numbers, correlations between them and the classification of these correlations into types by the idea of algebraic form, and geometry in turn, variable points, their conversion into a locus and the classification of loci into types by the idea of conditions of the same form, the essence of co-ordinate geometry is the identification of the algebraic correlation with the geometrical locus, so that it might also be called "graphical algebra" by contrast to its alternative designation as "analytic geometry". For this reason, then, we are correlating this branch of mathematics with Substrate's Inwardness at the stage of the Quantitative Relation.

The dialectical identification of the straight line with a curve (primarily with the circumference of a circle) as well as the determination of the position of a point in a plane by means of the reference to a pair of co-ordinate axes being already at our back, we find that the forms of the displayed Quantitative Relation on the side of the Ratio are now meant to explicate the dialectical Inwardness of the postulated unity of the straight line and curve on the side of the Exponent. The display of the Direct Relation obviously refers to the equation of a curve as immediately rectified. If x be any abscissa and y the corresponding ordinate, the equation is

$$y = x \tan a,$$

when a denotes the angle formed by the straight line with the x-axis. If we next plot the equation representing the standpoint of the Inverted Relation, namely, the equation

$$x \, . \, y = a,$$

where a is a constant, we get a rectangular Hyperbola referred to its asymptotes as co-ordinate axes. Finally, the graph of the equation typifying the Involved Inverted Relation,

$$y = x^2,$$

represents a Parabola, in record that the exponent does not at this stage immediately represent the Infinite in its conclusiveness, but only in an alternating determination with its finite aspect.

Since the equation $y = x^2$ obviously corresponds only to the primary version of the law of descent of a falling

body, we may further concern ourselves with the graph of the analytical version of this law, in which case the space traversed in any *one* time is at the same time to be treated as though the one time represented a Two. Namely, in distinct reproduction of the standpoint of the Inverted Relation within the Involved Inverted Relation $t^2 = s$ (treating the constant a as $= 1$), we would assume that t^2 represents originally the product of the neutral or common value of the two immediately variable sides of the Inverse Ratio, when the Inverted Relation is primarily embodied by means of the formula $v \,.\, t = s$, where s represents a fixed distance to be traversed by a mechanically moved body, whilst v stands for the velocity of this movement in inverse ratio with its duration. Now because the involution of this ratio implies that v is to be taken as equal to t, whilst s comes to represent the space traversed by a falling body, this space should be representable just as much by the formula $v^2 = s$ as by $t^2 = s$. Only as v in that case ceases to refer to the uniform velocity of a mechanically moved body, there arises the question as to its proper meaning. But, then, as the space traversed by a falling body embodies the standpoint of the Involved Inverted Relation, it involves the characteristic distinction of the Inverse Ratio, i.e. the progress towards the infinitely great and small; and in so far as the formula $t^2 = s$ immediately represents the progress towards the infinitely great, the alternative formula should, by contrast, refer to the progress towards the infinitely small, and hence to an analysis of the space traversed in any one time. This space always admits of being, and for the stated purpose must be conceived as if it had been traversed in two units of time, when the unit time has, of course, the sense of one half of the whole one time. And because v, too, must then be treated as a Two, it represents the velocity acquired by the body in the first unit time, i.e. the so-called Acceleration. Of course, the expression $v^2 = s$ implies the same standard unit as $t^2 = s$ only when s represents the space traversed in the *first two* units of time or $4 \times$ the space traversed in the first unit. But just because $v = 2$, $t^2 = s$ may be written

$$\frac{vt^2}{2} = s, \text{ or also } \frac{Vt}{2} = s, \text{ when } V = vt$$

$$= \text{Acceleration} \times \text{time} = ds/dt$$

As is seen, the analytical version of the law of descent of a falling body may be correlated with the *explicitly involved* Inverted Relation in supersession of the Quantitative Relation as such; as, of course, was from the very first obvious. In returning, now, to its graph: because the space covered in the first half of the original one time must itself be taken as if already covered in two units of time; if the equation $y = x^2$ originally corresponds to the equation $s = t^2$, and is next to stand also for the equation $v^2 = s$, its unit for the measure of the space traversed must be made four times smaller. Accordingly, the restated equation of the parabola is at first sight

$$4y = x^2,$$

or else, if we choose to plot t originally on the y-axis,

$$y^2 = 4x,$$

which is indeed its current form when the constant $a = 1$.

Now, the equation

$$y^2 = 4ax$$

may be written in the form

$$y^2 = 2ax + 2ax,$$

or
$$y^2 + x^2 + a^2 = (x + a)^2 + 2ax$$

or finally
$$\sqrt{y^2 + (x - a)^2} = x + a,$$

so that every point on the parabola is equidistant from a fixed point a called the *Focus* and a fixed line called the *directrix*. If the radius of rotation round any point is the focal radius, then the directrix becomes the tangent to every circle thus described. Therefore, the directrix represents the result of rectification of every possible circle having its centre on the parabola and passing through the focus. But because the focus *negates* the distinction between the infinitely many circular circumferences connecting it with the directrix all through the latter's extension, this connection should also be grasped only in the sense of One circular circumference, in representation of the dialectical truth of the directrix. And in so far as parabola is a curve such that any point on it is equidistant from the focus and the directrix: when this latter becomes the circle described round the focus with the radius $2a$, the parabola itself reduces to the circle described with the radius a.

But inasmuch as we are thus arriving at the conception of the circle in supersession of that of the parabola, the equation of this latter must prove itself to belong to the Inwardness of the circle, even though no longer in its original meaning. For that which is superseded does not vanish without a trace. In fact, as the equation of the circle is

$$y^2 = r^2 - x^2 = (r \pm x)(r \mp x),$$

if we proceed to restate this equation in correspondence to the fact that the equation of the parabola identifies the origin of co-ordinate axes with the vertex, x becomes $(r \pm x)$ and hence identical with either of the two factors into which the expression $r^2 - x^2$ admits of being resolved. When therefore the restated x, i.e. $r \pm x$, is identified with x in the equation $y^2 = 4ax$, $4a$ comes to stand for the other factor $r \mp x$ in the equation for the circle, with the consequence that $4ax$ represents the diameter in the sense of a sum of two factors in inverse ratio. That is to say, we reflect then upon the fact that the square on the perpendicular dropped from any circumferential point on the diameter is equal in area to the parallelogram formed by the two variable subdivisions of the diameter. As is seen, this already familiar result, by virtue of which we have grasped the circumference of a circle in the sense of the locus of the right-angled triangle in transition from elementary or Euclidean geometry into trigonometry, is now before us in restatement of the parabolic equation from within the circle's own Inwardness as the rounded-off parabola.

Seeing, then, that the trend of the just accomplished recapitulation of the measurable Quality under the head of the Quantitative Relation may be said to concern the bending of a straight line into the final shape of a circle, the circumference of this latter should next be envisaged as described from the centre. This means that, instead of representing the position of a circumferential point in terms of its horizontal and vertical distances along two standard lines of reference, we should now additionally define the position of the point by a length and a direction, namely, in terms of its *polar* co-ordinates—quite conformably with the fact that in connection with the standpoint of the

measurable Quantity, we reflect upon our subject-matter as determined from within its own self along with its remaining subjection to external influence. Thus, the two co-ordinates refer now to the rotating radius, called the *radius vector* and symbolized by r; whilst the direction refers to the angle of rotation, called the *vectorial angle.* The central point, by the way, is called the *pole,* and the original position of the radius vector the *initial line.* And as in trigonometry, the vectorial angle, usually symbolized by θ, is viewed as positive, if the direction of revolution is contrariwise to the motion of the hands of a clock.

In so far, then, as, in display of the Direct Relation, we concern ourselves again with the equation

$$y/x = \tan\theta,$$

we must read it conformably with the present attitude toward the circle as described from the centre. The direct ratio has now for its sides the arc and the radius; and in so far, therefore, as we continue to symbolize the amount which an arc is of the radian by θ, we refer to the circular system of measure. That is to say, the direct relation is now before us properly in the form

$$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1} z.$$

Now this equation need not be referred to one and the same circle, but always to another circle; in so far, namely, as the sides of the direct ratio symbolized by $z = y/x$ become variable in consequence of the shifting of the pole along the initial line in either direction. If the pole is conceived as shifted to an infinite distance, the arc coincides with the tangent, or else with respect to the shifting of the pole in the opposite direction, reduces to one of the infinitely many points, of which the tangent is analytically conceived as consisting. In this way, we are obviously putting on record the contradictory nature of the Direct Relation at the transitional stage into the Inverted Relation. We have here the Point as an element of a straight line, in so far as this straight line is at the same time also an element of the curve. The straight line has been from the very first presupposed as in direct unity with the curve (any straight line when referred to its co-ordinate axes, is called a "curve"); but now, after having explicitly bent the straight line into

the shape of a circle, its direct unity with a curve may be viewed as immediately established.

When we plot the equation $r\theta = a$, in display of the Inverted Relation, we get a curve such that the ratio of the distance of any two points from the pole is inversely proportional to the angles between their radii vectores; the so-called *hyperbolic spiral*. The curve keeps, on the one side, winding round the pole, and on the other approaching ever closer to the initial line. The approximation has, however, in either direction an unattainable beyond, because the Inverted Relation is not yet *involved*.

But as only immediately involved, the Inverse Ratio is not yet fully superseded. Its sides are equalized and hence appear to lose only their quantitative, not yet their qualitative distinguishedness. But that r and θ, whilst still retaining their previous meaning, the one as a straight line, the other as a curve, at the same time already reflect their meaning into one another, becomes plain when we plot $r\theta = A$, for $r = \theta = \sqrt{A}$. The result is the so-called *Archimedes spiral*, whose successive whorls are at the fixed distance of π from one another, when π represents 360 times the radius vector for $\theta = 1°$. In order to express this distance in terms of the arc $1°$, we must, of course, multiply the unit radius vector, and hence π, by 57·295, i.e. by the amount of degrees in one radian. The distance between the whorls may also be written $2a\pi$, when $\pi = 180°$ and a stands for the amount which a particular radius vector is of the arc subtending the corresponding vectorial angle.

In further deepening of the standpoint of the Involved Inverted Relation, when, that is, the display of this standpoint is emphasized with respect to its middle phase, in higher restatement of the previous display of the Inverted Relation, but so that this restatement acquires the character of an anticipatory reference to the Relation between the Original and the Derived Function (as has been exemplified in the method adopted for the calculation of the rate of a particular transformation in the same reacting system), the distances of the points in which the same radius vector cuts the successive whorls of the spiral must form a geometrical instead of the immediate arithmetic series in the case of the Archimedes spiral. Obviously, we are thus referring to the *logarithmic* or *equiangular spiral*, which suggested itself to us as a fitting

graphical illustration of the periodicity connected with the entrance of the Substrate into Presence by way of a Nodal Line of Measure Relations. The equation of the spiral is $r = a^\theta$, where a is constant. Hence $\log r = \theta \log a$ and $\log \frac{r_1}{r_2} = (\theta_1 - \theta_2) \log a$, i.e. the logarithm of the ratio of the distance of any two points on the curve from the pole is proportional to the angle between their radii vectores. The logarithm of the ratio of r_1 to r_2, r_3, . . . on the same straight line is given by $2\pi \log a$, $4\pi \log a$, . . . , seeing that $\theta_1 - \theta_2 = 2\pi = 360°$. Therefore, the spiral is made up of an infinite number of turns which extend inwards and outwards without limit. As already mentioned, the geometrical periodicity of the curve is a graphical representation of the periodicity perceived by the ear when a tone continuously rises in pitch, but it may also be used to illustrate the Newlands-Mendeléeff law of octaves, by arranging the elements along the curve in the order of their atomic weights.

As regards, finally, the curve embodying the concluding stage of the displayed Involved Inverted Relation, we cannot but identify it with the *ellipse;* not so much because this is the only curve that has not yet come up for treatment, but because we are now also concluding our embodiment of Substrate's Inwardness in abstract recapitulation of the measurable Quality and Quantity under each of the main stages of the dialectic of Quantity, and consequently have now arrived at a geometrical symbol of the Substrate in explicit correspondence to the Derived Function, in qualitative contrast to its original meaning as Absolute Indifference, which presented itself to us in conclusion of the dialectical whole of the measurable Quantity as a correspondence to the outcome of the dialectic of the Quantitative Relation as such, when the fundamental unity of Quality and Quantity is established only with an accent on Quantity and hence calls for an additional vindication of the implied return into Quality. We have from the very first made plain that our systematization of mathematics is to be viewed only in higher correspondence to the original explication of the to begin with only implicitly involved Inverted Relation.

At first, we might have been under impression that this task would include also higher mathematics, but systematic consistency compels us to drop our concern with mathe-

matics as soon as it outstrips the purely mathematical consciousness which alone corresponds to Absolute Indifference. Professional mathematicians may lament Euclid's limitation, in so far as "he never thinks of the line as an entity given once for all as a whole,"[1] or "always considers the whole circle as described".[2] But in so far as Infinitude remains quantitatively only a beyond of infinite approximation, it can become an object of mathematical consideration only as immediately finitized. What is it that we think about, if we are to "think of the whole of a straight line throughout its unending length in both directions," "in the place of a bit of a straight line"? Yet "this is the sort of general idea from which to start our geometrical investigations". But, then, is not an *endless* straight line *thinkable* only as a circle? Does not *endlessness* mean self-returnedness? That which is endless is also beginningless. Why blame Euclid on the ground that he "never starts with mere segments (i.e. bits) of circles, which are then prolonged"? In considering the whole circle as at once described, he actually did start from "the sort of general idea from which to start our geometrical investigations". Nevertheless, we are to believe that "his defective consideration of the straight line" was "the cause of the comparative sterility of the study of the science during so many centuries," because "the straight line, considered as a whole, is the root idea from which modern geometry starts" and "unfortunately the circle is not the true fundamental line in geometry"! Really, mathematicians are the last people to sit in judgment—as they so dearly love to sit—on *philosophical* comments on their own science, simply because they do not know what they are saying. "With hardly an exception, all the remarks on mathematics made by those philosophers who have possessed but a slight or hasty and late-acquired knowledge of it are entirely worthless, being either trivial or wrong. The fact is a curious one; since the ultimate ideas of mathematics seem, after all, to be very simple, almost childishly so, and to lie well within the province of philosophical thought."[3]

Well, since we entertain not the slightest shadow of doubt as to the appropriateness of this gracious admission, we

[1] "Introduction to Mathematics," Whitehead, p. 119.
[2] Ibid., p. 120. [3] Ibid., pp. 113-4.

submit that the *really* true fundamental line in geometry is *ellipse.* Of course, we do not expect *thoughtless* mathematicians to give up their own presumptions on the ground of anything we might say, any more than we expect to convince an unbelieving Thomas of God's existence; but since, after all, fundamental ideas embodied in the universe are primarily of *philosophical* import, we are really not concerned about the treatment our own exposition of Absolute Idealism is likely to receive at the hands of philosophical barbarians. We remember Lord Kelvin's indignation at Hegel's presumption to discuss scientific subjects without being a man of *empirical* science; but we say: Every cobbler to his last! Philosophy of any science and herewith also of mathematics is *our own* domain, where we bow only and solely to logical necessity in exposition of the nature of God as He is in His own Essence.

B. Indifference as Inverted Relation of its Factors.

In order to vindicate our attitude to the *ellipse* as a geometrical symbol of the Substrate, we proceed to concern ourselves with Substrate's Inwardness in supersession of its immediate correlation with the purely mathematical consciousness—in correspondence, that is, to the original supersession of the standpoint of the Quantitative Relation under the head of the Relation betwen the Original and the Derived Function. As mediated within itself with itself by the negation of all the determinations of Presence, Substrate is not merely Absolute Indifference, but a self-discerning, self-to-self-referent distinguishing. It also implicitly includes what it excludes, and therefore must itself stretch over its own phases, and thus present itself in correspondence to the Relation between the Original and the Derived Function. That its next designation as a sum of two factors in inverse ratio is indeed based on its own Inwardness will be seen from Hegel's own treatment of our present subject-matter:

"It is now to be seen, how this determination of Indifference is established in its own self and therewith as *Being-for-self.*

"(1) The Reduction of measure relations, valid at first as self-subsistent, establishes one Substrate of the same. This

is their continuity into one another; consequently, the inseparable self-subsistence which is *wholly* present in its distinctions. For this distinction, there are present the determinations contained in it, Quality and Quantity; and all depends wholly only on the *how* these are established in it. This, however, is determined by this, that the Substrate is primarily as result, and *in itself* as mediation. But as this latter is not yet explicitly established in it, the total self-subsistence is primarily only a Substrate and with respect to its phases as Indifference.

"The distinction is therefore in it essentially, first of all, the only quantitative, external one; and there are two distinguished Quanta of one and the same substrate which is in this manner the *sum* of the same, consequently itself determined as Quantum. The Indifference is, however, this fixed measure, virtually absolute limit, only with respect to those distinctions; so that it is not a Quantum in its own self, nor becomes opposed in any way, as sum, or also exponent, to others, be they sums or indifferences. It is only the abstract determinateness which falls into the Indifference; the two moments to be established as moments in it are variable, indifferent, greater or smaller against one another. Limited, however, by the fixed limit of their sum, they are just as much correlated, not externally, but *negatively*: this is their qualitative relation to one another. They are therefore *inversely proportional.* From the original formal Inverted Relation, the present one is distinguished by this, that here the whole is a *real substrate*, and each of the two sides is established as having itself to be *in itself* this whole.

"According to the stated qualitative determinateness, there is further present the distinction as of *two Qualities*, of which the one is superseded, at once excluded and included, by the other; but as held in one unity, which they constitute, neither is separable from the other. The Substrate itself is, as Indifference, equally in itself the unity of the two Qualities; each of the sides of the ratio contains, therefore, just as much both of them within itself, and is distinguished only by a more of the one Quality and a less of this other, or vice versa. The one Quality is through its Quantum on the one side only the *preponderant one*, the other on the other side.

"Consequently, each side is in its own self an inverted relation; This relation turns up formally on the two distinguished sides. These sides are thus themselves continuous into one another also with respect to their qualitative determinations; each of the two Qualities relates itself in the other to its own self, and is in each of the two sides only in a different Quantum. Their quantitative distinction is the said Indifference, according to which they continue themselves the one into the other; and this continuation is as identicalness of the Qualities in each of the two unities. The sides, however, each as the whole of both determinations and hence each as containing the Indifference itself, are thus at the same time established against one another as self-subsistences.

"(2) As this Indifference, Being is now a specification of measure as no longer in its immediacy, but in the just demonstrated developed manner;—Indifference as it is *in itself* the whole of the determinations of Being, which are resolved into this unity;—just as much Presence as totality of the established realization, in which the moments themselves are the totality-in-itself of the Indifference borne by this latter as their unity. But because the unity is only as *Indifference*, and therewith held fast only as it is *in itself*, and its moments are not yet determined in the sense of Being-for-self, i.e. as superseding their own self *in their own self* and *through one another* into a unity, there is therefore generally present the Indifference of the fundamental unity to its own self as developed determinateness.

"This inseparable self-subsistence is now to be considered closer. It is immanent in all its determinations and remains within them in unity with its own self, untroubled by them, but (*a*), as remaining *in itself* totality, contains the phases of its own Inwardness only as groundlessly emerging in it. The *In-itself* of the Indifference and this its *Presence* are unconnected. Modes of Being display themselves in it in an immediate manner, it itself being wholly present in each of them. Consequently, the distinction between them is to be treated, to begin with, as purely idealistic; that is to say, purely quantitative, not yet as involving Self-differentiation on their own part, seeing precisely that they are not to be invested with self-determination, but treated as *externally* determined as regards their Presence and its further specification.

"(β) The two moments are in an inverted qualitative relation;—an up and down of magnitude, determined, not by Indifference, seeing that this is precisely indifferent to this increasing or decreasing, but herewith purely externally. The principle of determination is here traced to something else than Indifference; to something outside it. The *Absolute* as Indifference has on this side the second defect of the quantitative form, in that the character of the distinction is not determined by it itself, just as it has the first defect in this, that distinctions on the whole simply *turn up* in it, that is to say, are not an outcome of its own mediation with itself.

"(γ) The quantitative determinateness of the moments which are now sides of the relation, constitutes the mode of their *subsisting*; their Presence is, on account of this their Indifference, freed from qualitative alteration. But in distinction from this their Presence, they also subsist implicitly, in that they are *in themselves* the Indifference itself, each being itself unity of the two Qualities, into which the qualitative moment disrupts itself. The distinction between the two sides is restricted to this that the one Quality is established on the one side with a more, on the other with a less, the other Quality being the reverse. Hence each side is in it the totality of Indifference. Each of the two Qualities, taken singly on its own account, remains the same sum which the Indifference is, it continues itself from the one side into the other and is not restricted by the quantitative limit which is thus established in it. Consequently, the determinations in question are found in immediate antithesis which develops itself into a contradiction now to be considered.

"3. Namely, each Quality enters *within* each side into reference to the other, and that in such wise that, as has been found, this reference too ought to be only a matter of quantitative distinction. If the two Qualities were self-subsistent, say, in the sense of mutually independent, sensuous stuffs, then the whole determinateness of Indifference would be torn to pieces; their unity and totality would reduce to empty words. They are, however, at the same time expressly determined as comprised in One unity, as inseparably united; each as having sense and reality only in this one qualitative reference to the other. Seeing, then,

that *their Quantitativity is directly of this qualitative nature, each reaches only as far as the other.* Viewed from the standpoint of the quantitative distinction between them alone, the one would go over beyond the other and have in its More an indifferent Presence in distinction from the other. But owing to their simultaneously qualitative correlation, each is only in so far as the other is. From this it follows that they are in *Equilibrium*; that by as much as the one increases or decreases, the other equally increases or decreases; and increases or decreases at the same rate.

"Therefore, on the ground of their *qualitative* connectedness, a *quantitative* distinction between them, or a *More* of the one Quality, is out of the question. The More, by which the one of the correlated moments is conceived as stretching beyond the other, would either remain *en l'air* or else be again only the other itself; in which case, however, we are reflecting upon an equality between them, which cannot be postulated for their Presence, in so far as this latter is to rest only on the inequality of their Quantum. Each of these would-be Factors vanishes, whether conceived as *beyond*, or as *equal* to, the other. This vanishing presents itself to quantitative conception in the sense of a disturbed equilibrium, so that the one factor becomes greater than the other, whose qualitative supersession, and consequent untenability, is thus rendered explicit. As the one factor acquires preponderance, the other factor decreases with accelerated velocity and is overpowered by the former which in this way constitutes itself into the sole self-subsistence; but so there are no longer two specific moments and factors but only the one Whole.

"As thus constituted by the totality of determinations whilst yet retaining the sense of Indifference, this unity (the one Whole) is an all-sided contradiction, and is therefore to be *established*, in supersession of this self-contradictory contradiction, in the sense of an independent Self-subsistence such that its result and truth is no longer the simply indifferent unity, but that immanently negative and absolute unity which is called *Essence*."[1]

Put concisely, paragraph by paragraph, the preceding unfoldment of the mediation which the Absolute Indifference is within itself amounts to this:—

[1] "Wissenschaft der Logik," I, 440-5.

(1) As regards its determinate Being, Indifference is primarily Substrate.

The distinction in it being thus at first purely quantitative, it presents itself in the sense of a sum of two Quanta in inverse relation.

Seeing, however, that each of the sides is in itself the whole, the distinction is just as much of qualitative kind: each side contains two Qualities of which the one or the other preponderates only quantitatively.

Each side is, consequently, in its own self, an inverted relation and, whilst being qualitatively continuous with the other, is also established against it as the whole Indifference.

(2) We have thus before us the Indifference as such, as indifferent to its own self-determining character; to its own self-representation.

Consequently, (α) it represents itself distinctively only by way of Self-differentiation pure and simple, i.e. distinctions on the whole simply *turn up* in it; and

(β) for that reason, the principle of determination presents itself in this respect as lying in something outside its own self.

(γ) But this external or quantitative determinateness of their mode of subsisting does not affect their qualitative continuity into one another.

(3) On the ground of their qualitative inseparableness, quantitative distinction between the sides of the inverse ratio now in question is out of the question: the sides are in Equilibrium.

But since their Presence at the same time rests only on the inequality of their magnitude, their equilibrium is just as much out of the question: each of the factors vanishes whether conceived as unequal or equal to the other.

That is to say, the determinate Being or Presence of the Indifference is an all-sided Contradiction.

Obviously, this Contradiction is the present reappearance of the vicious circle between the infinitized Finite and the finitized Infinite, whose supersession yields the notion of the true Being, to be called hereafter *Essence*. But this significance of the true Being will really become the proper object of our thought only in ultimate result of our intended circumstantially systematic treatment of the concluding section of the dialectical whole of Measurableness, in refer-

ence to the fundamental unity of the measurable Quality and Quantity. In fact, for the time being we are still only concerned with the transition into this circumstantial exposition of the ultimate transition into the next main department of the system of pure thought and its objective significance. In other words, we are immediately concerned only with the import of the all-sided contradiction, as which Indifference or Substrate proves itself to be in *immediate* outcome of its mediation within itself.

It will easily be seen that, in correlating the Substrate with mathematical consciousness, we have really also at once anticipated its present meaning as result of its mediation within itself. After all, mathematical consciousness does not step on the scene ready made, but must be formed; and this its formation is an instance of the resulting self-reduction to mere Indifference on the part of the simultaneously all-embracing one Whole. How difficult such a self-reduction is, comes home to us, in our school-days, when we have to force ourselves to manipulate empty numbers or their algebraical symbols in temporary oblivion of everything else. It is for this reason that counting is taught at first with the help of fingers, bullets or concrete images; and that exclusive concern with figures and generally pure mathematics converts mind into a mere thinking, or rather thoughtless, machine.

The prevailing impression that mathematics is a training in logical thinking *par excellence*, rests simply on a confusion of the abstract with pure thinking. The antithesis in this respect is of the same kind as the one between Indifference as such on the one side and the immanently negative absolute unity called Essence on the other. Not as if abstract thinking had no claim to cultivation—the present writer finds the study of mathematics most absorbing and delightful; all that is meant here is that everything should be viewed in its proper perspective. Even virtue turns into vice when carried too far. We must eat in order to live, but another thing is to live in order to eat. Pure thought itself insists on reducing itself to the level of abstract thinking, but that does not mean that it is ready to subordinate itself to the formal logic governing mathematical operations.

Mathematical consciousness presents itself to us, therefore, in the sense of a middle term between the purely

thinking consciousness and its obverse aspect in the ordinary empirical consciousness. Accordingly, too, its embodiment in mathematics admits of being envisaged either in the sense of a self-degradation on the part of pure thought, or else in the sense of a self-idealization on the part of the ordinary attitude towards objectivity. So comes it that a purely philosophical analysis of the fundamental ideas in mathematics finds itself confronted with *Principia Mathematica* of emperical rather than philosophical origin. We have here an instance of the returning movement characteristic of every dialectical whole, but so that it presents itself as if broken into two unconnected halves. Philosophy of mathematics aims at developing the primarily blank mathematical consciousness into a totality of its own determinations. Just because mathematical consciousness is analagous to the Substrate, distinctions on the whole simply *turn up* in it and their principle of determination is to be sought outside its own self—a reason, by the way, why Hegel, even though granting the possibility of a philosophy of mathematics, did not find its subject-matter sufficiently attractive for purely continuous thinking. Now, whereas we have found the principle of determination, welding together the successive branches of mathematics, in the successively repeated recapitulation of the Substrate's Inwardness under each of the main stages of the dialectical whole of Quantity beginning with Counting as such, a mathematician naturally correlates his *Principia Mathematica* with practical needs of the empirical consciousness and ends in identifying his generalizations with the fundamental ideas of the universe. He does not, therefore, take up the system of mathematics as it results from our consideration, or, which is the same, in direct development of the purely mathematical consciousness, under the already self-conscious or still self-oblivious sway of pure thought, but shifts the ground to the totality of measurable Being as given in experience and treats the system of mathematics as an outcome of mathematization of the universe. Nevertheless, mathematics cannot embody the essential nature of the universe. This essentially negative unity can become its subject-matter only in the form of abstract, lifeless unity. Under its sway, the universe with its absolute and inherent process of differentiation reduces to the condition of Oneness. As thus constituted

by the totality of determinations whilst yet retaining the sense of Indifference, mathematical consciousness proves itself as regards its Presence an all-sided contradiction.

"The relation of a whole," Hegel proceeds to remark, "which ought to have its determinateness in quantitative difference of factors determined qualitatively against one another, is applied to the elliptic movement of celestial bodies. In this case, we find primarily only two Qualities in inverse relation to one another : not two sides, of which each is itself the unity of both and their inverse relation. In so far, now, as theory [mathematical consciousness] simply clings to that primary empirical fact (to the inverted relation of two *simple* qualities), it passes unnoticed that, so far from being an adequate expression of this fact the theory really destroys it ; or rather, in that the basal fact is, as is proper, retained, that the theory proves itself empty as against it. Only the ignoring of this consequence makes it possible that the fact and the theory conflicting with it calmly co-exist.

"The simple fact is that in the elliptical movement of celestial bodies their velocity is accelerated on the journey towards the perihelium and retarded on the way towards the aphelium. The quantitative side of this fact has been exactly ascertained by the untiring diligence of observation and even reduced to its simple law and formula ; hence, all that can be truly expected from a theory has been done. But this did not seem sufficient to the reflecting intellect [mathematical consciousness]. For the purpose of the so-called explanation of the phenomenon and of its law, there are assumed a *centripetal* and a *centrifugal force*, as qualitative moments of the movement in a curved line. Their qualitative distinction consists in the opposition of their direction, and, from the quantitative standpoint, in this, that while being determined as unequal, the one should decrease as the other increases, and vice versa ; then also, further, that the relation of the same becomes again reversed : that after the centripetal force had for some time been increasing and the centrifugal force correspondingly decreasing, there enters a point, when the centripetal force begins to decrease and the centrifugal force to increase. This conception is, however, in contradiction with the mutual relation of their essentially qualitative determinate-

ness. Owing to this latter, they cannot at all be parted; each has sense only in reference to the other; so far, then, as one has a surplus over the other, so far would it have no reference to this latter and, in fact, would not be present.

"In connection with the assumption that the one is once greater than the other, and, as thus greater, still stands in reference to the smaller, there comes in what has been said above: that it would obtain absolute preponderance and the other disappear. This other is then established as a vanishing, vacant moment; and with respect to this its determination it makes no difference that the vanishing takes place only by degrees, or that, by as much as it decreases in magnitude, the other increases; this is ruined together with the other, seeing that it is what it is only and solely in so far as the other is. Surely, it is obvious that if, e.g. as is alleged, the centripetal force of a body, on its way towards the perihelium, should increase and the centrifugal force by contrast proportionately decrease, the latter *would no longer be able* to tear the body away from the grasp of the former and to remove it again from its central body; on the contrary, once the former gains preponderance, the other is overpowered, and the body brought back to its central body with accelerating velocity. Conversely, if the centrifugal force is gaining ascendency when approaching aphelium, it is equally contradictory that it should be overpowered by the weaker centripetal force at the aphelium itself.

"It is clear, further, that it would be a foreign force which would bring about this reversion; that is to say, that the now accelerated, now retarded velocity of the movement cannot be *grasped* (or, as it is named, *explained*) in terms of the assumed determination of those factors, although they have been assumed for the very purpose of explaining this distinction. The implied disappearing of the one or the other factor, and hence of the elliptical movement on the whole, is ignored and concealed because, as a matter of fact, this movement continues and goes over from the accelerated velocity into the retarded one. The assumption that the weakness of the centripetal force turns at aphelium into a preponderating strength against the centrifugal force, and conversely at perihelium, contains, *partly*, what has been developed above: that each of the sides of the in-

verted relation is in its own self this whole inverted relation. The side of the movement from aphelium to perihelium—when the centripetal force is supposed to preponderate—is still assumed to imply the centrifugal force, but on the decrease as the former increases; whilst on the side of the retarded movement, it is the preponderating, and in preponderance ever increasing, centrifugal force, that is in just the same relation to the centripetal force. Consequently, on neither side has one of them disappeared, but only grows smaller until it begins in its turn to preponderate over the other. There recurs thus in each side only that which is the defect in this inverted relation. So far as each force is taken self-subsistently for itself, and both are viewed as merely externally joined in a movement (as in the parallelogram of forces), the unity of the Notion, the nature of the thing itself, is left out of sight; but inasmuch as both are qualitatively related to one another by the Notion, neither can be given an indifferent, self-poised subsistence against the other, depending only on a More. The form of intensity, the so-called dynamic element is here to no purpose, because it has itself its determinateness in Quantum, and hence can imply force, i.e. can be at all only in so far as its effect is balanced by the opposite force. But the self-conversion of preponderance into its contrary implies also an alternation in the qualitative determination of the sides as either positive or negative, the more of the one meaning so much less of the other. The inseparable qualitative coherence of this qualitative contrast is distorted by the theory into a *succession by turns;* but just so an *explanation* of this alternation, as well as more particularly of this distortion itself, remains a desideratum. The semblance of unity which is still implied in the increasing of the one side by so much as the other decreases, disappears here completely; the assumed purely *external* succeeding of the one side after the other contradicts only the inference to be drawn from their qualitative coherence, namely, that preponderance of either argues vanishing of the other [with the consequence that the quantitative aspect of their mode of subsistence cannot come up for treatment at all except in utter contradiction with the theory which yet pretends to explain it]."

But along with illustrating the nature of the mathematical

consciousness as an all-sided contradiction, the quoted remark serves also to force on our attention that this contradiction displays itself just as much objectively in the planetary motion. The idea of absolutely free movement is postulated in conclusion of the dialectical whole of Matter and Movement under the head of the measurable Quality, and it is proper that the dialectical whole of the measurable Quantity should bring us back to it. But now we have it before us in transition into the concluding section of the total dialectical whole of Measurableness, having for its object-matter the fundamental unity of the measurable Quality and Quantity, as is obvious from the trend of the dialectical progress in the two preceding sections. In so far as the standpoint of the measurable Quality concerns *physical* change of Matter, and that of the measurable Quantity, by contrast, *chemical* change, it would suggest itself at first sight that the concluding section should reflect on *vital* process, especially as this process may indeed be said to be distinctly foreshadowed in the properties of the radio-active Matter. But a reflection on this *essential* nature of Matter lies still beyond the scope of our present progress in knowing. Just as we treated physical or chemical change as a mere display of the qualitative Quantum, in simultaneous recapitulation of what we have already designated as the original element of our progress in knowing, i.e. the dialectical whole of Quality and Quantity; so we are finally meant to treat the no longer inert or dead matter as mere display of the fundamental unity of Quality and Quantity in concluding recapitulation of its embodiment in the two foregoing sections of Measurableness. And so comes it that Life assumes, so far, only the significance of Absolutely Free Movement.

That this movement cannot be explained from the standpoint governing our ordinary handling of Matter is obvious. So far, Matter must needs come to our notice only in its immediate significance, as something excluded out of our own self and for that reason devoid of the principle of spontaneity, coming to the front only partially in the conditionally free movement. Before we can grasp Absolutely Free Movement, we must altogether supersede our ordinary external attitude to objectivity, the standpoint occupied by the mathematical consciousness, or in a word, Intellect, and

hence advance to that standpoint, at which the totality of determinations implicitly belonging to the own Inwardness of the fundamental Oneness (or All-oneness) is also explicitly inwardized, so that the principle of determination is no longer traced to something else outside, but to its own *essential* mode of subsistence. Just because Intellect is our capacity for abstract thinking, it takes its stand by the *principium identitatis* and the rest of abstract maxims which it dignifies by the name of *laws of thought*, though they have as much right to this name as Newton's principles for measuring forces have to their current designation as *laws of motion.* But *principium identitatis* is only another term for Absolute Indifference in higher restatement of the immediate attitude towards the Infinite as what is utterly incompatible with the Finite. Just as in this case, it is not immediately realized that, owing to the implied exclusion of finitude, the Infinite is itself straightway deposed to the rank of another Finite; so it equally passes unnoticed that, in excluding distinction, Identity or Absolute *Indifference* is from the very first also pregnant with *Difference*, and that its true nature is so far anticipated in the sense of a Third—that very third which presumably there is not! To deny this third its *raison d'être* amounts to denying the *true* Being—and, as a matter of fact, Intellect does convert Absolute Truth into a figment of *its own* fancy; into the Unknowable, quite unaware (in keeping with its own character as an all-sided contradiction) that it thus places its own pursuit of knowledge *en l'air*.

In order, however, fully to supersede this intellectual, but not yet intelligent, attitude towards objectivity, we must just as much aim at gaining an *experience* of the consciousness resulting from the deepening of the mathematical consciousness into that self-determining totality of distinctions which is to be called hereafter Essence. The fact that pure thinking only too easily—to those who are not yet capable of self-conscious thought—assumes the appearance of a bacchanalian dance of bloodless categories, is manifestly due only to an inability to estimate their bearing on actuality; and this inability is generally due to the narrow limits of one's own experience. Pure thinking is truly also an inwardization of every possible experience; the universe with all its contents is its own Inwardness,

even though this Inwardness is *pari passu* displaying itself in the guize of Thoughtlessness. To find ourselves confronted with a world, not only apparently taking no heed of our own thinking, but even ridiculing it, is no anomaly to us. Pure thought itself insists that it cannot be otherwise, because such is the nature of the true Being. Unless he relapses into a fit of thoughtlessness, a pure thinker communes in objectivity purely with his own self in the guize of Otherwiseness. The World has nothing to offer him but what is *implicitly* his own. The dialectic of progress with its clash of opinions and even arms, is his own dialectic. Whilst apparently living a secluded life, a philosopher finds himself at the very centre of universal strife, with his hand on the very pulse of civilization. *Ideas govern mankind*, and he is the preacher of ideas and ideals *par excellence!*

Now, we have already made plain to ourselves that mathematical or intellectual consciousness stands midway between the purely thinking consciousness and the ordinary sensuous, or we might also say, observational consciousness. Its character as an all-sided contradiction is simply due to the fact that it is the meeting ground of the idealizing tendency which does not permit us to remain wholly under the sway of our sensuousness, and of the materializing tendency on the part of the purely thinking consciousness which in its own turn presses for self-embodiment. On both sides, there is an evidence of the self-discerning activity of the true Being as the unity of Thought and Being. Sensuous consciousness misses the idealistic element of thought, whilst this latter misses in turn its own implicitly presupposed Objectivity. As an only abstract middle term, mathematical consciousness excludes, therefore, both the sensuous as well as the purely thinking element, and hence stands for materialistic Idealism or idealistic Materialism. This is contradictory; but, then, this contradiction is just the very character of the mathematical consciousness. Materialism negates Idealism only in such wise that it itself is infected by it; it cannot get away from it, because its own foundation lies in it. And the same applies in reverse sense to Idealism. Many a professed materialist puts to shame, by his idealistic tendencies, many a professed idealist; and vice versa. Matter cannot be separated from its

fundamental property, Heaviness; yet in being heavy, Matter proves itself from the very first as pure Ideality. In order to supersede this immediate conflict between the two tendencies brought together in mathematical consciousness, we must deepen their immediately only direct unity into the negative one representing the true Being itself. And we find that the *experience* of this process of deepening is procured in the simplest manner by a training in visualization of Mr. Hinton's block of cubes.

"I found myself—and many others I find do so also—I found myself in respect to knowledge like a man who is in the midst of plenty and yet who cannot find anything to eat. All around me were the evidences of knowledge—the arts, the sciences, interesting talk, useful inventions—and yet I myself was profited nothing at all; for somehow amidst all this activity, I was left alone, I could get nothing which I could know.

"The word 'sceptical' has a certain unpleasant association attached to it, for it has been used by so many people who are absolutely certain in a particular line, and attack other people's convictions. But to be sceptical in the real sense is a far more unpleasant state of mind to the sceptic than to any one of his companions. For to a mind that inquires into what it really does know, it is hardly possible to enunciate complete sentences, much less to put before it those complex ideas which have so large a part in true human life.

"Every word we use has so wide and fugitive a meaning, and every expression touches or rather grazes fact by so very minute a point, that, if we wish to start with something which we do know, and thence proceed in a certain manner, we are forced away from the study of reality and driven to an artificial system, such as logic or mathematics, which, starting from postulates and axioms, develops a body of ideal truth which rather comes into contact with nature than is nature.

"Scientific achievement is reserved for those who are content to absorb into their consciousness, by any means and by whatever way they come, the varied appearances of nature, whence and in which by reflection they find floating as it were on the sea of the unknown, certain similarities, certain resemblances and analogies, by means of which they

collect together a body of possible predictions and inferences, and in nature they find correspondences which are actually verified. Hence science exists, although the conceptions in the mind cannot be said to have any real correspondence in nature."

We quote this passage from Mr. Hinton's "A New Era of Thought," 1900, pp. 8-10, because it strikes us as a pertinent illustration of the craving on the part of the mathematical consciousness to supersede the contradiction hemming it in on all sides and in sequel, pp. 11-13, throws a further light on the problem of finding the truly solid ground on which to build up our knowledge of the universe :

"Asking, then, what there was which I could know, I found no point of beginning. There were plenty of ways of accumulating observations, but none in which one could go hand in hand with nature.

"A child is provided in the early part of its life with a provision of food adapted for it. But it seemed that our minds are left without a natural subsistence, for on the one hand there are arid mathematics, and on the other, there is observation, and in observation there is, out of the great mass of constructed mental images, but little which the mind can assimilate. To the worker at science, of course, this crude and omnivorous observation is everything ; but if we ask for something which we can know, it is like a vast mass of indigestible material with every here and there a fibre or thread which we can assimilate.

"In this perplexity I was reduced to the last condition of mental despair ; and in default of finding anything which I could understand in nature I was sufficiently humbled to learn anything which seemed to afford a capacity of being known.

"And the objects which came before me for this endeavour were the simple ones which will be plentifully used in the practical part of this book. For I found that the only assertion I could make about external objects, without bringing in unknown and unintelligible relations, was this : I could say how things were arranged. If a stone lay between two others, that was a definite and intelligible fact, and seemed primary. As a stone itself, it was an unknown somewhat which one could get more and more information about the more one studied the various sciences. But

granting that there were some things there which we call stones, the way they were arranged was a simple and obvious fact which could be easily expressed and easily remembered.

"And so in despair of being able to obtain any other kind of mental possession in the way of knowledge, I commenced to learn arrangements, and I took as the objects to be arranged certain artificial objects of a simple shape. I built up a block of cubes, and giving each a name I learnt a mass of them.

"Now I do not recommend this as a thing to be done. All I can say is that genuinely then and now it seemed and seems to be the only kind of mental possession which one can call knowledge. It is perfectly definite and certain. I could tell where each cube came and how it was related to each of the others. As to the cube itself, I was profoundly ignorant of that; but assuming that as a necessary starting point, taking that as granted, I had a definite mass of knowledge.

But I do not wish to say that this is better than any kind of knowledge which other people may find come home to them. All I want to do is to take this humble beginning of knowledge and show how inevitably, by devotion to it, it leads to marvellous and far-distant truths, and how, by a strange path, it leads directly into the presence of some of the highest conceptions which great minds have given us.

". . . I do not think that it is possible to get anything simpler, with less of hypothesis about it, and more obviously a simple taking in of facts, than the study of the arrangement of a block of cubes.

"Many philosophers have assumed a starting point for their thought. I want the reader to accept a very humble one and see what comes of it."

Quite in agreement with Mr. Hinton's remark that "it ought not to be any objection to an inquiry that it begins with obvious and common details," the present writer adopted his so movingly advocated path to knowledge—as a suitable complement, on the side of experience, to the beginning of knowledge in pure thought. It will be seen that, on leaving the standpoint occupied by Intellect, under the growing suspicion and finally overwhelming certitude

of the futility of intellectual solutions of the riddle of the universe, our thirst of knowledge advances us either in the direction of pure thought or in that of fuller psychological self-realization. The element of mediation between the two tendencies immediately, and for that reason only contradictorily, united in the intellectual consciousness brings them into the relation obtaining between the infinitized Finite and the finitized Infinite, and though this distinction ultimately falls to the ground, at first sight it has an appearance of dualism offering a choice between its sides. Of course, the ultimate result is the same whichever side is chosen: pure thinking presses for self-embodiment in experience and concrete self-realization is developing the capacity of pure thought; the capacity of grasping the nature of the true Being; of God as He is in His Essence, in supersession of the created universe.

Mr. Hinton's block of cubes consists of 27 variously coloured cubes, and the object of the practice with it is to render oneself so thoroughly familiar with every aspect of it as to become able to visualize instantaneously and effortlessly, not only any particular position in which it may be presented to view *en bloc*, but any particular combination of the component cubes, on the understanding, of course, that their inter-relation is once for all fixed. This is an enormous task, especially to him whose native element is pure thought. The present writer came, by what seems a pure chance, across Hegel's "Great Logic," and even though he had never before read, or troubled to read, any philosophical work, he found it a fairly easy reading—and unspeakably thrilling with all-round interest. But the assimilation of Mr. Hinton's block of cubes!

Notice that the determination of any particular position of the block, as it lies there before us on one of its six possible bases, necessitates an indication of at least two coloured cubes in a spatial relation such that they do not form a straight line with the permanently implied central cube, which, of course, is not affected by any change in the position of the block. Therefore, given a particular circumferential cube, the position of the block is determinable by coupling the given cube with any of the twenty-four cubes which remain after the exclusion of the central cube and of the one beyond it in a straight line with the given cube. But

this latter is one of twenty-six. Consequently, on reducing the determinableness of a particular position of the block to its lowest terms one must qualify himself to identify it from the standpoint of any of the $(24 \times 26)/2 = 312$ ways in which the adopted principle of determination displays itself. We divide the product 24×26 by two, because otherwise the same element of determination would be counted twice. In so far, then, as the block may be built up in twenty-four different aspects, it follows that the identification of a particular aspect is really attempted from the standpoint of $312 \times 24 = 7488$ ways of combining two circumferential cubes.

But, now, the block consists of twenty-seven cubes and must not, therefore, be treated only as a single object of visualization. In order to assimilate it in fully adequate manner, we must aim at becoming able to visualize effortlessly any single cube forming a group of three circumferential cubes with the two circumferential cubes of any of the 7488 elements of determination. Seeing, however, that the third cube in question is, in reference to the given aspect of the block, one of possible twenty-four, and the element determining the given aspect of the block one of 7488, a group of three circumferential cubes involving an element of determination (or rather, three such elements) is one of possible $24 \times 7{,}488 = 179{,}712$ ways of forming the specified group of three. It is next to be realized that when the third cube is grouped with the two circumferential cubes belonging to an element of determination, there equally enters a reflection upon the order in which these two cubes are visualized. Abstractly considered, the two cubes are a single object of visualization, but when they are to be combined with a third cube, it is soon found that this third cube must be assimilated twice. If the two given cubes are symbolized by the letters *a* and *b*, we find that, from the standpoint of pure visualization, the formation of the group *abc* is of no direct help in the formation of the group *bac*. It follows that there are really 359,424 ways of combining 7488 elements of determination with a third circumferential cube. And if, finally, we take into consideration that the circumferential cubes lying on a straight line or axis with the permanently fixed central cube must, for completeness' sake, be equally included within the scope

of visualization, we realize that before we can claim to be able to *visualize* any possible group of three cubes in an instantaneously effortless manner, we must have assimilated roundly some 360,000 ways of combining them.

Notice that we have italicized the word "visualize". In the training we are describing, there are no simplifying devices; no short-cuts; no appeal to intellectual ingenuity. Intellect finds a way to orientate itself in the maze of possible combinations in the same places of the block in no time. The present writer thought he had mastered the whole task in a week when he had only *committed to memory* one position of the block, in order to convert it into a permanent background to every other position as well as the inter-relation between the component cubes. In this way, training in visualization remained conspicuous by its utter absence, the whole task being treated only as a matter of intellectual rapidity of recognizing in every case one and the same complex of fixed relations. Of course, absence of any particular result from this kind of intellectual gymnastics very soon puts a stop to it.

In order to yield the expected reward, the assimilation of the block of cubes must be the work of visualizing capacity alone. That is to say, every one of the thousands upon thousands of elementary inter-relations between the component cubes, irrespective of the possible position of the total block, must be treated as utterly independent objects of visualization; every one must be remembered, not on the ground of an intellectual reflection, but only and solely *on the ground of having been seen* in utter absence of any mediation whatever. The position of the total complex of the cubes is here of no consequence, except as one of the many objects of visualization.

In so far, then, as success in this training depends on deliberate exclusion of mental activity, it will be seen how repugnant it must seem to him whose life-interest centres in the element of thought; to whom mere staring at things appears the climax of all boredom. But unpleasantness is no argument against a task recommending itself on excellent a priori grounds. *Noblesse oblige.* Does not our own beginning in knowledge appear just as repugnant to those engrossed in merely material interests?; to those taking their stand on empirical facts alone? They shrink from

what to them appears emptiness of all real content, and we shrink from what to us appears emptiness of all real content. Is it fair to reproach them with their one-sidedness if we ourselves flatly reject their pathway to reality? True, pure thought is not truly antagonistic to empiricism, but is supposed to contain the result of every possible experience; does not, however, this its truth call also for vindication? It is pure thought itself that makes it incumbent upon us to seek to experience its own process of self-realization; lest we pay only lip-service to the truth of the unity of Thought and Being.

It is in the nature of the true Being that its own self-exposition at the level of pure thought should still leave room for the widening of one's experience in the search of an adequate embodiment of the standpoint of Absolute Idealism. Even though the conclusions forced upon the pure thinker in his progress in knowing are irrefutably true, their full breadth and width may escape his grasp, because he is as yet unable to experience their objective bearing on concrete existence. For instance, although Hegel's own writings may be construed into a conclusive proof of post-mortem existence, of clairvoyance, of reincarnation, and in fact of all the tenets of Occultism, it is by no means certain that he himself personally would have cared to be saddled with such convictions. As a matter of fact, his most competent modern follower, Professor Bolland of Leyden, flatly rejects reincarnation as a kind of pitiable nonsense, whilst Professor McTaggart, with all his shortcomings as an exponent of Hegel, yet defends it and to that extent is on our side.

That Professor Bolland is a pure thinker *par excellence*, we would nearly say, a Hegel *redivivus*, is a patent fact to all those who have read his inspiring and magnificently eloquent "Zuivere Rede en Hare Werkelijkheid" (Pure Reason and its Actuality); yet his very enthusiasm and the resulting *apparently* so overbearing tone, in which he chides empiricism and occultism, brings home to us that even pure thought is subject to the three phases characteristic of every dialectical whole. That is to say, the true Being is already properly grasped even under the head of the immediate phase of pure thought, but without yet being explicitly mediated with its own self in the guise of the

then equally from the very first already implicated idealistic otherwiseness: its objective significance as embodied in the universe and the full compass of concrete self-realization. So comes it that whilst already hailing the standpoint of Absolute Idealism as the only true standpoint, one may remain content to hold this standpoint as a purely mental conviction. Naturally, one is, to begin with, so taken up with bringing all one's mental acquisitions into line with this dazzling truth, that one has no time, nor inclination, to trouble about anything else, with the consequence that one comes to feel as if estranged from one's, and the world's, concrete mode of existence; one simply turns one's back on it, or else proclaims highly the little one thinks of it.

As is seen, the shortcoming characteristic of the immediate attitude towards the Infinite makes thus its appearance under the very head of the true Being; in the very realm of pure thought. Nevertheless, the said shortcoming is then in principle also already superseded. Directly one has succeeded in reorganizing one's mentality, practical interests awake. The truth is universal; it must be proclaimed from the house tops, and, above all, it must be *lived* in one's own person. No, there is no *dolce far niente* for him who knows the truth! One feels entrusted with the philosopher's stone. Let one's voice be a *vox clamantis in deserto:* it must be sounded. In any case, there is the never-ending task of grasping the import of the true Being yet deeper; of elaborating the sweep of its self-revelation yet more thoroughly; of bridging over any gap that may still have escaped one's notice. The initial estrangement from the concrete mode of divine self-embodiment is thus gradually waning, giving place to a growing capacity of self-recognition in every aspect of human striving. The attitude of a school-master towards the lower aspects of mental activity is gradually submerged in a growing realization of their substantial Inwardness, as, in their turn, equally autonomous modes of the true Being itself.

The thoroughness with which human mind insists on examining every detail of manifested existence, is in itself a guarantee that no error is lasting. Love of truth is the essential nature of mankind; aye, even God's own Being is denied, whenever it seems to be denied, out of love of truth. We *must* insist on taking nothing for granted. It is the

own nature of eternal truth to prevail. Therefore, a pure thinker ceases to agitate himself at the appearance of thoughtlessness in the sphere of empirical science or elsewhere. The fact that mankind appears to recognize the most elementary truths in human relationship, such as national self-determination, freedom of conscience, universal solidarity, etc., only until forced to do so by the calamities resulting from their denial, reminds us at once of the nature of our own progress in knowing, in the course of which we equally refuse to consider as properly established an otherwise from the very first obviously safe position—say, that Quantity is inseparably united with Quality—until we are forced to do so by dialectical necessity.

The distinction between anticipation and realization, between Being-*in-itself* and Being-*for-itself*, between subjectivity and objectivity, or generally Thought and Being, asserts itself spontaneously as a characteristic feature of every development. Even though every one of us readily gives assent to the desirability of such and such *modus vivendi*, the implied recognition of an elementary verity remains subjective until it acquires collective endorsement and reaches its Being-for-self in a publicly instituted law. So comes it that public life, relation between nations, appears to lag behind the level reached by private individuals; but it cannot be otherwise, because the institutions characteristic of a civilization remain a measure of the general level reached by human self-consciousness. Individual striving is only the womb, in which the embryonic Objective Spirit is awaiting its birth, which becomes our own, but *second*, birth. Civilization is our own baby, our own self confronting us in the form of Nature, but of our own *human* Nature, so that we may re-live in its objective development the trend of our own subjective growth and thus bring home to ourselves what we really are, or rather were, in our childhood or hot-headed youth.

And it is, then, this need to envisage oneself also objectively, that finally leads a pure thinker out of his initial self-absorption and saddles him with the task of retracing his progress in knowing also phenomenologically, in order to experience as fully as possible what he already knows to be true. With this end in view, the initial disinclination to subject himself to the discipline involved in the assimilation

of Mr. Hinton's block of cubes is speedily superseded. The needed suppression of mental activity converts itself at once into an analogy of the immediate conversion of Quantity into Magnitude as such. Or rather, in so far as the manifoldly varied inter-relations between the component cubes correspond to the totality of distinctions into which the Substrate or Indifference differentiates itself on entering into Presence, assimilation of these distinctions apart from mental co-operation, or on the ground of sensuous consciousness alone, manifestly brings one back to the stage of babyhood with a view to gaining an experience of the inwardization which visual impressions suffer in the image-making consciousness.

As a rule, we do not particularly trouble about the various modes in which our mind (speaking of our total consciousness) displays itself, the so-called faculties, but are content to use them as our birth-right. In any case, we cannot divest ourselves of them, any more than Matter can be divested of its fundamental property. So comes it that an endeavour to tear one particular faculty out of its qualitative coherence with the rest of our mind, may indeed appear an impossibility. Attention, however, may be drawn to the fact that we have endeavoured to tear Quantity out of its fundamental unity with Quality only in order to vindicate this its truth. The endeavour to break or disrupt our mind into distinctly hypostatized faculties is to us equally only a way to vindicate their fundamental unity. But as thus vindicated, the fundamental unity of our mind ceases to be a purely subjective mind, but acquires the primarily missed element of objectivity. It becomes then explicitly an all-embracing mind, in supersession of its initial subjectivity, when it appears to be only a matter of cerebration and in any case remains conditioned by objectivity.

Indeed, unless buoyed up by these anticipations, it would hardly be possible to undertake, or rather, which means much more, to *persevere* in, the enormous task implied in the assimilation of Mr. Hinton's block of cubes. The feeling of helplessness to speed up one's capacity to retain and freely reproduce images running into thousands upon thousands, is at first well-nigh unbearable. The present writer cannot conceive a greater test of Job-like patience and endurance. It is a revelation to find oneself so utterly deficient

in the use of what one does not hesitate to call *one's own* mind! One learns to be humble—and to persevere. One does not grudge time for meals, or the morning-bath, sandwiched in between Müller's exercises, or the work in the garden—very well, one hour a day must be found also for visualizing exercises. And in that case, one must be prepared to keep at it regularly the rest of one's life as a matter of duty, or else lose one's self-respect. After all, what do we live for, if not to experience the full depth of our own Being?

But, then, it does not take so very long before the visualizing capacity begins to answer to repeated effort. The at first faint image is acquiring surprising vividness and particular combinations of cubes float into the field of inner vision at will. The meddling intellectual consciousness learns finally that it is not wanted, and the training proceeds in a strangely peaceful atmosphere. The necessity of inducing in oneself a properly receptive attitude for the images to be retained, is becoming gradually a matter of habit, and one begins to feel one's grip on their accumulating mass. It is a regular joy to find that they are becoming part and parcel of one's own Being. Of course, one anticipates as much from the very first; but the difference between anticipation and realization begins now properly to come home.

The element of objectivity characteristic of the more and more distinctly hypostatized faculty becomes a source of never-ending surprise. There is arising a feeling of dualism in one's own Inwardness. On the one side, there am I in the position of a master giving orders to my own self on the other side, but so that this my self presents itself to me in the sense of a slave executing my orders with the most commendable alacrity. This feeling of dualism does not accompany an ordinary indulgence in recollection. In conjuring before my inner vision particular scenery I have seen, I feel that the image is there only through my own effort. In this case, however, I feel as if, after ordering a particular combination of cubes to enter into Presence, I turned myself into a passive onlooker awaiting with curiosity the execution of my order. Sometimes the automatic working of my recollection seems disarranged, but in that case I find that I am not "collected" enough. My slave is unwilling to

serve a distracted master. But on those rare occasions when concentration on the task in hand becomes perfect, it comes home to one with astounding fulness that an image of anything one has ever seen is recoverable from the fathomless pit of the recollecting consciousness. The enormous amount of images one has to learn to reproduce in connection with the assimilation of Mr. Hinton's block of cubes ceases to frighten one, because, as time goes on, one finds oneself in a position to recollect them with cumulatively increasing ease.

The whole secret lies in acquiring the capacity of throwing oneself at will into that perfectly undistracted state of mind which is related to the entrance of images into consciousness in the same way as the Substrate is to its own entrance into Presence. The arising feeling of dualism spoken of above is the way in which one is experiencing the totality of given distinctions as *pari passu* reducing itself to Indifference. Recollected images float into the inner field of vision as if ushered in by an external agency, because I, who am their Substrate, maintain myself in an attitude of Indifference; they are analogous to the points of which the circumference of a circle may be conceived as consisting, whilst my recollecting consciousness is the central point.

The ultimate result of the whole training may now be foreseen: the recollecting consciousness proves itself in the end to be just as much self-integrated as self-differentiated. The initial endeavour to assimilate the block of cubes piecemeal rests, of course, on the expectation that the fundamental unity of the manifold aspects, in which the block may present itself to view, will finally equally enter into consciousness. It is to be noticed, however, that in that case reference is made to any and every possible position of the block. When every element of determination with respect to a particular position of the block can be summoned before the recollecting consciousness at a moment's notice, the block admits of being reconstructed from its elements and hence ceases to be a single object of visualization, as it would be assimilated to begin with. It is only because any position of the block may be also recollected simply *en bloc*, i.e. as a single object, that the task of assimilating it appears at first sight a simple affair. In that case, however, we are only able to break up the block into its elements of deter-

mination, but not to reconstruct it again. Before we can do this, we must assimilate every element of determination as an independent object of visualization.

When we are familiar with the positions of the block simply *en bloc*, we cannot straightway identify them from their elements; the identification remains dependent on a rapid survey of the familiar positions with a view to finding the one containing the given element. But because this kind of identification is not the work of the recollecting consciousness pure and simple, it cannot procure us an experience of its own self-integrating aspect. This aspect must assert itself independently of any co-operation on the part of our intellectual or analytical consciousness, and that this its independent self-assertion is conditioned by a thorough assimilation of all the elements determining any and every possible position of the block, is obvious. In that case, given a particular element, i.e. a particular combination of two superficial cubes, we are instantly in a position to *visualize* any of the cubes filling up the remaining spaces in the block and hence make the identification of the latter's position dependent on its reconstruction from within the recollecting consciousness itself.

But the "re-collecting" of the primarily purely separately assimilated images does not stop at the reconstruction of a particular position of the block. Elements of determination fall into groups having a common term in reference to two, three and even four positions of the block. For instance, the nearest left-hand corner cube is common to three positions, and therefore any element of determination implying it has two alternatives, which are equally a matter for recollection and the ensuing reconstruction of the corresponding position of the block. Since, with practice, the process of reconstruction is becoming practically instantaneous, especially as every position of the block is equally assimilated *en bloc*, we find ourselves finally in the position of reconstructing two, three—in fact, all the aspects of the block, in a single effort of the recollecting consciousness.

It is only when one is approaching this climax of the whole training that the insufficiency of the initial tendency to visualize the positions of the bloc only *en bloc* becomes patent. In that case one may well be able to pass in review the twenty-four positions of the bloc with breathless

rapidity of succession, or else fancy oneself as if looking at one and the same position in twenty-four successively different ways, but these ways of looking at the same position, or else the twenty-four positions, remain discrete Ones, refusing to blend in one total impression. But when one is able to reconstruct *effortlessly* any particular position, then the alternative positions implying one and the same cube in the same place present themselves to vision with the same ease as the cubes needed for the reconstruction of a particular position from a given combination of a few among them. The transition from one position into another one loses in this way its initial character of abruptness; instead of a change *per saltum*, we have then a *continuous* process of transformation. One has only to abandon oneself to a lazy contemplation of a particular position to find that the recollecting consciousness at once hurries to reconstruct an alternative position from the standpoint of a particular element of determination, subjecting the new position to the same treatment, and so on, endlessly it would seem, if one did not step in and check this otherwise very delightful display of the accumulated mass of interrelated images, all presenting themselves to inner vision with all the vividness of actual seeing.

In this way, it comes home to us that fixedness of a particular position does not concern the recollecting, but only our ordinary sensuous consciousness. In order to *see* the block in a different position, we must take it to pieces and rebuild it in the desired position, or else look at it sideways, from the opposite side, from the top, etc. In recollecting consciousness, one may equally reproduce these three-dimensional attitudes towards the block from the standpoint of a particular fixed position; but in this case, every adopted attitude simultaneously comes to be accompanied by a feeling that one is always looking at the block from one and the same frontal position. Whilst fancying oneself to be looking at it from behind—and looking means then always reconstructing—this impression, unless maintained by an effort, converts itself into a feeling that one is still looking at the block in front; but the effected substitution of one position for another one comes to one's consciousness in a peculiar feeling of muddleheadedness. On repeating the same kind of experiment

several times in reference to different attitudes towards the block which is *pari passu* also under a constant process of reconstruction, one feels the familiar distinctions of front and back, right and left, or top and bottom, all mixed up in utter confusion.

That is to say, the characteristic feature of our sensuous consciousness to envisage things only in a fixed spatial relationship does not apply to the hypostatized recollecting consciousness. This consciousness *envelops* what it contemplates and hence enables one to look at a thing from all sides at once and right through. At any rate, this is manifestly the ultimate result of the practice with Mr. Hinton's block of cubes. Just as the strength developed in the course of a particular physical exercise remains serviceable in any practical respect: so the ultra-spatial or supersensuous consciousness, developed in the course of the described training in visualization, does not remain limited to a mere recollection of the block of cubes, but stabilizes itself in the so-called clairvoyant or astral consciousness—that consciousness which normally asserts itself only after our bodily death.

Mr. Hinton did not apparently carry the assimilation of his own block of cubes to its full climax, for he could not have failed to encounter many curious clairvoyant experiences, which would have taught him that the difference between the supersensuous world and this world is not a matter of a spatial dimension, but that it is the difference between Presence and Essence. The present writer is grateful to him for the suggestion of a very pleasant (now, not at first) way of turning many an idle moment to the greatest possible advantage; but the study of his tessaract is another matter. The assimilation of the block of cubes is meant to deepen the mathematical consciousness into a correspondence to the standpoint of Essence, not simply to return back to it in a monstrously exaggerated form.

C. The Nodal Line of the Mean Planetary Distances from the Sun.

In order to complete our treatment of the transition into the fundamental unity of the measurable Quality and

Quantity, we must concern ourselves with the Substrate also in explicit correspondence to the Relation between the Original and the Derived Function, but so that, in keeping with our previous mode of procedure, this Relation equally acquires the form of a summary recapitulation of the dialectic of the measurable Quality and Quantity.

Now, as the Measurable is reflected upon more particularly in terms of the Derived Function, having to be grasped as the immediate result of Self-differentiation, the distinction between the measurable Quality and Quantity is to be symbolized on the side of the derived Function in reference to its equality or inequality with the Original Function. This latter being a square, the derived Function can be in the first case only equal to 4, whilst in the second case it refers to $d(x^2)/dx$, when x is equal at least to 3.

As is seen, we make the number 4 symbolize the standpoint of the measurable Quality, because there suggests itself nothing else for a purely quantitative symbolization of its homogeneity except the equality between the Original and the Derived Function. Similarly, the heterogeneity characteristic of the measurable Quantity cannot be symbolized purely quantitatively except in so far as we refer to the distinction between the two correlated Functions, in which case the Original Function must be at least a square of 3.

As regards the many present Ones of the measurable Quantity, the simplest way to symbolize their Nodal Line in one and the same Substrate is to keep duplicating the preceding term. For, the successive terms of the arising series ought to arise from the preceding ones in the same constant manner, and this manner of determination has its exemplification in the fact that the first term, in which the Substrate as such enters into Presence, is represented by the product 2×3, the number 3 being the present symbol of the Substrate as such in connection with the standpoint of the measurable Quantity.

As a matter of fact, we ought to avail ourselves only of as many terms of the series formed by successive duplication as are required for the simplest representation of the two qualitatively distinct groups under the head of the displayed Inverted Relation. In so far as each group must consist at least of two members, the number of terms re-

quired would be four. But in order to symbolize that the two groups are in qualitative relationship, we must group them round a middle term at once joining and disjoining them. Therefore, we are concerned with the series 6, 12, 24, 48, 96, or at most with the series 3 6, 12, 24, 48, 96, 192—if, that is, we include in it, for completeness' sake, also the number 3, with which the series may be said to begin abstractly, and the corresponding term on the other side of the middle term.

Now, as the series decreases or increases, on the one side, as many times as it increases or decreases on the other, the terms symmetrically disposed round the middle term yield the same fixed product, i.e. 24^2; and at first sight it would seem that this product is all we have to fall back upon for the symbolization of the measurable Quantity at the next stage of the Involved Inverted Relation. But the involved series contains the middle term, which presents itself as originally given along with, or additionally to, the fixed product, and which, therefore, should now be conceived as originally deducted from the number symbolizing the reached completeness of the measurable Quantity; consequently, this number is $24^2 + 24 = 600$.

In so far, further, as the completed measurable Quantity is to be viewed as in unity with the measurable Quality, and this latter has been symbolized by means of the equalization of the original with the derived function, the sum $24^2 + 24$ should be also treated in the sense of a derived function; that is to say, in the sense of a sum of two quanta such that each increases or decreases by as much as the other decreases or increases. We did not emphasize this inwardness of the derived function in connection with the number 4, because this number is meant to symbolize the measurable Quality in its simple immediacy as against the measurable Quantity. Now, however, the measurable Quality is to be symbolized expressly with respect to its unity with the measurable Quantity as the Involved Inverted Relation, and the number 600 carries, therefore, with it the distinct significance of the major axis of an ellipse.

As a symbol of the still simple character of the measurable Quality, the number 4 implies, by contrast, a simple length having its starting-point, by virtue of the involved

centrality, in the middle-point of the axis and thus cutting off 4 of the 300 units which fix the limit of linear extension on either side away from the middle-point. And as the series representing the measurable Quantity has its starting-point where the length representing the simple measurable Quality ends, its terms refer to points at a distance of 7, 10, 16, 28, 52, 100, 196 units from the common centre. Thus, the Substrate presents itself to us in the image of the major-axis of a series of ellipses, in so far as each of its halves has the specific meaning of a Nodel Line of the following measure relations: 4, 7, 10, 16, 28, 52, 100, 196, 300. As is well known, this series represents mean planetary distances from the Sun.

SECTION III.

THE UNITY OF THE MEASURABLE QUALITY AND QUANTITY: THE SOLAR SYSTEM.

A.

THE STANDPOINT OF THE MEASURABLE QUALITY: THE LAWS OF PLANETARY MOTION.

CHAPTER I.

THE STANDPOINT OF THE DIRECT RELATION: KEPLER'S FIRST LAW.

IN making the series of the mean planetary distances from the Sun the object of our further consideration, we must at once emphasize that its terms have no meaning apart from one another. The series must consist of several members, but we are concerned with them only collectively, and must even then keep in mind that the series symbolizes the nature of the Substrate of all that is measurable. For, we have now definitely left the sphere of Physics or Chemistry behind us, so that Quality and Quantity, simple or measurable, are before us as moments of a transcendental meaning and therefore must be interpretated accordingly. The plurality implied in the series has the same relation to this latter's transcendental meaning as the present Ones have to the original one One; a relation of the same kind as that between the manifold features of a work of art and beauty.

Now, because the fundamental meaning now in question immediately concerns the direct representation of the Substrate as such by means of the series of the mean planetary distances from the Sun, and the last term of this series has been ushered on the scene in the sense of the major semi-

axis of an ellipse, it follows that this sense applies to every other term. Therefore, instead of a nodal line of simple distances from a common starting-point, we have now before us a series of ellipses which have for their semi-axes the corresponding terms of the original series, and which, we must now add, have additionally a focus in common—only *one* focus, because our present standpoint does not require that the ellipses should have one and the same fixed distance between their foci. Namely, we are recapitulating the standpoint of the measurable Quality, and, in this respect, extension concerns us only as concentrated at a point, the centre of gravity, which is now to be interpreted in the sense of the common centre of the system of universal gravitation. Thus it becomes obvious that, at the outset of our discussion of this system, we postulate Kepler's first law of planetary motion, the orbital movement of the planets being the immediate specification of the absolutely freely falling body. For, according to this law, the orbit described by every planet is an ellipse of which the centre of the Sun occupies one of the foci. Of course, planets, as well as the Sun, are so far only viewed as mere points; the central points of their extension, which latter must await its determination at a later stage.

It follows that the present display of the Direct Relation refers to the orbital velocity which may be represented by the ratio s/t, inasmuch as, from the standpoint of the resulting series of comparative numbers, the common factor 2π on the side of the orbit becomes superfluous. But since we manifestly are not yet in a position to undertake the determination of the series in question, we must concern ourselves with orbital velocity only to the extent to which our present standpoint permits us to go. Now, seeing that our present standpoint does not justify us in assuming that orbital velocity is a fixed Quantum for all the planets, because in that case we would efface the from the very first to be postulated implicitness of a qualitative distinction between the individual planets, as their common nature will have to be interpreted when our present recapitulation of the dialectic of Measure will reach the standpoint corresponding to the measurable Quantity: the orbital velocity is before us properly at once in illustration of the transition from the standpoint of the Direct into that of the Inverted

Relation, and hence varies not only from one planet to another, but also in reference to one and the same planet.

That we cannot postulate uniformity in this connection follows after all from this, that the concept of uniform velocity can concern only a mechanically originated and sustained movement and hence, as has been explained, ceases to apply already to the conditionally free movement. Inasmuch as we are now dealing with the absolutely free movement in the shape of a self-concluding or self-returning motion, the velocity of such a movement must be alternately accelerated and retarded. And it is in further explication of this its contradictory nature that we proceed to deal with the orbital movement in correspondence to the display of the Inverted Relation.

CHAPTER II.

THE STANDPOINT OF THE INVERTED RELATION: KEPLER'S SECOND LAW.

It is because the measurable Quality or physical Quantity concerns us now only as from the very first completely idealized, and hence no longer in its original meaning, and the present recapitulation of the trend of its dialectical development must all the same immediately embody its already completed idealization in such wise, that its new meaning necessarily calls for another exemplification of the progress towards complete idealization and that not only with respect to the immediate restatement of the dialectical whole of the measurable Quality, but also with respect to the necessary transition into the next ensuing recapitulation of the dialectical whole of the measurable Quantity—it is for this reason that the present object of our thinking acquires, in marked correspondence to the first stage of the dialectical whole of the measurable Quality, the sense of the orbital movement with an alternately increasing and decreasing velocity. For, although we are in this way from the very first superseding the standpoint of physical and chemical change, we are yet, by virtue of our familiarity with facts, immediately aware that we are thus touching the merest fringe of all that is implied in the system of the absolutely free movement.

As a matter of fact, the import of the present stage, in marked correspondence to the display of the Inverted Relation, leaps to the eye. We are meant to explicate the contradiction forced on our attention in transition into the present stage, which, of course, equally means that we must at the same time at once reflect upon the ultimate supersession of this contradiction. For, a contradiction can be rendered explicit only from the standpoint of its resolution,

as otherwise one remains unaware of the vicious circle constituting its characteristic feature.

Now, seeing that the contradiction in question concerns the nature of orbital velocity as alternately increasing and decreasing, in order to make this self-returning process properly explicit, we must remind ourselves that orbital velocity is describing an elliptically shaped orbit round the centre of the Sun in one of its foci. Although we have immediately been aware of this its nature, we have yet left it out of sight in simply defining the orbital velocity as a direct ratio s/t, when s stands for the space traversed and t for the time elapsed. Of course, we have at once reminded ourselves that the expression $s/t = v$ really is inadequate; but we did not yet make explicit to ourselves that the space traversed should really be treated in the sense of an *area* rather than of a mere curve. Yet this is precisely what at once follows, if the connectedness of the moving orbital point with the centre of the Sun is forced on attention by means of an imaginary line, called *radius vector*.

The space traversed may, of course, be treated simply in in the sense of the elliptical curve which is the orbit; but in that case we would make no progress towards the postulated idealization of the subject-matter given at the outset of our discussion. In order, however, to embody the advance to be made in marked correspondence to the middle stage of the dialectical whole of the measurable Quality, we must explicate the qualitativity of the distinction between the space traversed and the time elapsed. That is to say, since the sides of the direct ratio s/t must now present themselves as interchangeable moments of a fixed product, the element of time must now be given just as much a spatial significance, instead of having for its representative a mere number; and inasmuch as both sides of the inverse ratio are to be treated as mere moments of one whole and hence neither of them can represent the space traversed apart from the other, this space can be represented only by the side of the exponent standing for an area.

In any case, progress towards idealization just as much means progress towards materialization, which latter concerns in the present connection the already stated dialectical necessity for grasping the planetary orbit in the explicit sense of a plane, as against its primary meaning as a curve.

But the area represented by the exponent of the inverse ratio is not meant to be identified at once with the whole plane orbit. Since it is to serve as a background to an exemplification of the inverse ratio between the radius vector and the arc described, and such an exemplification presupposes Alterableness in the shape of a fixed area, seeing that either of its two determining moments should be increasing or decreasing proportionately to the decrease or increase in the other moment: the fixed area can represent only a particular part of the whole plane orbit.

Inasmuch, therefore, as the plane orbit is thus conceived as subdivided into an arbitrary amount of segments, identical in area if not in shape, and each segment represents the space swept over by the radius vector in the same fixed time, the inequalities in the arc or the distance from the Sun being a consequence of the elliptical shape of the orbit and the alterable velocity of the orbital movement, there follows Kepler's second law of planetary motion, according to which a line drawn from a planet to the sun sweeps over equal areas in equal times.

CHAPTER III.

THE STANDPOINT OF THE INVOLVED INVERTED RELATION: KEPLER'S THIRD LAW.

In becoming an area, the space traversed acquires with respect to the time elapsed the relation typified by the law of descent of a falling body. Of course, the fact that the conditionally free movement is now reflected upon only as never getting beyond the first unit of time, so that its continuity comes to appear only by way of a repeated description of the same area in the same time—this modification of the law of descent of a falling body directly follows from its present necessary subordination to the standpoint of the orbital movement round the centre of the Sun. Just because this is a self-returning movement, time acquires the significance of a rounded-off or discrete unit, and in so far as the space traversed is described by the radius vector, the space corresponding to the unit time becomes a sector. But even though the standpoint of the orbital movement is then equally reflected upon, its characteristic nature as an *absolutely* free movement, nevertheless, does not yet explicitly come to speech. Therefore, although the elements of space and time are on the one side already implied as moments of the Inverted Relation, they have yet on the other side once again relapsed into a purely direct unity and thus confront us in the guise of a contradiction between their Direct and their Inverted Relation. And since in order to supersede this contradiction we are meant, first of all, to remove the remaining restriction which we have still been imposing upon the orbital movement as a display of the *absolutely free* movement, it is now soon brought home to us that the curve described by the centre of the planets must really be conceived as though traced on the surface of a volume, as against our immediate identification thereof with the periphery of a *fixed* plane orbit. For what ground

have we to assume that the orbital movement must be purely two-dimensional?

We have just made plain to ourselves that the two laws of planetary motion already derived do not yet explicitly concern its characteristic feature as an absolutely free movement. They remain valid even though the planetary orbit be itself subjected to a further movement in the third dimension; and the possibility of such a movement is also from the very first implied in this, that planets concern us so far only as orbital points moving round the *centre* of the sun, not perhaps round a given section of the sun as a three-dimensional body. In any case, we cannot restrict an *absolutely* free movement to two dimensions. Accordingly, too, planetary orbits are variously inclined to one another; though in so far as this their distinctiveness cannot yet go beyond the stage of the present Ones, just because planets are not yet before us as individualized, the common plane of reference should not be identified with the Ecliptic, i.e. with the plane of the Earth's orbit or of the Sun's apparent motion among the stars. From our present attitude towards the orbital movement, it follows that the empirically determined inclination of plane orbits to the ecliptic is not fixed once for all. All we may, or rather must, postulate in this connection is a permanently fixed common plane of reference for the changes in the mutual inclinations between the planetary plane orbits. Now, that there does exist such a plane was established by the investigations of La Place, and in his "Essays and Addresses on Theosophy and Science" (Bombay, 1906), Mr. G. E. Sutcliffe asserts that the Invariable Plane in question is *exactly* at right angle to the plane of the great circle of Sirius and Vega. Inasmuch as the conception of the Invariable Plane for the Solar System is quite rational, we quite agree with Mr. Sutcliffe that its stated relation is hardly a mere coincidence.

When we, then, turn our attention to the completion of our task, we see that the space traversed must, finally, be associated with the conception of a volume in simultaneous supersession of the analytical attitude. That is to say, we no longer imply a splitting up of the orbital movement into parts, but must identify the unit time to the space traversed with a unit sidereal period. Consequently, the space

traversed would seem at first sight to reflect upon the volume swept out by the revolution of a plane orbit. But since the pictorial consciousness must not be allowed wholly to usurp the place of our thinking consciousness, and from this latter's standpoint we must refuse to represent the space traversed in terms of a cube of the mean orbital radii, because the determination of these radii already implies an elementary evidence of planetary individuality, i.e. the distance between the foci, the spatial unit to the cube in question must be identified with the mean distance from the sun.

The dialectical element comes, however, properly to the front only in connection with the form of the Involved Inverted Relation between the space traversed and the time elapsed. Space and Time are now to be grasped at once in the Inverted and the Direct Relation. Along with being already only moments of a fundamental unity, each of them just as much remains indifferent to the other, continuing to retain its own distinctive meaning apart from the other. Seeing, then, that for the purpose of our task we are meant to proceed as though we had no knowledge of Kepler's third law, we must usher it on the scene as an embodiment of the properly grasped and *eo ipso* also already solved contradiction under treatment.

Now, to grasp space and time in a unity which is at once direct or immediate and at once also implies mediation obviously means to grasp them in the sense of the two hypostatized aspects of the self-discerning true Being. We have before us simply another instance of the vicious circle between the finitized Infinite and the infinitized Finite, or, as we may now also express ourselves, between Quality and Quantity. The two aspects of the true Being, whether this latter is taken in its purely abstract sense or whether it is identified with its manifold concrete embodiments, have truly no separate being apart from one another except in the sense of Ideality; but then until one fully grasps the meaning of Ideality, as a higher kind of Reality in contrast with which what ordinarily passes for Reality becomes only an appearance devoid of a Being *sui generis*, of a Being, hence, having its Being in its Not-Being, or in a word, the Finite as such—until then the two aspects in question present themselves as two utterly distinct and incompatible

Beings. So comes it, then, that Space immediately appears to have no connection with Time. It is easy, one imagines, to conceive Space on its own, and just as much Time on its own. The former simply *is*, the latter, however, is only in its *not*, and Being as such is, of course, credited with an utter incompatibility with its own Negation. In truth, this assumed incompatibility stands only for that kind of qualitative distinction which is postulated between the members of the Nodal Line of Measure Relations in one and the same Substrate. And this its meaning is, in fact, acknowledged when we speak of two so-called incompatibles as giving rise to a contradiction. A contradiction requires two mutually exclusive meanings in our mind, and hence in one fundamental Being which is then their third and stands for their true Being. Therefore, seeing that the absolutely free movement bears witness to the nature of the true Being, as the principle of any happening whatever, it is not surprising that the law meant to put on record the full nature of the orbital movement should reflect upon the relation between Space and Time directly from the standpoint of the true Being and hence as qualitatively distinct Beings mediated in one fundamental unity. In so far, then, as the primary indefiniteness of their conception is thus left behind, seeing that the specification of either as an explicit aspect of the true Being necessitates that each be reflected upon in the form concluding its own dialectical whole: on the side of Space we get the general conception of Volume, which becomes analytically the Cube of the mean distance from the sun, whilst on the side of Time we get the Square of the sidereal period, because, as what is in itself dimensionless, Time acquires, in distinction from Space, the significance of the One of Quantity which reaches its concluding form in the Square.

CHAPTER IV.

TRANSITION INTO THE STANDPOINT OF THE MEASURABLE QUANTITY: (A) NEWTON'S LAW OF UNIVERSAL GRAVITATION.

By virtue of its significance, as the middle term at once joining and separating the two main specifications of the absolutely free movement, the present Transition should anticipate the standpoint occupied in the sphere of the measurable Quantity, where the standpoint of fundamental Oneness steps on the scene in the explicit sense of the self-differentiated One, but so that the present Ones acquire the rank of material somethings and hence in the present application of individual planets. Accordingly, we proceed to reconsider the already familiar analytic mode of interpretation of the planetary motion with a view to bringing out the formal alteration which Kepler's third law suffers in consequence of the analytical effort to present it in embodiment of the immediate attitude towards the planets, when, that is, they are conceived in the image of ordinary material things.

Therefore, in initiating our present task by a reflection upon the orbital movement, first of all, from the standpoint of Newton's first law of motion, we are more particularly concerned with the magnitude of the velocity of the implied uniform movement in a straight line; of the kind of movement planets are to be supposed as having been imparted by that original and perpetually lasting impulse which in the analytical mode of interpretation disguises the otherwise ignored element of freedom in the planetary motion. In this respect, we are referring to the average orbital velocity, represented in the case of individual planets by the expression $2\pi RT = v$, when $2\pi R$ represents the orbit in the image of a circle; for, the curbing of the original movement in a straight path into a circular one is not supposed to affect the magnitude of its uniform velocity.

As regards, next, the determination of the magnitude of the centripetal force, and herewith also of the centrifugal force, postulated in connection with the circular movement, we are adopting the standpoint of Newton's second law of motion, because this law indicates the manner in which forces acting under similar conditions are to be measured. If impressed forces set equal masses of matter in motion and act for equal times, their magnitude becomes proportional to the changes of velocity produced.

Now, inasmuch as the original straight path of a body moving with uniform velocity under the simultaneous influence of an impressed centripetal force is to be reflected upon in the sense of a surviving tendency to assume tangential direction at all points of the resulting circular motion and the change in the direction of this perennially intended, but at the same time just as perennially forcibly prevented movement at any two points of the circular orbit finds its diagrammatic representation in the relation between two lines enclosing the angle a subtended by the arbitrarily chosen arc at the centre; the magnitude of the centripetal force on unit mass comes to be symbolized by the ratio B/t, when B stands for the base of an isosceles triangle whose sides are made equal to v, whilst enclosing the angle a. For, from the standpoint of the parallelogram of forces, B equally represents the velocity which must be added in order to produce the change in the tangential direction at the two limiting points of the arc described in the time t with the velocity v, and this added velocity is traced to the influence of the centripetal force. As regards, finally, the evaluation of the ratio B/t in terms of v and R, there is the similarity between the already defined isosceles triangle with the triangle having for its base the arc vt (which for an indefinitely small t may be treated analytically as a straight line) and for its sides the radius of the circular orbit. Since in that case $B/v = vt/R$, we get $B/t = v^2/R$.

The same result is also reached by conceiving the circular motion as a case of a body falling under the sway of the centripetal force into the centre whilst simultaneously compelled to move along the tangent applied at any and every circumferential point; because, however, the law of descent of a falling body is thus prevented from entering into existence, its operation must be viewed as if always restricted

only to the first unit of time; in token, as it were, that its introduction has in this respect the significance of a mere device. The arc vt comes thus to be treated as the hypotenuse of a right-angled triangle, in which the side a lying on R represents the space traversed by the falling movement towards the centre in the first unit of time indicated by t. Accordingly $a = \frac{1}{2} ft^2$, and the implied acceleration having to be identified with the centripetal force f, which we have previously symbolized by the ratio B/t, its magnitude is now evaluated from the familiar geometrical relation symbolized by $v^2t^2 = 2R \times \frac{1}{2} ft^2$.

Of course, as the entire circumference $2\pi R$ is traversed at the velocity v in the time T, we have

$$2\pi R = vT,$$
$$\text{that is,} \quad v = 2\pi R/T,$$
$$\text{hence finally } f = 4\pi^2 R/T^2,$$

a formula representing also the force of attraction on a unit mass at a planet's distance, in reflection upon the previous symbolization of the planetary orbit with $2\pi R$ and of T with the sidereal period.

Now, "let it next be supposed that several planets at different distances from the Sum represented by R, R′, R″ . . . are revolving round him in different times T, T′, T″ . . . we shall have in each case

$$f = 4\pi^2\frac{R}{T^2},\ f' = 4\pi^2\frac{R'}{T'^2},\ f'' = 4\pi^2\frac{R''}{T''^2}.$$

But, by Kepler's third law, in each case the squares of the times of revolution T^2, T'^2, T''^2 are proportional to the cubes of the distances from the Sum R, R′, etc. Calling this law L, we have in each case

$$L = \frac{R^3}{T^2},\ L = \frac{R'^3}{T'^2},\ L = \frac{R''^3}{T''^2}.$$

Dividing the former equations by these, we get

$$f = 4\pi^2\frac{L}{R^2},\ f' = 4\pi^2\frac{L}{R'^2},\ f'' = 4\pi^2\frac{L}{R''^2};$$

that is, in each case f, or the attraction on the unit mass, varies in the inverse ratio of the square of the distances." [1]

[1] "Elementary Lessons in Astronomy," Sir N. Lockyer, 1909, Art. 623.

As is seen, the Newtonian explanation of the "hidden meaning" of Kepler's third law amounts to its mere transformation from the standpoint of the two forces postulated in connection with the circular motion, when this motion is treated as a resultant of the tangential and centripetal movement. The expression s^3/t^2 is thus reflected upon in the form of the inverse ratio $s^2 \times \frac{s}{t^2}$, where the factor s/t^2 is identified with the formula for the force of attraction on a unit mass at a planet's distance when interpreted in the sense of a series because in that case the common factor $4\pi^2$ disappears.

In so far as, in explicit reflection upon Newton's third law, the attracted body must be equally grasped as reacting on the attracting body, the Inverse Square Law acquires ultimately the form of Newton's Law of Universal Gravitation, whose formula is

$$fa\frac{m_1 m_2}{d^2},$$

where m_1 and m_2 refer to the masses of two bodies at a distance d from one another. For, inasmuch as the magnitude of attraction depends also on the mass, if the distance between the two bodies is constant and very large in comparison with the dimensions of either, the attraction will be doubled if we double the mass of one body, and quadrupled if we double the mass of both.

B. The Stellar System and the Belt of Minor Planets.

In rounding off our discussion of the analytic attitude towards the absolutely free movement, we must finally bring into relief the significance attaching to the self-differentiating aspect of the one true Being in explicit connection with our present transition from the previous correlation of the standpoint of the measurable Quality with the planetary system as simply Sun-centred into the next correlation of the standpoint of the measurable Quantity with this system as a system consisting of individually self-centred planets, and hence as a system of Planetary Individuality.

Now, just because the standpoint of Planetary Individuality has with us still the significance of an anticipation, and

the self-differentiated Oneness, consequently, is not yet to be explicitly correlated with the system of self-centred planets, we must immediately interpret it purely abstractly, in which respect we obviously reflect upon the dialectical background of the so-called Stellar system. Inasmuch as it is the one true Being that thus hypostatizes the self-differentiated aspect of its own all-embracing Inwardness, and the true Being communes with its own self even when assuming the guise of presupposed Immediacy, even the relation between the fixed stars should not be wholly bereft of the self-integrating aspect. So comes it that although the brightest stars are scattered very irregularly, "if we look at the smaller ones, we find that they gradually increase in number as their position approaches the portion of the sky occupied by the Milky Way," [1] which fact "enables us to get a rough idea of the shape of our system as we might get a rough idea of the shape of a wood from some point within it by seeing in which direction the trees appeared densest and thickest together, and in which direction it was most easy to pierce its limits" (ib., art. 32). Next, quite conformably with the character of finitude attaching to the sphere of the presupposed Immediacy, the fixedness of the stars concerns only their relative position from the standpoint of their collective motion, in which respect they have been divided by Eddington into two groups each of which has a distinct proper motion drift (ib., art. 44). It is also further noticeable that the stars are distributed into groups called *constellations* and conveniently divided into *northern* and *southern* in reference to the zodiacal constellations through which the sun appears to pass in his annual course. Moreover, a careful examination of the stars with powerful telescopes reveals the remarkable fact that some of them "which appear simple to the unassisted eye, appear double, triple or quadruple; and in some instances the number of stars revolving round a centre common to all is even greater" (ib., art. 47). And that along with this kind of correlatedness of the stars with the standpoint of physical change, there also necessarily must be some sort of correlatedness with the standpoint of chemical change, follows from the fact that both these standpoints belong to the Substrate's own Inwardness.

[1] Sir Lockyer's "Astronomy," art. 29.

Nevertheless, the fundamental unity of the self-differentiating and self-integrating aspect in the stellar system comes properly to view in the Solar System. In making this assertion, we find ourselves once again at variance with the analytic standpoint which insists on viewing the Sun only as one of the stars or else associates with every star the conception of a solar system. But in connection with the demonstrableness of the true Being as Absolute Ideality, the fundamental Oneness must be emphasized in the explicit sense of *one* Oneness; and we have taken notice already, on the occasion of our discussion of Oneness in original transition into the standpoint of Quantity, of the dialectic inadmissibility of associating this comprehensive sense of Oneness with the present Ones representing its immediate aspect as the self-differentiating One.

Obviously, the stellar system and the opening which it provides for another recapitulation of the dialectical whole belonging to Substrate's Inwardness in simultaneous reduction of celestial bodies to mere analogy of ordinary things, must be envisaged by us as coming under the head of the external aspect of the mediation forming the middle term between Absolute Indifference as such and the demonstrableness of the true Being as Absolute Ideality. Since, however, we are now really concerned with the inwardization of this external aspect of mediation, in so far, that is, as the self-differentiated Oneness is reflected upon in the explicit sense of a transition from the common Sun-centredness on the part of the planets into their individual Self-centredness, we must reduce the original stellar system to a mere moment of the Solar System, having to connect the original central point with the System of Planetary Individuality. The fact that this system is to be identified with the planets originally representative of the standpoint of the measurable Quantity, leads to the further inference that the indefinite manifoldness immediately characteristic of Self-differentiation should have its exemplification in connection with that term in the series of the mean planetary distances which was introduced in the sense of a background to the distinction between the two planetary groups under the head of the measurable Quantity i.e. with the term falling between the comparative numbers for Mars

and Jupiter. And that we thus refer to the belt of Minor Planets, is obvious.

The middle term between the Sun and the system of Planetary Individuality should have the form of indefinitely many moving bodies on *à priori* grounds. That, however, these bodies should not yet be distinguished against one another in the same way as individual planets, follows from their position in the solar system. Conformably with the notion we are attaching to them, we must just as much negate their analogy with individual planets and hence postulate in their connection a surviving exemplification of the standpoint of the measurable Quality. Just because they stand for the middle term between the two characteristic departments of the dialectical whole of Measurableness, they should share the present general meaning attaching to these departments without yet properly coming under the head of either. As a matter of fact, whilst, on the one side, already possessing every appearance of individual planets, they yet remain, on the other side, inseparably bound up together in such wise that "If their orbits are figured under the form of material rings, these rings will be found so entangled that it would be possible by meansof one among them, taken at random, to lift up all the rest" (ib., art. 285).

B

THE STANDPOINT OF THE MEASURABLE QUANTITY: THE SYSTEM OF THE PLANETARY INDIVIDUALITY.

From the standpoint of our familiarity with facts, we have from the very first been aware that planets revolve round the sun only whilst at the same time rotating upon their own axis. Inasmuch, now, as this conception of the planetary nature manifestly stands for the final meaning of the fundamental unity of Quality and Quantity, and the aspect of the orbital movement has been dealt with in recapitulation of the dialectical whole of the measurable Quality, the ensuing recapitulation of the dialectical whole of the measurable Quantity should concern the determination of the features connected with the aspect of the axial rotation. In any case, in vindication of our attitude towards the solar system, as the absolutely idealistic system of Measurableness, we must satisfy ourselves that planetary equators and sidereal days admit of an *à priori* determination. Hence, if we are right in correlating the two aspects of the planetary nature with the two aspects of Measurableness, the further determination of the features now in question cannot but be part and parcel of the present reinterpretation of the dialectic of the measurable Quantity. In so far, however, as the aspect of planetary Self-centredness remains all through in fundamental unity with the aspect of the common Sun-centredness, and the threefold subdivision of the ensuing dialectical whole, in display of the forms of the Quantitative Relation, reflects also upon a threefold specification of the fundamental unity of the two aspects, the aspect of axial rotation comes to the front explicitly only in transition into the middle stage. Just because the emphasis immediately falls still on the qualitative aspect, the feature to be correlated with the display of the standpoint of the Direct Relation must be identified with what we have already defined as the elementary evidence of the planetary individuality, i.e. with the distance σ between the foci of the elliptically shaped orbits.

CHAPTER I.

THE STANDPOINT OF THE DIRECT RELATION: σ OF THE PROPER PLANETS.

Inasmuch as the aspect of the orbital movement is now emphasized within the system of the planetary individuality, and this system is to be primarily exemplified in connection with those planets which are originally representative of the standpoint of the measurable Quantity, our task concerns immediately the determination of the distance between the foci in connection with the orbits of the Earth, Mars, Jupiter, and Saturn,—planets, quite consistently with our own attitude towards their position in the solar system, currently qualified as *proper* planets. The implied emphasis on the orbital movement obviously means that σ for these planets is to be treated as equivalent to spaces traversed with corresponding orbital velocities. If we symbolize the time in which planets cover the space equivalent to their σ by the letter ϑ, we get the equivalency

$$\sigma/\vartheta \equiv s/t,$$

where the ratio s/t stands, of course, for the series of orbital velocity v.

The fact, now, that the mere statement of this equivalency is of itself of no assistance in the carrying out of our present task, simply reminds us that the original import of the Direct Relation immediately calls for a reinterpretation from the standpoint of the measurable Quantity. Whereas under the head of the measurable Quality the moment of Amount in the Direct Ratio stands for an Extensive Quantum and the moment of Unity for an Intensive Quantum, under the head of the measurable Quantity the sides of the Direct Ratio exchange this their original significance, because we refer them to an internal rather than a merely external change.

Accordingly, in contrast to the original dependence of t

on s, it is now σ that should be viewed as dependent on ϑ. That is to say, we really aim at the determination of the series σ for the proper planets in terms of their ϑ, or are primarily concerned with the determination of the series ϑ. In any case, in order that the relation $s/t = v$ acquire the significance of a premise to further development under our present heading, it must be conceived as pregnant with its own negation, i.e. with another than its own original significance in simple reference to orbital velocity; and we cannot but connect this its other significance with the *time* required by the proper planets to traverse spaces equivalent to the distance between their foci.

As regards now the problem of the conversion of the series v into the series ϑ, we need a middle term between the two aspects of the planetary nature now before us with an accent on the orbital motion. But the postulated middle term can refer only to the function derived from Kepler's third law, i.e. to $2at/3s^2$. For this expression stands for the series of orbital velocities on the basis of $v_{♁} = 6\cdot\dot{6}$, whilst at the same time being already pregnant with the standpoint of planetary individuality.

Therefore, in putting before ourselves, at the outset of the ensuing process of mediation, the relation $s/t = v$, we do so only in order to restate it in incorporation of the element of negation implied in its postulated double significance, but without actually setting aside its original significance. That is to say, we reflect primarily only on the equalization of the ratio s/t with the expression $2at/3s^2$.

Since, in this respect, we must modify either the ratio s/t, so as to make it represent the series v on the basis $v_{♁} = 6\cdot\dot{6}$; or else the ratio $2at/3s^2$, so as to make it represent the series v on the basis $v_{♁} = 10$; and the two series presuppose the series v on the basis $v_{♁} = 1$: the equalization of the two alternative expressions for v becomes explicit in a set of three equations:—

$$s/10t = at/s^3 \quad . \quad (1) \text{ on the basis } v_{♁} = 1$$
$$2s/3t = 2at/3s^2 \quad . \quad (2) \quad ,, \quad ,, \quad ,, \quad v_{♁} = 6\cdot\dot{6}$$
$$s/t = at/s^2 \quad . \quad (3) \quad ,, \quad ,, \quad ,, \quad v_{♁} = 10$$

Viewed solely from the standpoint occupied in the sphere of the measurable Quality, these equations present them-

selves simply in the sense of a purely formal transformation of the formula for Kepler's third law. In reflection, however, upon the standpoint of the measurable Quantity, which we must now bring to the front in their connection, the distinction between them must be treated as *qualitatively fixed.* And in this respect, we cannot but correlate them with the moments in the system of the proper planets, when this system is subdivided under three heads conformably with the nature of a dialectical whole.

In that case, the equation (1) obviously refers to the Earth and the equation (3) to Saturn, because s from the standpoint of the original significance of the ratio s/t acquires its full value and this its full value should now be reinterpreted in the sense of the maximum value for the mean distance from the sun in the system of the proper planets. Accordingly, the equation (2) should refer to Mars and Jupiter as falling under one head; which they well may, seeing that from the standpoint of the original subdivision of the proper planets, in connection with the determination of the nodal line of the mean planetary distances, Mars represents the moment of totality in the group on the side of immediacy and Jupiter, by contrast, the moment of unity in the group on the side of totality, so that their distinctive meaning admits of being treated only as aspect of one and the same fundamental meaning.

Inasmuch, now, as the stated correlation of the threefold modification of Kepler's third law is to serve as a clue to the determination of the series ϑ for the proper planets, and ϑ is to be treated as taking the place of σ in the equivalency $s/t \vdash$, which means that ϑ is to be ushered on the scene from the standpoint of its implicit identificableness with σ and hence also with s: the concluding stage in the present process of mediation simply concerns the final identification of the series ϑ with the modified values of s as indicated on the left side of the three equations. In so far, namely, as $s_{♁}$ is implied as reduced to 1, $s_{♂}$ and $s_{♃}$ as multiplied by $2/3 = 0{\cdot}\dot{6}$, and $s_{♄}$ as retaining its full value, we get:—

$$\vartheta_{♁} = 1,$$
$$\vartheta_{♂} = 16 \times 0{\cdot}\dot{6} = 10{\cdot}\dot{6}\,\vartheta_{♁},$$
$$\vartheta_{♃} = 52 \times 0{\cdot}\dot{6} = 34{\cdot}\dot{6}\,\vartheta_{♁},$$
$$\vartheta_{♄} = 100\,\vartheta_{♁}.$$

Indeed, if we appeal for corroboration to the results of observation, as adduced in the tables appended by Sir Lockyer to his Astronomy, we find that, on the basis $\vartheta_{♁} = 1 : \vartheta_{♂} = 10{\cdot}45$, $\vartheta_{♃} = 34{\cdot}13$ and $\vartheta_{♄} = 98{\cdot}25$. As regards the discrepancy between these and our own results, it is to be realized, firstly, that observational results are not necessarily quite exact, and, secondly, that, neither can we, so far, claim finality for our own results. This comes home to us at once more particularly in connection with the terms for Mars and Jupiter. Even though these two planets admit of being implicitly identified under the same meaning, they remain explicitly two distinguishable aspects; and in reflection upon this their distinguishableness, it is reasonable to anticipate a later need for an alternative mode of determination for their ϑ.

CHAPTER II.

THE STANDPOINT OF THE INVERTED RELATION: EQUATOR AND AXIAL ROTATION OF THE PROPER PLANETS.

WE are now meant to view s and t in the sense of variable factors of a fixed product. But as these factors are at the same time to be interpreted in application to the planetary individuality, represented primarily by the system of the proper planets, and these planets are now to be ushered on the scene in the sense of celestial bodies revolving at once round the sun and their own axis: the factor s ceases to represent the mean distance from the sun, but, as unity to t, which itself is now to be interpreted in reference to the axial rotation, acquires the significance of the Equator. Therefore, inasmuch as the product $s \,.\, t$ is now meant to stand for a particular amount of planetary equators in simultaneous reference to the mean distance from the sun, the present rendering of the standpoint of the Inverted Relation is found in the formula

$$2\pi r \mathrm{T} = \mathrm{S},$$

where T stands for the element of time, the sidereal period, but as subject to further specification, and S for the space described with equatorial velocity in terms of the mean distance from the Sun when equally suitably specified.

We can postulate no other interpretation of the space represented by the expression $2\pi r\mathrm{T}$, because the exponent of the Inverse Ratio embodies a return to Quality in its purely immediate sense. Also, we are under dialectical necessity of determining the *actual* value of the mean distance from the Sun as against, not only its round notionally derived series, but also its evaluation from the standpoint of observation. For, in so far as we must also reflect upon our normally to be presupposed familiarity with observational astronomy, we do so only in order to remove the immediate appearance of our dependence on what is given;

or rather, in order to prove to ourselves that facts and knowledge are interdependent, our fundamental premise being precisely the unity of Thought and Being. Now, it is with a view to the necessary repayment of our debt to observational astronomy (in the shape of a removal of the immediate unconnectedness between its results and their in this way implied improvement or at least certification in the light of pure thought) that the deviation of the actual values for the mean distances from the sun from the value assigned to them in the notionally derived series is to be treated as an expression of the planetary individuality. So comes it that we propose to connect the space $2\pi r\mathrm{T}$ with the actual value of S.

The nature of the specification to which T and S must be subjected when the general formula $2\pi r\mathrm{T} = \mathrm{S}$ is applied, first of all, to the system of the proper planets, follows at once from the original discernment of this system into two groups. Inasmuch as these groups are to be contrasted in such wise that the Earth and Mars stand on the side of Unity, and Jupiter and Saturn on the side of Totality, the original meaning of S presents itself as a middle term to its twofold specification. Obviously, on the side of Unity, S is to be emphasized in the sense of an abstract unity to its comparative value in the series of the comparative numbers for the mean distance from the sun, in so far as these numbers, 10 and 16, have also the sense of an immediate Totality. On the side of this latter, S becomes, by contrast, a unity to the Totality completing the spatial relationship of Jupiter and Saturn to the Sun. That is to say, whilst the right side of the formula $2\pi r\mathrm{T} = \mathrm{S}$ is meant to represent on the side of unity the amount which the actual value S of the mean distance from the Sun is of the comparative value assigned to it in the series s, in application to Jupiter and Saturn it stands for their orbit as represented by the expression, $2\pi\sqrt{\mathrm{S}^2 - \frac{\sigma^2}{8}}$, where the expression under the sign of evolution represents, of course, the mean orbital radius ρ.

That, however, this double specification of S remains subject to further specification in reflection upon the distinction to be drawn between the members of either group, must be obvious. But as the nature of this further speci-

fication can come to the front only in the course of a circumstantial interpretation of the meaning to be recorded by means of the general formula in its application to either side, we must leave it for the time being undiscussed and turn our attention to the equally needed specification of the left side.

Seeing that, in this respect, $2\pi r$ is to be identified with the Equators of the planets coming up for treatment, the duality of meaning implied in the specification of a common meaning can come to the front only in connection with T. Now, since the distinction to be made between the planets belonging to each group inwardizes the main distinction between the two groups, the general reference of T to the sidereal period should be emphasized in the sense of either unity or totality. But as these terms apply to the sidereal period as a self-differentiated totality of days, the distinction between unity and totality should be interpreted in reflection upon the distinction between the mean solar and the sidereal day. In grasping T as an amount of either mean solar or sidereal days, we may be said to specify it in reflection upon the two characteristic aspects of the planetary individuality: the sidereal day, which let us agree to symbolize by the letter δ, emphasizes the aspect of Self-centredness as against the simultaneously obtaining aspect of the common Sun-centredness, embodied in the mean solar day d.

A. The Group on the Side of Unity.

1. *The Earth and Mars.*

Inasmuch as the planetary group on the side of unity embraces the Earth and Mars, and these two planets are themselves to be contrasted as extremes of unity and totality, it is at once plain that the expression $2\pi r$T should, in application to the Earth, represent the space equivalent to as many of its equators as there are *mean solar days* in its year; whilst, in application to Mars, the space in question becomes *exactly* equivalent to the space traversed with his equatorial velocity in his year, T having to represent his sidereal period as an amount of his *sidereal days.*

Our problem on this side concerns, therefore, mainly the further specification of these spaces in the sense of unities

to the comparative numbers for the *actual* mean distance from the sun of the two planets. But for this fact, the space represented by $2\pi r\mathrm{T}$, i.e. by either $(2\pi rt/d)_{♁}$ or $(2\pi rt/\delta)_{♂}$, would have to be treated as equivalent to one and the same fixed product, having for its factors $2\pi r_{♁}$ and $(t/d)_{♀}$ on the one side, and $2\pi r_{♂}$ and $(t/\delta)_{♂}$ on the other. In other terms, we would have to postulate the equation

$$(2\pi rt/d)_{♁} = (2\pi rt/\delta)_{♂} \ ;$$

and although we could then determine $2\pi r_{♁}$ from the given $t_{♁}$ and the fixed product, equally then to be assumed as included in the data, on which, to begin with, we find ourselves dependent, the magnitudes corresponding to $2\pi r_{♂}$ and $(t/\delta)_{♂}$ would remain indeterminate unless we made ourselves dependent on the given magnitude of one of them, which, however, runs counter our intention to reduce, if possible, rather than to increase our dependence on given data.

But, then, it is not primarily in reflection upon this our intention that we insist on the twofold significance of the fixed product represented by the right side of our general formula for the standpoint of the Inverted Relation, but because the group on the side of unity is meant to inwardize its own distinction from the group on the side of totality. In so far as the two groups are to be treated only as moments of one system, they can be contrasted only on the understanding that the distinction between the aspect of unity and totality remains on either side qualified by a reference to the same kind of distinction on the other side. From this it at once follows that the unit of the mean planetary distance from the Sun represented by the right side of our formula in application to the side of unity, is not to be specified simply as the tenth or the sixteenth part of the Earth's or Mars' actual mean distance from the Sun, but rather as the 52nd and 100th part of Jupiter's and Saturn's S respectively. As a matter of fact, even this inference calls for further qualification in reflection upon the fact that the Earth is to be emphasized (for the purpose of the specification of the right side of our formula) in the sense of an *extreme* of unity and hence in negation of its

immediate meaning as the In-itself of the system of the proper planets, when the Earth is at once also implied as a totality. Now, since the to be emphasized aspect of abstract immediacy in connection with the Earth can only refer to its identification with Venus, it suggests itself to substitute for the 52nd part of Jupiter's mean distance, the 196th part of Uranus' mean distance.

Therefore, given the mean planetary distances in question and additionally the Earth's sidereal period, we get for the determination of the Earth's and Mars' equators and Mars' sidereal day the equations:—

$$(2\pi rt/d)_{\text{♁}} = S_{\text{♅}}/196 \quad . \quad . \quad . \quad (\mathrm{I})$$

$$(2\pi rt/\delta)_{\text{♂}} = S_{\text{♄}}/100 \quad . \quad . \quad . \quad (\mathrm{II})$$

That is to say

$$2\pi r_{\text{♁}} = S_{\text{♅}}/196t_{\text{♁}} \quad . \quad . \quad (1)$$

$$2\pi r_{\text{♂}} = S_{\text{♄}}\delta_{\text{♂}}/100t_{\text{♂}} \quad . \quad . \quad (2)$$

$$\delta_{\text{♂}} = 100(2\pi rt)_{\text{♂}}/S_{\text{♄}} \quad . \quad . \quad (3)$$

In order, however, to solve the second and third of these equations without increasing our dependence on given data, we must additionally evaluate the expression $(2\pi rt)_{\text{♂}}$. In this respect, we are plainly meant to bring to our consciousness the fact that the double specification of the unit of planetary distance on the right side of our fundamental formula does not affect its significance as a fixed product of the two variable factors on the left side. Since, however, $(2\pi rt)_{\text{♁}}$ is not immediately equal to $(2\pi rt/\delta)_{\text{♂}}$, and $2\pi r$ cannot but retain its reference to either the Earth or Mars, the only way to present $2\pi r_{\text{♁}}$ and $2\pi r_{\text{♂}}$ in the sense of variable factors of one and the same fixed product is to identify the meaning of the abstract unity to the amount represented by the other factor. That is to say, we postulate that

$$(2\pi rt/d)_{\text{♁}} = (2\pi rt/d)_{\text{♂}} \quad . \quad . \quad (\mathrm{III})$$

when

$$\delta_{\text{♂}} = 100S_{\text{♅}}/196S_{\text{♄}}$$

and

$$2\pi r_{\text{♂}} = S_{\text{♅}}/196t_{\text{♂}}.$$

But, now, we cannot expect that the figures given for $S_{\text{♅}}$ and $S_{\text{♄}}$ have the same degree of exactness as the figures given for $2\pi r_{\text{♁}}$ and $\delta_{\text{♂}}$. Again, with a view to our

endeavour to reduce the margin of indefiniteness clinging to the given results of observational astronomy, we ought to base the testing of our results on the most reliable data. Therefore, it is advisable that we should make ourselves dependent, on the one hand, on the figure resulting for the Earth's equator from the given figures for its longer and shorter equatorial diameter rather than on the given figure for $S_{♅}$, and, on the other hand, on the figure resulting for $(t/\delta)_{♂}$ from the given figures for $t_{♂}$ and $\delta_{♂}$ rather than on the given figure for $S_{♄}$.

Now, the figures given by Sir Lockyer for the Earth's longer and shorter equatorial diameter are respectively 41,853,258 and 41,850,210 English feet. Accordingly,

$$2\pi r_{♁} = 24{,}901 \cdot 655430 \text{ statute miles.}$$

As regards $t_{♁}$, we find it, according to the best authorities (Leverrier or Hansen), a little over 365·256362 m.s.d. Therefore,

$$(2\pi rt/d)_{♁} = 9{,}095{,}488 \cdot 06 \text{ miles}$$

$$\therefore S_{♅} = 1{,}782{,}715{,}660 \text{ miles,}$$

as against Sir Lockyer's 1,783,383,000 „
or Chambers' 1,781,944,000 „

On dividing the resulting figure for $(2\pi rt/d)_{♁}$ by the amount of mean solar days corresponding to $t_{♂}$ according to best authorities, i.e. by 686·979826, we find that

$$2\pi r_{♂} = 13{,}239 \cdot 8183 \text{ miles,}$$

so that, as against Sir Lockyer's figure for Mars' equatorial diameter 4,185 miles (though on p. 81 of his "Elementary Astronomy" we find it also given as 4,211 miles), we get roundly 4,214 miles. Chambers, by the way, adduces in his Astronomy 4,999 miles.

Finally, assuming the given figure for $\delta_{♂}$ (= 24 h. 37 m. 23 s.) to be fairly exact, we find that

$$(t/\delta)_{♂} = 669 \cdot 61024,$$

$$\therefore (2\pi rt/\delta)_{♂} = 8{,}865{,}516 \cdot 3066 \text{ miles.}$$

$$\therefore S_{♄} = 886{,}551{,}630 \cdot 66 \quad „$$

as against Sir Lockyer's 886,779,000 „
or Chambers' 886,065,000 „

2. *The Earth and Venus.*

Since the Earth remained so far only as an abstract middle term having no explicit qualitativity of its own, its immediate meaning, as the first proper planet, having been associated with Mars and its own abstract immediacy with Venus: we must also grasp the concrete character of its immediacy in supersession of the immediately arising distinction between its two aspects. Accordingly, we proceed to put on record the Earth's identity with Venus and Mars in simultaneous distinction from both of them. Before, however, the Earth can be presented as objectively distinct *also* from Venus (seeing that its objective distinction from Mars is from the very first implied in this, that Mars is also another proper planet), Venus must be raised to the rank of a self-centred planetary individuality in supersession of her immediate significance as the extreme of abstract immediacy of the first proper planet.

Now, Venus' relation to the Earth being at first the relation of the Finite to its own Infinitude, the establishing of the Earth in explicitly negative unity with her becomes an instance of the dialectic between the Finite and the Infinite. The infinitization of the Finite means now that, in so far as Venus does not immediately present herself in the character of a self-centred planet, this character must be conferred upon her by means of her identification with the Earth. Accordingly, it suggests itself that, in application to Venus, the equation (I) becomes

$$2\pi r_{♀} t_{♁} / \delta_{♀} = (2\pi rt/d)_{♁} \quad . \quad . \quad \text{(IV)}$$

since in reflection upon the negative element in the postulated identity of the Earth with Venus and the implied emphasis on the latter's self-centredness, $t_{♁}$ should be expressed as an amount of $\delta_{♀}$. In other terms, we postulate that the ratio between the Earth's and Venus' equators is equivalent to the ratio between $d_{♁}$ and $\delta_{♀}$.

That, however, the left side of the equation (IV) cannot, in reference to the concrete character of the Earth's immediacy, simply return to its original form, is obvious. As an Other to its own Other in Venus, the Earth must be put on record as self-discerned, i.e. as identified with Venus in such wise that this identity is in the same breath negated;

and inasmuch as this negation is meant to restore its own immediate self-reference, its record should concern the substitution of $2\pi r_{♁}$ for $2\pi r_{♀}$. In so far, further, as the right side of the equation is then no longer meant to stand for $S_{♅}/196$, and neither, of course, for $S_{♄}/100$, it suggests itself to make it equal to $3\sigma_{♁}$. For, in dealing with the Earth as an explicit third to Venus and Mars, we are emphasizing its planetary individuality in return to its own abstract immediacy in explicit negation of the latter's reference to Venus and hence in return to its own elementary evidence in σ, which, however, acquires then the significance of an abstract unity to that tri-unity which becomes characteristic of the Earth's *concrete* immediacy, as a purely self-referent planet in supersession of the external aspect of its self-discernment.

But, then, in putting on record the Earth's immediacy in *pure self-reference*, we cannot stop at the implied transformation of the equation (IV) into

$$2\pi r_{♁} t_{♁}/\delta_{♀} = 3\sigma_{♁} \quad . \quad . \quad . \quad \text{(V)}$$

but must finally also remove the remaining reference to Venus. We find ourselves driven to the inference that the amount which $t_{♁}$ is of $\delta_{♀}$ admits of being expressed in terms of measurable features belonging to Earth alone. In this respect, it suggests itself at once to correlate $t_{♁}$ with $2\pi\rho_{♁}$ and $\delta_{♀}$ with $\sigma_{♁}/2(=e_{♁})$, seeing that σ implies the sense of a Two, whilst the feature corresponding to $\delta_{♀}$ should have the character of an abstract unity. In further reflection, however, upon the fact that the ratio $(2\pi\rho/e)_{♁}$ is immediately correlated with the standpoint of the orbital movement, and hence is also negative of the standpoint embodied in the ratio $t_{♁}/\delta_{♀}$, it occurs to us, further, to postulate between them the distinction obtaining between d and δ, seeing, precisely, that this distinction has been already used by us in embodiment of the contrast between the orbital and axial movement. If we are right, it follows that

$$t_{♁}/\delta_{♀} = (2\pi\rho/e)_{♁} + 1,$$

so that, finally, the equation (V) becomes

$$2\pi r_{♁}[(2\pi\rho/e)_{♁} + 1] = 3\sigma_{♁} \quad . \quad . \quad \text{(VI)}$$

where, as is seen, every reference to the rest of planets is excluded.

3. *The Probable Value of $S_{♁}$ and $\sigma_{♁}$.*

It becomes now obvious that, in further combining the equation (VI) with the expression resulting for ρ in terms of S and σ, i.e. with the equation

$$\rho = \sqrt{S^2 - \frac{\sigma^2}{8}},$$

we find ourselves in a position to estimate the margin of indefiniteness clinging to a given set of figures for the Earth's mean distance S and its orbital eccentricity *e*. If, for instance, *e* is identified with Sir Lockyer's 1,559,000 miles, we find that the corresponding S amounts only to 92,962,576 miles, instead of the given 92,965,000 miles. But that the margin of indefiniteness in connection with Sir Lockyer's figures is much smaller than in connection with Chambers' figures is plain from the fact that the latter's *e* amounts to 1,560,000 miles, whilst S = 92,890,000 miles.

That, if $S_{♁} = 92{,}965{,}000$ miles, $e_{♁}$ should be 1,559,021·5 miles, is proved by the evaluation of $S_{♁}$ on the basis of this figure for *e*: we get for $S_{♁}$ 92,965,162 miles. But when we further proceed to ascertain this distance also from the data given for Venus and Mars, then on the basis of

$t_{♁}^2 = 133{,}412{\cdot}2100\ (= 365{\cdot}2563621^2)$,

$t_{♀} = 224{\cdot}7008$ (practically the mean of Leverrier's and Hill's figures),

$t_{♂} = 686{\cdot}979826$ m.s.d.,

and $s_{♁} = 1$,

we find that

$$s_{♀}^3 = 0{\cdot}378454487,$$
$$\therefore\ s_{♀} = 0{\cdot}723332350$$
$$s_{♂}^3 = 3{\cdot}537466925.$$
$$\therefore\ s_{♂} = 1{\cdot}5236929.$$

Now, inasmuch as the given figures for $S_{♀}$ and $S_{♂}$ are respectively

67,245,000 and 141,650,000 miles,

we find, further, that, along with its given value of 92,965,000 miles, $S_{♁}$ is also implied as equal to 92,965,564 miles in connection with $S_{♀}$ and to 92,964,927 miles in connection with $S_{♂}$. Accordingly, there arises the question as to whether the figure for $S_{♁}$, based on $e_{♁} = 1{,}559{,}021{\cdot}5$ miles, approximates its actual value by excess or defect.

In this respect, we have immediately Sir Lockyer's figure for $\delta_{♀}$ to fall back upon. Namely, $\delta_{♀}$ is given by him as = 23 h. 16 m. 19 s. ?, as against Chambers' 23 h. 21 m. 23 s. Having already found that Sir Lockyer's figure for $e_{♁}$ and $S_{♁}$ are an improvement on those adduced by Chambers, we feel at first sight disposed to treat his figure for $\delta_{♀}$ as more reliable. As a matter of fact, we find that in this case, at any rate, Sir Lockyer's figure errs, comparatively speaking, grossly by defect. We may take for granted that $S_{♁}$ does not reach the value corresponding to it on the basis of $e_{♁} = 1{,}559{,}025$, i.e. 92,965,714 miles. From the equation (V) it, however, follows that $\delta_{♀}$ would even then amount only to 1400·180981 m., so that in order to reach Sir Lockyer's figure, $e_{♁}$ would have to be over 1,562,190 miles, which is clearly out of the question.

Nevertheless, since Sir Lockyer's error is undoubtedly due to an extreme of caution in the desire to improve on the previously given figure, and $\delta_{♀}$ on the basis of $e_{♁} = 1{,}559{,}021{\cdot}5$ miles amounts to 1400·1841 m. as against 1400·180981 m. on the basis of $e_{♁} = 1{,}559{,}025$ miles, it is reasonable to assume that the given figure for $S_{♁}$, or even its value 92,965,162 miles, errs by defect rather than excess. And in this respect, it seems further plausible (from the standpoint of the consideration likely to govern the rounding off of the figures for S in terms of 1000 miles) to assume that its actual value is anything up to 92,965,500 miles. Accordingly, we propose to base it on the highest round value in miles that $e_{♁}$ can reach up to this figure,

i.e. on $e_{♁} = 1{,}559{,}024$ miles, the result of our evaluation being as follows:—

$$
\begin{aligned}
S_{♁} &= 92{,}965{,}433{\cdot}5 \text{ miles.}\\
\rho_{♁} &= 92{,}958{,}924 \text{ miles}\\
t_{♁}/\delta_{♀} &= 375{\cdot}64345978\\
\delta_{♀} &= 0{\cdot}9723485\, d_{♁} = 1400{\cdot}181868 \text{ m.}\\
&= 23 \text{ h. } 20 \text{ m. } 11 \text{ s.}\\
\vartheta_{♁} &= 2807{\cdot}8384 \text{ m.}\\
2\pi r_{♀} &= 24{,}213{\cdot}087 \text{ miles}\\
2r_{♀} &= 7{,}707{\cdot}265 \text{ miles}
\end{aligned}
$$

as against Sir Lockyer's 7919 miles
or Chambers 7480 „

$$S_{♂} = 141{,}650{,}770 \text{ miles.}$$

as against Lockyer's 141,650,000 „
or Chambers' 141,536,000 „

A conclusive vindication of the correctness of these results must, of course, remain a matter of further tests in connection with the manifold additional relationships, into which they may be expected to enter in the sequel.

b. The Group on the Side of Totality.

1. *Formulæ for Jupiter's and Saturn's Equators.*

We already know that, on the side of Totality, the space traversed with equatorial velocity should represent the orbit. But just as this space did not, on the side of Unity, simply refer to $S_{♁}/10$ or $S_{♂}/16$, so neither now can it be expected to refer to $2\pi\rho_{♃}$ or $2\pi\rho_{♄}$ without any further specification. In fact, seeing that the left side of our formula implies two unknown magnitudes, and our dependence on given data is to be reduced to its minimum: the space now in question should be specified doubly, once with respect to the determination of the Equator aud another time with respect to the determination of the Axial Rotation. The only way to obviate the necessity of this double specification would be to treat the formula from the standpoint of the Involved Inverted Relation, i.e. to make $2\pi r = t = \sqrt{2\pi\rho}$. But, then, our present consideration comes still mainly under the head of the Inverted Relation.

Nevertheless, inasmuch as on the side of totality the standpoint of the Inverted Relation should present itself as in transition into the next higher standpoint, the form of the Involved Inverted Relation should, by anticipation, be part and parcel of the present determination of Jupiter's and Saturn's equators,—in the first place of Saturn's equator, because it is more particularly in connection with the concluding member of the system of the proper planets that the return from the measurable Quantity into the measurable Quality should be properly ushered on the scene.

Inasmuch, however, as the left side of our formula should not be converted into $(2\pi r)^2$ without any more ado even in reference to Saturn, the implied specification manifestly having to take the shape of a co-efficient a on the right side, which becomes thus $a \,.\, 2\pi\rho_{\text{♄}}$: in reference to Jupiter, it suggests itself *additionally* to decrease $(2\pi r)^2$ as many times as $2\pi\rho$ is to be increased in consequence of its necessary specification. For, although the mode of determination for his equator should already anticipate the standpoint of the Involved Inverted Relation, we are also meant to put on record its distinction from the adopted determination for $2\pi r_{\text{♄}}$, in which case we surely ought to give prominence to the feature properly characteristic of the Inverted Relation. Whereas, therefore, the formula for Saturn's equator is :—

$$(2\pi r_{\text{♄}})^2 = a \,.\, 2\pi\rho_{\text{♄}} \quad . \quad . \quad . \quad (1)$$

the formula for Jupiter's equator becomes

$$(2\pi r_{\text{♃}})^2/\beta = \beta \,.\, 2\pi\rho_{\text{♃}} \quad . \quad . \quad . \quad (2)$$

As regards, now, the coefficients a and β, we cannot but view in them an embodiment of the present specification of the right side of our formula in contrast with the corresponding specification on the side of Unity. Whereas the specification consisted in this case in the reduction of a given totality to unity, the right side of the formula standing once for $S_{\text{♅}}/196$ and another time for $S_{\text{♄}}/100$, now the specification ought to consist in the treatment of the orbits as unities to be multiplied. And inasmuch as the coefficients in question must also belong to the own Inwardness of the orbits to be specified, it commends itself to our dialectical instinct that they should represent comparative numbers for the mean orbital radii. As regards the

standard to be used in this respect, it is in keeping with our grasp of the Earth, as the In-itself of the planetary system, that the standard should be identified with $S_{♁}$—with $S_{♁}$ rather than $\rho_{♁}$, because the reference to orbit concerns the side of totality alone.

But, then, since we must also reflect upon the distinction to be drawn between the members of the group on the side of totality, the standard too must be specified conformably with its application to either $\rho_{♃}$ or $\rho_{♄}$. Now $S_{♁}$ counts also as a totality of 10 units, and its further specification undoubtedly concerns the abstract unity to this totality. At any rate, we cannot think of any other way to embody the double reference to be symbolized by $S_{♁}$ except to substitute for $S_{♁}/10$ the unit to S of those two planets which represent the extremes of unity and totality to the whole system of the proper planets. Those planets are, of course, Venus and Uranus. Accordingly, the coefficient for Jupiter's orbit would represent the amount which $\rho_{♃}$ is of $10 \times S_{♀}/7$, whilst in the case of the coefficient for $2\pi\rho_{♄}$, $\rho_{♄}$ becomes the amount of $10 \times S_{♅}/196$. On substituting these values for α and β in the equation (1) and (2), we find finally that

$$2\pi r_{♃} = 0{\cdot}7\rho_{♃}\sqrt{2\pi\rho_{♃}}/S_{♀} \quad . \quad . \quad \text{(VII)}$$

$$2\pi r_{♄} = \sqrt{19{\cdot}6\rho_{♄} \,.\, 2\pi\rho_{♄}/S_{♅}} \quad . \quad \text{(VIII)}$$

2. *Side-issues Involved in their Evaluation.*

a. The Actual Value of $\vartheta_{♂}$ and $\vartheta_{♃}$.

According to our previous result, $\vartheta_{♃} = 34{\cdot}6\vartheta_{♁}$. But we have at once pointed out that, in arriving at this result, we have left out of consideration the element of mediation between Mars and Jupiter. Consequently, we proceed now to reflect upon the distinction to be drawn between them within their fundamental unity.

Inasmuch, now, as this their unity implies a direct correlation between the standpoint of unity and that of totality, the distinction to be drawn between them obviously should be due to an emphasis on either the one or the other of the two standpoints. That is to say, we are meant to

inwardize the relation between Mars and Jupiter under the head of either, on the understanding that the side of unity gives prominence to the standpoint of Axial Rotation, and the other side to the standpoint of Orbital Movement. And in so far, therefore, as the distinguishment between the two aspects of the planetary individuality should find its characteristic embodiment in connection with the moments of unity and totality on the opposite sides (each side being assumed as a unity of both moments), it suggests itself to identify the moment of unity on the side of Mars with his sidereal day and the moment of totality on the side of Jupiter with his sidereal period.

With respect to the other two moments on the opposite sides, it suggests itself at once that the moment of unity on the side of Jupiter should be identified with his ϑ; for, since the standpoint of axial rotation should on this side remain subordinated to the standpoint of orbital movement, it cannot go beyond the elementary evidence of planetary self-reference. As regards the moment of totality on the side of Mars, it would suggest itself at first sight to identify it with t/δ. But as this inference rests on the tacit assumption of a simple analogy between the meanings to be associated with the two moments on either side, the opposite moment of unity (as against $\delta_{\mars}$ forming the moment of unity to t/δ) would have to be identified with $\vartheta_{\mars}/\vartheta_{\earth}$, this obviously being the form assumed by ϑ in consequence of its specification on the side of Mars. That is to say, we are assuming that

$$\vartheta_{\mars}/\vartheta_{\earth} : (t/\delta)_{\mars} = \vartheta_{\jupiter}/d_{\earth} : (t/d_{\earth})_{\jupiter}.$$

In so far as this equation may be said to represent the aspect of fundamental unity between the two contrasted sides and hence is not meant to lose its validity when, in more explicit embodiment of the contrast between the two sides, the moment of unity on the side of Mars is (conformably with our original suggestion) identified with $\delta_{\mars}$, we find that the corresponding moment of totality on this side stands for $t_{\mars}$ as an amount of $\vartheta_{\mars}/\vartheta_{\earth}$. Hence, we are transforming the inferred equation into

$$\delta_{\mars} : t_{\mars}\vartheta_{\earth}/\vartheta_{\mars} = \vartheta_{\jupiter} : t_{\jupiter} \quad . \quad . \quad \text{(IX)}$$

With respect to the *second* equation needed for a complete determination of $\vartheta_{\text{♃}}$ and $\vartheta_{\text{♂}}$—in reflection upon the qualitative distinction obtaining between the contrasted sides, it commends itself to our dialectical instinct to anticipate the standpoint of the Involved Inverted Relation in connection with the ratio between the moments of totality. That is to say, we postulate that

$$t_{\text{♂}}\vartheta_{\text{♁}}/\vartheta_{\text{♂}} = \sqrt{t_{\text{♃}}} \quad . \quad . \quad . \quad (X)$$

with the consequence that

$$\sqrt{t_{\text{♃}}} = \vartheta_{\text{♃}}/\delta_{\text{♂}} \quad . \quad . \quad . \quad (1)$$

and

$$\vartheta_{\text{♃}}\vartheta_{\text{♂}}/\vartheta_{\text{♁}} = t_{\text{♂}} \, . \, \delta_{\text{♂}} \quad . \quad . \quad . \quad (2)$$

On putting these equations to the test in terms of the given figures for

$$t_{\text{♃}} = 4332{\cdot}5848d_{\text{♁}},$$
$$t_{\text{♂}} = 686{\cdot}979826d_{\text{♁}},$$

and

$$\delta_{\text{♂}} = 1{\cdot}025{,}960{,}648{,}148d_{\text{♁}},$$

we find that

$$\sqrt{t_{\text{♃}}} = 65{\cdot}8223731,$$
$$\vartheta_{\text{♂}}/\vartheta_{\text{♁}} \text{ or } t_{\text{♂}}/\sqrt{t_{\text{♃}}} = 10{\cdot}436874175,$$
$$\vartheta_{\text{♃}} \text{ or } \delta_{\text{♂}} \, . \, \sqrt{t_{\text{♃}}} = 67{\cdot}531164570,$$
$$\therefore \vartheta_{\text{♃}}\vartheta_{\text{♂}}/\vartheta_{\text{♁}} = 704{\cdot}94257 \ldots,$$

as against

$$t_{\text{♂}} \, . \, \delta_{\text{♂}} = 704{\cdot}8142675. \ldots$$

The discrepancy between the last two results need cause us no anxiety as regards the objective validity of our reasoning, because it is obviously due to an incorrectness in the given figure for $t_{\text{♃}}$. At any rate, we have no reason to suspect an appreciable margin of indefiniteness in connection with the given figure for $t_{\text{♂}}$, seeing that $S_{\text{♂}}$, resulting from it, according to Kepler's third law, is practically identical with the given $S_{\text{♂}}$. True, in so far as, in reflection upon the discrepancy between Sir Lockyer's figure for $S_{\text{♄}}$ and our result representing $100 \times (2\pi rt)_{\text{♂}}/\delta_{\text{♂}}$, the correctness of the given figure for $\delta_{\text{♂}}$ invites some misgiving, the discrepancy now in question is very probably also

traceable to this source; but that its main reason must be connected with $t_{♃}$ comes home to us when we calculate from it $S_{♃}$: we get roundly 483,527,000 miles as against Sir Lockyer's 483,678,000 miles.

β. *The Actual Value of* $t_{♃}$ *and* $\delta_{♂}$.

When, in attempting to evaluate the actual value of $t_{♃}$ purely tentatively, we first of all treat the given figure for $\delta_{♂}$ as quite correct, we find that, in order to secure an agreement to six decimal places between the values corresponding to $t_{♂}\delta_{♂}$ on the one side and to $\vartheta_{♃}\vartheta_{♂}/\vartheta_{♁}$ on the other, the figure for $t_{♃}$ must be based roundly on $s_{♃} = 5{\cdot}2095051 s_{♁}$. For, in that case

$$
\begin{aligned}
t_{♃} &= 4334{\cdot}83260809\ldots, \\
\sqrt{t_{♃}} &= 65{\cdot}8394456848\ldots, \\
\vartheta_{♂}/\vartheta_{♁} &= 10{\cdot}43416782829\ldots, \\
\vartheta_{♃} &= 67{\cdot}548680358738\ldots, \\
\therefore\ \vartheta_{♃}\vartheta_{♂}/\vartheta_{♁} &= 704{\cdot}81426714\ldots, \\
\text{as against}\qquad (t \times \delta)_{♂} &= 704{\cdot}81426754\ldots.
\end{aligned}
$$

But as we cannot be quite sure that the given figure for $\delta_{♂}$ is quite exact, and the margin of indefiniteness in this respect is brought to our notice primarily by the discrepancy between the figure resulting for $S_{♄}$ from our mode of determination and Sir Lockyer's figure, it suggests itself at first sight to compute $\delta_{♂}$ on the assumption that the given figure for $S_{♄}$ is after all fairly correct. The fact, however, that $\delta_{♂}$ would in that case have to be reduced by 23s. throws doubt on this assumption. Indeed, abundance of distinct markings on Mars' surface and the consequent ease of ascertaining the correct figure for his sidereal day, makes us reluctant to tamper with the given figure at all. Consequently, any deviation in this connection can commend itself to us only if it rests on a distinctly dialectical element.

Now, inasmuch as the determination of the correct

value of $\delta_{♂}$ is necessitated for the ultimate purpose of evaluating $2\pi r_{♃}$, and this evaluation brings to the front the Inverted Relation between Jupiter and Mars, it suggests itself that this Relation should also play a part in the solution of our present problem. In so far, however, as the already derived connection between $\vartheta_{♄}$ and $\vartheta_{♂}$ on the one side, and $\delta_{♂}$ and $t_{♂}$ on the other, is then of no help, because $\delta_{♂}$ is in this respect presupposed as already known, the Inverted Relation between Jupiter and Mars must now be reinterpreted conformably with the present end in view. Accordingly, we propose to exemplify the Inverted Relation in reference to the spaces covered by a particular amount of Mars' and Jupiter's axial rotations; when, that is, the space covered is viewed from the standpoint of the fundamental equation $2\pi r\mathrm{T} = \mathrm{S}$. It commends itself to our dialectical instinct that when the equator refers to Mars, T refers to Jupiter, and *vise versâ;* whilst, in so far as we are meant to compare the described spaces, it suggests itself, in absence of any other clue, to make them directly proportional to $\mathrm{S}_{♂}$ and $\mathrm{S}_{♃}$. That is to say, we postulate the equation :—

$$\frac{2\pi r_{♃}\mathrm{T}_{♂}}{\mathrm{S}_{♃}} = \frac{2\pi r_{♂}\mathrm{T}_{♃}}{\mathrm{S}_{♂}} \quad . \quad . \quad . \quad (1)$$

As regards the meaning to be associated with T; in reflection upon its connection with the standpoint of axial rotation, it suggests itself at first sight to identify it with t/δ. The question, however, is whether the standpoint of the Inverted Relation is to be exemplified also in this connection; whether, that is, $t_{♂}$ is to be grasped in terms of $\delta_{♃}$, and $t_{♃}$, by contrast, in terms of $\delta_{♂}$? But, then, since the standpoint of the Inverted Relation in the first place concerns only the product $2\pi r\mathrm{T}$, the further specification of the immediate significance of T as t should be viewed independently of the direct proportionality of the final products with the corresponding S, and in that case present us with an opportunity to put on record the double aspect of the relation between Mars and Jupiter. In so far, that is, as their relation remains direct, $t_{♃}$ may well be grasped as an amount of $\delta_{♂}$, because totality should then be connected

with Jupiter and unity with Mars. In so far, however, as this their direct relation is to be at the same time negated, and Mars becomes in that case on his own side negatively self-referent, it suggests itself, in reflection upon the original relation between the Earth and Mars, that $t_{♂}$ should become an amount of $\delta_{♁}$. In this way, the equation (i) assumes the shape

$$\frac{2\pi r_{♃} t_{♂}/\delta_{♁}}{S_{♃}} = \frac{2\pi r_{♂} t_{♃}/\delta_{♂}}{S_{♂}} \quad . \quad . \quad (2)$$

or else

$$\frac{2\pi r_{♃} s_{♂}}{t_{♃}/\delta_{♂}} = \frac{2\pi r_{♂} s_{♃}}{t_{♂}/\delta_{♁}} \quad . \quad . \quad . \quad (XI)$$

As is seen, given the values for $2\pi r_{♃}$ and $t_{♃}$ and $s_{♃}$, in result of the already known mode of determination on the basis of a tentatively fixed figure for $\delta_{♂}$, the condition embodied in the last equation enables us to test the degree of its correctness. That its given value approximates its correct value by excess, becomes now directly demonstrable.

Since in that case $s_{♃} = 52{\cdot}0295051$, the ratio $\dfrac{2\pi r_{♂} s_{♃}}{t_{♂}/\delta_{♁}}$ becomes equivalent to

$$\frac{52{\cdot}0295051 \times 13239{\cdot}8183}{688{\cdot}8608} = 1000{\cdot}0005,$$

whilst, on determining $2\pi r_{♃}$ from its formula on the basis of the figures resulting for $t_{♃}/\vartheta_{♃}$*, we find that

$$\frac{2\pi r_{♃} s_{♂}}{t_{♃}/\delta_{♂}} = \frac{277{,}355{\cdot}89 \times 15{\cdot}236929}{4225{\cdot}14510} = 1000{\cdot}214.$$

* $t_{♃}/\vartheta_{♃} = \dfrac{4334{\cdot}83260809}{67{\cdot}54868036} = 64{\cdot}1734610504 = a$

$$\rho = \frac{Sa\sqrt{8}}{\sqrt{4\pi^2 + 8a^2}}$$

$$a^2 = 4116{\cdot}3131031872058713 40$$
$$4\pi^2 = 39{\cdot}4784175 1413609$$
$$\sqrt{4\pi^2 + 8a^2} = 181{\cdot}5763840454$$
$$a\sqrt{8} = 181{\cdot}4676412628$$
$$S_{♃} = 92{,}965{,}433{\cdot}5 \times 5{\cdot}20295051$$
$$= 483{,}694{,}550 \text{ miles}$$
$$\frac{a\sqrt{8}}{\sqrt{4\pi^2 + 8a^2}} = 0{\cdot}99946555394$$

γ. *The Unit Length Postulated by our Standpoint and its Coincidence with the British Statute Mile.*

It is also to be realized that the suggested mode of determination for the measurable features now under discussion rests on the tacit assumption that the unit length postulated by our formulæ for $2\pi r_{♃}$ and $2\pi r_{♄}$ is identical with the British Statute Mile. That, if the results of observation, serving us as a test of the objective validity of our inferences, were given in terms of the metric system of France, our present fairly close agreement between the values resulting for the two sides of the equation (XI) would be conspicuous by its absence, follows from the fact that the computed value for $2\pi r_{♃}$ depends on $\sqrt{2\pi\rho_{♃}}$. Seeing that the value for $2\pi r_{♄}$ equally depends on $\sqrt{2\pi\rho_{♄}}$, it follows that our result for the equators of the two planets can be correct only on condition that the magnitudes involved in the one ration are given in terms of a unit length to be ushered on the scene by our own reasoning. We ought to have made allowance for this reflection from the very first by writing the formula, say, for Jupiter's equator in the form

$$2\pi r\mu = 0{\cdot}7\rho\mu\ \sqrt{2\pi\rho\mu}/\mu S_{♀},$$

or simply $$2\pi r\sqrt{\mu} = 0{\cdot}7\rho\ \sqrt{2\pi\rho}/S_{♀},$$

the coefficient μ standing of course for the ratio between the unit length adopted in connection with the results of observation and the dialectically necessary unit postulated by our reasoning. But, then, as the thus implied comparison is not really part and parcel of the dialectic embodied in our formulæ, but concerns only a subsequent test of their objective validity, our omission to introduce the coefficient μ calls in truth for no apology.

But the task of placing the discovered coincidence on a

$$\begin{aligned}
\therefore\ \rho_{♃} &= 483{,}436{,}251 \text{ miles} \\
S_{♀} &= 67{,}244{,}905{\cdot}5 \quad \text{,,} \\
\therefore\ 0{\cdot}7\rho_{♃}/S_{♀} &= 5{\cdot}032431277 \\
\therefore\ \sqrt{2\pi\rho_{♃}} &= 55{,}113{\cdot}697 \\
\therefore\ 2\pi r_{♃} &= 277{,}355{\cdot}89 \text{ miles.}
\end{aligned}$$

dialectical foundation is still before us. Now, seeing that the sides of the equation (XI) may be said to embody a kind of Involved Inverted Relation between Space and Time, and their value unmistakably is meant to amount to a round 1,000, it suggests itself to grasp either of them as a restatement of Kepler's third law, according to which s^3/t^2, when $s_{\text{♁}} = 10$ and $t_{\text{♁}} = 1$, equally amounts to 1,000. In any case, a return to the standpoint typified by this law is in place in connection with the evaluation of $2\pi r_{\text{♃}}$, seeing that we must in this respect also already anticipate a return from the standpoint of the measurable Quantity (i.e. Axial Rotation) into that of the measurable Quality (i.e. Orbital Movement). In so far, now, as the element of Space comes to be represented by an Inverted Relation between $2\pi r$ and s, in place of the original s^3, and the qualitativity in connection with the element of Time equally loses its original purely quantitative symbol, on account of the now necessary reflection upon the standpoint of Axial Rotation, the fundamental identity with Kepler's third law can come to the front only implicitly in connection with its constant value on the basis of $s_{\text{♁}} = 10$ and $t_{\text{♁}} = 1$; when, that is, the constant represents $s_{\text{♁}}{}^3$.

At any rate, in so far as we must postulate an absolutely fixed unit length, we cannot but base its determination on a dialectically necessary relation between comparative numbers of equally absolutely fixed astronomical magnitudes; and in that case it is surely in place that the dialectically necessary relation should, in connection with the evaluation of $2\pi r_{\text{♃}}$, be identified with the suggested significance of either of the two sides of the equation (XI). Accordingly, we postulate that,

firstly, $$2\pi r_{\text{♃}} = s^3_{\text{♁}} t_{\text{♃}} / s_{\text{♂}} \delta_{\text{♂}} \quad . \quad . \quad \text{(XII)}$$

secondly, $$2\pi r_{\text{♂}} = s^3_{\text{♁}} t_{\text{♂}} / s_{\text{♃}} \delta_{\text{♁}} \quad . \quad . \quad \text{(XIII)}$$

and thirdly, $$2\pi r_{\text{♁}} = s^3_{\text{♂}} t_{\text{♁}} / s_{\text{♃}} \delta_{\text{♁}} \quad . \quad . \quad \text{(XIV)}$$

seeing that, by virtue of Kepler's third law,

$$s^3_{\text{♁}} t^2_{\text{♂}} = s^3_{\text{♂}} t^2_{\text{♁}},$$

and, further, by virtue of the equation (III),

$$2\pi r_{\text{♂}} t_{\text{♂}} = 2\pi r_{\text{♁}} t_{\text{♁}}.$$

If, in further demonstration of the already realized untenability of any appreciable difference between our own unit length and the British Statute Mile, we now additionally evaluate $2\pi r_{♁}$ from the equation (XIV), whilst still identifying $s_{♃}$ with its value on the basis $\delta_{♂} = 1477·383$ m., then since

$$s^3_{♂} = 3537·4669255,$$
$$s_{♃} = 52·0295051,$$
$$t_{♁}/\delta_{♁} = 366·256362,$$

we find that $2\pi r_{♁} = 24901·6354$ miles.

As is seen, although the figure for $s_{♃}$ certainly is above its correct value, the discrepancy from the magnitude resulting for $2\pi r_{♁}$ from the given results for the Earth's larger and smaller equatorial diameter (24,901·655430 miles) amounts barely to 0·02 mile. And since the given figure for $\delta_{♂}$ exceeds its correct value only by a few seconds, so that the actual value of $s_{♃}$ cannot be much less than 52·0295051, we infer that by basing it on

$$2\pi r_{♁} = 24{,}901·655430 \text{ miles},$$

any margin of indefiniteness still clinging to it practically becomes negligible. In this way, then, we find that

$$s_{♃} = 52·0294634$$
$$\text{or } S_{♃} = 483{,}695{,}162 \text{ miles},$$

as against Sir Lockyer's	483,678,000	„
or Chambers'	483,288,000	„

3. *Their Resulting Values.*

a. $2\pi r_{♃}$.

We find now that

$$t_{♃} = 4334·82624329 \,.\, d_{♁}\,;$$
$$\therefore \sqrt{t_{♃}} = 65·8393973,$$
$$\therefore \vartheta_{♂}/\vartheta_{♁} = 10·4341755.$$

As regards $\vartheta_{♃}$, we know that

$$\vartheta_{♃} = \delta_{♂}\sqrt{t_{♃}} \quad . \quad . \quad . \quad . \qquad (1)$$

and further that the value for $\delta_{♂}$ must at the same time also satisfy the equations :—

$$2\pi r_{♃} = 0{\cdot}7\rho_{♃}\sqrt{2\pi\rho_{♃}}/S_{♀} \quad . \quad . \quad (2)$$

and

$$2\pi r_{♃} = 1000\ t_{♃}/s_{♂}\delta_{♂} \quad . \quad . \quad . \quad (3)$$

Reduced as we are in this respect to tentative methods, we find, after a rather laborious round of calculations, that the postulated $\delta_{♂} = 1477{\cdot}210$ m. $= 1{\cdot}025840\ d_{♁}$. For in that case :—

$$\vartheta_{♃} = 67{\cdot}5406873\ d_{♁},$$
$$(t/\vartheta)_{♃} = 64{\cdot}1809613936 = a,$$
$$a \,.\, \sqrt{8}/\sqrt{4\pi^2 + 8a^2} = 0{\cdot}999401\,5369,$$
$$\rho_{♃} = 483{,}404{,}689 \text{ miles},$$
$$0{\cdot}7\rho_{♃}/S_{♀} = 5{\cdot}03210,$$
$$\sqrt{2\pi\rho_{♃}} = 55{,}111{\cdot}8974\,;$$
$$\therefore\ 2\pi r_{♃} = 277{,}328{\cdot}578 \text{ miles},$$

as against $2\pi r_{♃} = 277{,}328{\cdot}571$ „

resulting from the other formula for

$$t_{♃}/\delta_{♂} = 4{,}225{\cdot}6357748$$
$$\text{and}\quad s_{♂} = 15{\cdot}236929.$$

If, for the purpose of comparison, we calculate the equatorial diameter, we find that

$$2r_{♃} = 88{,}276{\cdot}42 \text{ miles},$$

as against Sir Lockyer's 87,680 „
or Chambers' 88,439 „

β. $2\pi r_{♄}$.

On the now established assumption of the practical identity of the absolutely fixed unit length with the British Statute Mile, the evaluation of the formula for $2\pi r_{♄}$ presents no further difficulty.

Given $S_{♅} = 1{,}782{,}715{,}656$ miles,

the formula becomes $2\pi r_{♄} = 0{\cdot}0002628312\ \rho_{♄}\,;$

and as regards the evaluation of

$$\rho = Sa\sqrt{8}/\sqrt{4\pi^2 + 8a^2},$$

we find that, given our result for $2\pi r_{♂}(= 13{,}239{\cdot}8183$ miles) and $\delta_{♂}(= 1477{\cdot}210$ m.),

$$\begin{aligned}
S_{♄} &= 886{,}638{,}074 \text{ miles};\\
\therefore\ s_{♄} &= 95{\cdot}37287578,\\
\therefore\ t_{♄} &= 10758{\cdot}088345\ d_{♂},\\
\vartheta_{♄} &= 100\ \vartheta_{♂} = 280783{\cdot}84 \text{ m.};\\
\therefore\ a &= 55{\cdot}172859010,\\
\therefore\ \rho_{♄} &= 0{\cdot}999190417\ S_{♄}\\
&= 885{,}920{,}267 \text{ miles}\\
(2\pi\rho_{♄} &= 5{,}566{,}401{,}198{\cdot}586475 \text{ miles}\\
\sigma_{♄} &= 100{,}890{,}207{\cdot}6 \text{ miles});\\
\therefore\ 2\pi r_{♄} &= 232{,}847{\cdot}5 \text{ miles},\\
\therefore\ 2r_{♄} &= 74{,}117{\cdot}5 \quad ,,
\end{aligned}$$

as against Sir Lockyer's 73,713 ,,
or Chambers' 75,036 ,,

4. $\delta_{♃}$ *and* $\delta_{♄}$.

Seeing that the formula $2\pi r\mathrm{T} = 2\pi\rho$ does not yet apply to Jupiter and Saturn without the need of further specification, the determination of their sidereal day has its starting point in the inequality $\mathrm{T} \neq \rho/r$,
where $\mathrm{T} = t/\delta$.

Now, T is determinable either quite independently of the already known value of the ratio ρ/r, or else by means of its equalization with this ratio. The two sides of the original inequality concern Time and Space, and hence two qualitatively distinct aspects of their fundamental unity, which can come to the front only through a specified relation between them. Inasmuch, therefore, as the two alternative modes of determination of T are based on the immediate affirmation and the equally implied suppression of the distinction between Time and Space, they are dialectically necessary; and we cannot be in doubt that the immediate affirmation of the distinction concerns the determination of $(t/\delta)_{♃}$, and its supersession the determination of $(t/\delta)_{♄}$.

In so far, now, as the ratio ρ/r is to play no part in the determination of $(t/\delta)_{♃}$, the clue to this determination must be sought in the meaning attaching to Jupiter in the system of the proper planets. In this respect, we know that

Jupiter is to be grouped either with Saturn or with Mars. In order to exemplify this his double meaning, it suggests itself to substitute for $t_{\text{♃}}$ the sidereal period of that planet, to which Jupiter is immediately related as the moment of unity, and for $\delta_{\text{♃}}$ the siderial day of that planet, to which Jupiter is *pari passu* related as the moment of totality. In this way, we come to postulate that

$$(t/\delta)_{\text{♃}} = t_{\text{♄}}/\delta_{\text{♂}} = 10487{\cdot}1016386 \quad . \quad . \quad \text{(XV)}$$
$$\therefore\ \delta_{\text{♃}} = 595{\cdot}22155 \text{ m.} = 9 \text{ h. } 55 \text{ m. } 13{\cdot}4 \text{ s.},$$

as against Sir Lockyer's 9 h. 55 m. 28 s.,
or Chambers' 9 h. 55 m. 21 s.

In so far, further, as $\delta_{\text{♄}}$ is to be determined through the removal of the inequality

$$(t/\delta)_{\text{♄}} \neq (\rho/r)_{\text{♄}},$$

by means of a specifying number x, we postulate that

$$(t/\delta)_{\text{♄}} : x = (\rho/r)_{\text{♄}} \quad . \quad . \quad . \quad (1)$$

In order to evaluate x, we have immediately only the relationship between Jupiter and Saturn to fall back upon. But since, by virtue of this relationship, the previous mode of determing $\delta_{\text{♃}}$ should equally belong to the Inwardness of the now sought mode of determinating $\delta_{\text{♄}}$, the ratio $(\rho/r)_{\text{♄}}$ should represent also a particular amount of the already evaluated ratio $t_{\text{♄}}/\delta_{\text{♂}}$. Accordingly, we postulate that x stands for a specified value of $\delta_{\text{♂}}$. Seeing, now, that, in contrast to Mars, as the moment of totality in the group on the side of Unity, the Earth stands for the moment of unity only as *pari passu* identified with Venus it suggests itself to make x equal to the ratio $\delta_{\text{♂}}/\delta_{\text{♀}}$. The equation (1) becomes thus

$$(t/\delta)_{\text{♄}} : \delta_{\text{♂}}/\delta_{\text{♀}} = (\rho/r)_{\text{♄}} \quad . \quad . \quad \text{(XVI)}$$

where
$$(\rho/r)_{\text{♄}} = 23{,}905{\cdot}8324,$$
$$\delta_{\text{♂}}/\delta_{\text{♀}} = 1{\cdot}0550126\,;$$
$$\therefore\ (t/\delta)_{\text{♄}} = 25{,}220{\cdot}9544060,$$
$$\therefore\ \delta_{\text{♄}} = 0{\cdot}4265535\, d_{\text{♁}}$$
$$= 614{\cdot}2370 \text{ m.}$$
$$= 10 \text{ h. } 14 \text{ m. } 14{\cdot}2 \text{ s.},$$

as against Sir Lockyer's 10 h. 14 m. 24 s.
or Chambers' 10 h. 29 m. 17 s.

CHAPTER III.

THE STANDPOINT OF THE INVOLVED INVERTED RELATION

a. The Polar Circumference of the Earth and Venus.

We have now before us the planetary individuality as immediately established with respect to its return into its original immediacy. Thus, the formula $2\pi r\mathrm{T} = \mathrm{S}$ refers once again to the Earth, but so that the right side represents now also the orbit, and hence has a threefold meaning :—

(1) that of the 196th part of Uranus' mean distance from the sun (I) ;

(2) that of the distance between the Earth's foci increased three times (V) ;

(3) that of the Earth's orbit.

From the fact, however, that emphasis comes now on the completed grasp of the Earth's immediacy, it follows that the first two meanings are now to be viewed only as inwardized moments of the third meaning.

First of all, then, we are meant to put

$$2\pi r = \mathrm{T} = \sqrt{2\pi\rho_{♁}},$$

in which case it is obvious that $2\pi r$ cannot refer to the Earth's equator. Because it is meant to represent the root of the Earth's orbit, $2\pi r$ should refer to the Earth's aspect of abstract immediacy; hence, to Venus. Yet its meaning cannot even then stand for the Equator. Just because the standpoint of the original immediacy is now taken up in return into the standpoint of the orbital movement, and hence, analogously to the notion of Measure as the third to Quality and Quantity, acquires a third meaning: it suggests itself to interpret $2\pi r$, in its present application to Venus, in the sense of her Polar Circumference, which we propose to symbolize by $\mathrm{P}(2\pi r)_{♀}$.

That we are now indeed bringing to the front the standpoint of Measure, comes to our notice in the significance now to be attached to the left side of our formula. Namely, in postulating along with the equation

$$P(2\pi r)_{♀}T = 2\pi\rho_{♁} \quad . \quad . \qquad (1)$$

the further equation $\quad P(2\pi r)_{♀} = T,$

we imply that

$$P(2\pi r)_{♀}^{2} = 2\pi\rho_{♁} \quad . \quad . \qquad (\text{XVII})$$

That is to say, *the unit length postulated by our reasoning is the same part of Venus' polar circumference as this circumference is of the Earth's orbit.*

Accordingly, our present task involves also a further test of the already established coincidence of the dialectically postulated unit length with the British Statute Mile. Now, if $2\pi\rho_{♁}$ is roundly 584,070,145 miles, the figure resulting for $P(2\pi r)_{♀}$ is roundly 24,167·710 miles. Seeing that the otherwiseness characterizing the polar circumference in distinction from the equator implies a negation of the latter's elliptical shape, we cannot test our resulting figure for $P(2\pi r)_{♀}$ from the standpoint of the given figure for Venus' polar compression. But, then, inasmuch as the equations :—

$$(2\pi rt)_{♁} = S_{♅}/196 = 9{,}095{,}488 \text{ miles} \quad . \qquad (2)$$

$$(2\pi rt)_{♁}/\delta_{♀} = 6e_{♁} = 9{,}354{,}144 \text{ miles} \quad . \quad . \qquad (3)$$

are now to be treated as dialectically bound up with the equation (1), and consequently as no longer simply retaining their original form, we undoubtedly are meant to substitute 24,167·71 for $2\pi r_{♁}$ in one of the two inwardized equations, with a view to determining the alternative value for T in (1) to be postulated for it, partly, in explication of its simultaneous *qualitative* distinctiveness from its direct *quantitative* equality with $P(2\pi r)_{♀}$, but partly also for the purpose of determining the magnitude belonging to the alternative meaning attaching to $2\pi r_{♁}$ in the other inwardized equation. In so far as this alternative meaning is surely meant to brush aside the character of abstract immediacy associated with its primary immediacy and hence to stand for $P(2\pi r)_{♁}$, the proper test for the

correctness of our figure for $P(2\pi r)_{♀}$ is to be sought in the correctness of the figure resulting for $P(2\pi r)_{♁}$, in which respect it becomes now also plain that the equation to be used for the determination of T is the equation (2) or (I), since the identification of $2\pi r$ in (3) or (V) with $P(2\pi r)_{♁}$ is in agreement with the import of its original concrete meaning. Accordingly, the alternative value for T is given by the ratio $(2\pi rt)_{♁}/\sqrt{2\pi\rho_{♁}} = 376{\cdot}3487$; and by substituting this value for $t_{♁}/\delta_{♀}$ in the equation (3), we get

$$P(2\pi r)_{♁} = 3\sigma_{♁}\sqrt{2\pi\rho_{♁}}/(2\pi rt)_{♁} \quad . \quad \text{(XVIII)}$$
$$= 24{,}854{\cdot}9875 \text{ miles.}$$

And this result is, indeed, practically identical with the magnitude assigned to $P(2\pi r)_{♁}$ in connection with the fixation of the unit of the metrical system of France, this unit, one Metre, being meant to represent the ten-millionth part of the quadrant of the meridian. Seeing that one metre is equal roundly to 39·370432 inches,

$$P(2\pi r)_{♁} = 40{,}000{,}000 \times 39{\cdot}370432$$
$$= 24{,}855{\cdot}\dot{0}\dot{7} \text{ miles.}$$

An Attempt at an Explanation of the Coincidence of the British Statute Mile with our own Unit Length.

As is seen, the correctness of our identification of the absolutely fixed unit length with the British statute mile may now be viewed as conclusively established. In explanation of this coincidence it would suggest itself at first sight that the empirical fixation of the British statute mile might be directly traceable to a particular astronomical relationship. For, along with the relations that have already come to our knowledge, there may be additional, and perhaps even simpler relations between astronomical magnitudes endorsing our identification of the British statute mile with the unit length postulated by our own standpoint; and in its endeavour to eliminate the element of pure arbitrariness in the fixation of its standard length, observational astronomy may well have stumbled upon one of these relations. But that this our immediate inference

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does not hold good, is seen from the following quotation from Whitaker's Almanack ready to hand (1913, p. 452):—

"The *Mile* was originally 1,000 Roman paces, or 5,000 Roman feet, each = 11.67 inches. With the establishment of the English foot it became 5,000 English feet; under Elizabeth it was fixed at 5,280 feet, or 1,760 yards, so as to make it = 8 furlongs."

This makes plain that the astronomical significance of the British statute mile must be traced to the astronomical import to be postulated in connection with the more elementary unit length lying at the back of its fixation at 5,280 English feet. But that in this respect too the postulated astronomical significance may have been secured quite undesignedly, comes home to us at once in connection with the Roman foot.

Namely, one degree of the Earth's Equator happens to be equal to 375,552 Roman feet, as though one Roman foot had been intended to represent the same part of the 360,000th part of $2\pi r_{♁}$ as $\delta_{♀}$ is of $t_{♁}$. Of course, the ratio $2\pi r_{♁}/360{,}000$ should really be equal to 375·643459, instead of roundly to 375·552; but the discrepancy may be traced to the roundness of the given figure for one Roman foot, the figure necessitated by our own standpoint being 11·6671, instead of roundly 11·67 inches.

Yet, since it is quite certain that our figure for $\delta_{♀}$ was not known to Roman astronomers, the fixation of their foot must admit of an alternative explanation, which is, indeed, at once suggested by the fact that the Roman Mile represented 1000 paces, each = 5 Roman feet. The present writer happens to be exactly 6 Roman feet (i.e. 5 English feet, 10 inches), and he finds that the space measured off by one and the same foot in the course of his normal stride amounts exactly to 5/6 of his height (i.e. exactly to one Roman pace). Therefore, one Roman foot acquires the stated astronomical significance by being based on the difference between the height and pace on the part of a man of 6 Roman feet. At any rate, inasmuch as the human frame is differentiated into a system of natural measures, the possibility of a correlation of the various aspects of this system with the measurable features of the solar system may be expected on *à priori* grounds. Namely,

in reflection upon the significance assigned to the solar system by our progress in knowing, as the culminating embodiment of the true Being, ultimately of the Spirit, in the sphere of Presence, and upon the fact that in the sphere of the properly individualized Spirit, to be dealt with in due time, this embodiment presents itself immediately in the shape of a human body, it is to be expected that there should be some kind of correlatedness between the measurable features of the solar system and the system of measures embodied in our body.

When, therefore, we find further that one English foot represents almost exactly the 365·2563rd part of the 1000th part of one degree of the Earth's equator, we are at once inclined to put this surprising coincidence to the account of the just postulated connection between primitive limb-measures and the measurable features of the solar system. As a matter of fact, the fixation of the English foot appears to be of Egyptian origin, and, according to Whitaker's Almanack, the primitive limb-measures were, in ancient Chaldea and Egypt, given legal standard :

"Man's arms afforded the primitive measures: the Cubit (fore arm), Span, Palm (4 finger-breadths); also the Fathom = 4 cubits or 8 spans. The cubit was 24 finger-breadths. When a standard of the cubit was fixed, two-thirds of it were taken as a secondary measure and called a Foot. This was a convenient length, as, while the span was 12 finger-breadths, the foot could be divided either into 16 of these, or into 12 thumb-breadths, afterwards called ounces or Inches. Thus arose the duodecimal and sexdecimal series of measures, each with its advantages, the latter series generally more convenient."

That the Foot in question is indeed the origin of the English foot may be ascertained in most cases by measuring one's fore arm and finger or thumb breadth. At any rate, in the case of the present writer, the results exactly tally with the quoted figures.

From the fact that the base of the Great Pyramid is exactly one half the mean meridian mile, it may be inferred that Egyptians had also standards of purely astronomical or rather geometrical origin. As a matter of fact, the mean meridian mile was with them equal to 1000 fathoms, each = 6 feet, each = the 360,000th part of one mean

degree of latitude. As against this, the so-called Egyptian or Greek foot of distinctly geographical application, the other foot representing the 365256th (or properly only the 365224th) part of one degree of the equator appears to have been used only in the determination of measures of capacity. At any rate, "if the vessel containing 1000 Egypto-Roman, or Early English, ounces of water, which was the basis of our measures of capacity, were constructed of exactly cubic dimensions, its inner measurement would be exactly a foot, and its content would be a cubic foot". In so far, by the way, as the standard measure of capacity, the Alexandrian *talent*, all the same was equivalent to 1500 ounces of water, this, the so-called *royal* cubic foot, obviously was meant to be related to the simple cubic foot, containing 1000 ounces of water, as the cubit to the foot.

But in order fully to explain the possibility of the coincidence of the British statute mile with our own dialectically postulated unit length, we must properly account for the presumably non-astronomical origin of the furlong = $\frac{1}{8}$ mile. On consulting again Whitaker's Almanack, we find that the *Furlong* is 40 rods, whilst as regards the *Rod*, we read as follows:—

"The Saxons appear to have brought the Rhineland foot = 12·356 inches, to Britain. It survived for centuries in Scotland, the Scots Ell being = 37·068 inches or 3 Rhineland feet, practically 37 inches of English standard. The only trace of the Rhineland foot in England is in the statute rod, pole, or perch, which was fixed, not later than the time of Edward I., at $5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet, a length almost exactly equal to 16 Rhineland feet."

Accordingly, we are finally concerned with the origin of the Rhineland foot. After a vain search for a suitable limb-measure that would account for this origin, we find the only likely clue in the already mentioned identification of ounces with inches. As a matter of fact, assuming that the Egyptian wheat was not exactly $\frac{4}{5}$ of the weight of the same volume of water, but a little less, as is the case with average wheat, the measure for the Alexandrian talent of wheat would contain as many *ounces* of water as there are cubic *inches* in the cube formed by the Rhineland foot. In any case, even though in absence of precise data the origin of this foot remains a matter of speculation, the possibility of

its fixation, and herewith also of the Furlong and finally of the British statute mile, apart from any direct reflection upon astronomical relations, is, we hope, established.

b.

1. *The Formula* $2\pi r T = S$ *in Application to Uranus and Neptune.*

Seeing that the identity of the Earth and Venus corresponds, in our present circumstantial recapitulation of the dialectic of the measurable Quality and Quantity, to the stage reached in the Compound Body, and the next stage concerns the analysis of the thus still only immediately displayed unity of the measurable Quality and Quantity: we are now meant to reconsider the identity of the Earth with Venus with a view to a distinct embodiment of its double aspect.

The other sense, in which we must now emphasize this identity, in contrast to the immediate emphasis on Venus, obviously shifts the accent back to the Earth. But in order to get a clue to the necessary distinct embodiment of these two senses, we must further realize that the implied contrast properly concerns the planetary individuality with respect to its meaning as the In-itself of the planetary system. Or rather, inasmuch as the establishing of the In-itself of the whole planetary system comes up for its proper treatment only in the concluding portion of the present display of the Involved Inverted Relation, the contrast now under discussion concerns only the two main groups of the whole planetary system, i.e. on the one side, the system of the planets coming under the head of the measurable Quantity, hence the system of the proper planets inclusive of Venus and Uranus, and, on the other side, the two remaining planets representive, by contrast, of the standpoint of the measurable Quality, once in its simple immediacy, another time with respect to its self-recovery in return from the standpoint of the measurable Quantity. But as in connection with either of these groups we must reflect on the implicit unity of both, and hence associate with them the standpoint of totality, our dialectic instinct leads us to seek the present distinct embodiment of the two aspects of the planetary individuality now under discussion in connection

with the two remaining superior planets which still await their treatment, i.e. in connection with Uranus and Neptune.

Of course, in applying the formula $2\pi r\mathrm{T} = \mathrm{S}$ to these planets, we aim at the determination of their equators and axial rotations. The fact that $2\pi r$ concerned, in connection with the just superseded identity of the Earth with Venus, their polar circumferences, and T consequently equally lost its original meaning, as an amount of axial rotations—this fact presents itself to us now in the light of a pertinent analogy of the qualitative change undergone by chemical substances through chemical combination into a salt. Therefore, we may say that $2\pi r$ and T assume now once again their original reference to the equator and axial rotation, in correspondence to the reappearance of the original nature of the constituents of a salt in the course of chemical analysis.

In so far, now, as this return to the original meaning of the formula at the same time concerns its interpretation on the side of totality, its application to Uranus and Neptune should reproduce the main features of its application to Jupiter and Saturn. But since the implied reproduction is really meant to correspond to the distinct re-embodiment of the Involution of the Inverted Relation, as part and parcel of the Inwardness of the already reached standpoint of the Involved Inverted Relation, the application of the formula to Uranus and Neptune *pari passu* becomes also a matter of a suitable specification of its application to the Earth and Venus.

Accordingly, we postulate immediately that

$$(2\pi rt/\delta)_{♅} = \alpha \,.\, 2\pi\rho_{♅} \quad . \quad . \quad . \quad (1)$$
$$(2\pi rt/\delta)_{♆} = \beta \,.\, 2\pi\rho_{♆} \quad . \quad . \quad . \quad (2)$$

where α and β represent the present modification of their original significance in connection with the specification of the right side of the formula in its previous application to Jupiter and Saturn. As regards the import of this modification, we cannot but connect it with the middle term of the now arisen contrast between the present and previous reading of the left side of the formula. Since it is primarily in reflection upon the just superseded identity of the Earth

with Venus that the specified orbit is equated with the space traversed with Uranus' and Neptune's equatorial velocities in the course of their sidereal periods, and these two planets are now before us in the express sense of distinctly embodied aspects of this identity, it commends itself to our dialectical instinct that this their fundamental identity should have its distinct record in connection with the now necessary alteration of the original value of the specifying numbers α and β.

Now, the simplest way to secure such a record is to identify the numerators of the ratios represented by α and β, these ratios being respectively $\rho_{♃}\Big/\frac{10S_{♀}}{7}$ and $\rho_{♄}\Big/\frac{10S_{♅}}{196}$. Therefore, consistently with the fact that the accent in connection with the once again emphasized identity of the Earth with Venus falls on Venus, it suggests itself to substitute for $\rho_{♃}$ and $\rho_{♄}$ simply $\rho_{♀}$. But because the distinction belonging to the Inwardness of the emphasized fundamental unity should not vanish without a trace, it commends itself to us, on further thought, to substitute for $\rho_{♃}$ and $\rho_{♄}$ either the least or the greatest distance of Venus from the Sun. And if

$$\alpha = (S_{♀} - e_{♀})\Big/\frac{10S_{♀}}{7},$$

and

$$\beta = (S_{♀} + e_{♀})\Big/\frac{10S_{♅}}{196},$$

the equations (1) and (2) acquire the form :

$$(2\pi rt/\delta)_{♅} = 2\pi\rho_{♅}(S_{♀} - e_{♀})\Big/\frac{S_{♀}}{0{\cdot}7} \quad . \quad \text{(XIX)}$$

$$(2\pi rt/\delta)_{\psi} = 2\pi\rho_{\psi}(S_{♀} + e_{♀})\Big/\frac{S_{♅}}{19{\cdot}6} \quad . \quad \text{(XX)}$$

2. *Side-issues in the Evaluation of the Right Side of the Resulting Equations.*

a. The Evaluation of $e_{♀}$.

It suggests itself at once that, in so far as Venus forms the extreme of abstract immediacy to the system of the

proper planets, her ϑ, or the time, in which she traverses with her orbital velocity the space equivalent to her $\sigma = 2e$, should be determinable from $\vartheta_{♁}$. While forming the unity in the series ϑ for the proper planets, $\vartheta_{♁}$ should, in reference to $\vartheta_{♀}$, have the character of a simple self-differentiable One of Measure. Inasmuch, now, as in connection with such an One we are at the same time placing on record the unity of the Original and Derived Function, the inference lying nearest to hand is that

$$\vartheta_{♁} = 4\vartheta_{♀} \quad . \quad . \quad . \quad \text{(XXI)}$$

$$\therefore \vartheta_{♀} = 701{\cdot}9596 \text{ m.}$$

Now, given $t_{♀} = 224{\cdot}7008 d_{♁}$,

it follows that $(t/\vartheta)_{♀} = 460{\cdot}951245$.

Therefore
$$\rho_{♀} = \frac{S_{♀} \times 460{\cdot}951245\sqrt{8}}{\sqrt{4\pi^2 + 8 \times 460{\cdot}951245^2}}$$
$$= 0{\cdot}999986854 \times 67{,}244{,}905{\cdot}5$$
$$= 67{,}244{,}018{\cdot}5 \text{ miles.}$$
$$\therefore e_{♀} = 458{,}298{\cdot}6 \quad \text{,,}$$

as against Sir Lockyer's

459,000 ,,

or Chambers 460,000 ,,

Accordingly $\alpha = 0{\cdot}6952292443457 \ldots$

$\beta = 0{\cdot}7443603191440 \ldots$

β. *The Evaluation of* $e_{♅}$.

The fact that Uranus forms the extreme of totality in *qualitative* opposition to Venus, suggests that his orbital eccentricity is a dialectically determinable amount of $e_{♀}$. The clue to this determination obviously must be sought in connection with the system of the proper planets. For, this system is enveloped by Venus and Uranus, and hence may be said to belong to the Inwardness of the relation between them.

Accordingly, we propose to treat the ratio $e_{♅}/e_{♀}$ as a product whose factors represent the ratios formed by the orbital eccentricities of the two proper planets on the side of totality with the orbital eccentricities of the two proper planets on the side of unity, in so far as Mars stands for

the unity to Jupiter and the Earth to Saturn. The two factors are therefore immediately identified with the ratios $e_{\text{♃}}/e_{\text{♂}}$ and $e_{\text{♄}}/e_{\text{♁}}$. But, on further reflection, it occurs to us that the qualitative contrast between the proper planets, as thus grouped, should be equally taken into consideration, seeing that the contrast is also an essential moment of the system of the proper planets. In order to put it on record, it suggests itself at once to correlate the two ratios with the element of Space and Time respectively, in which case the factor representing the group Jupiter-Mars should be expressed in terms of space and the factor representing the group Saturn-Earth in terms of time. That is to say, we postulate that

$$\frac{e_{\text{♅}}}{e_{\text{♀}}} = \frac{e_{\text{♃}}}{e_{\text{♂}}} \times \frac{\vartheta_{\text{♄}}}{\vartheta_{\text{♁}}} \qquad . \quad . \quad \text{(XXII)}$$

Now, we know already that

$$(t/\vartheta)_{\text{♃}} = 64\cdot1809613936,$$

and that $\rho_{\text{♃}} = 483{,}404{,}689$ miles,

so that $2\pi\rho_{\text{♃}} = 3{,}037{,}321{,}236$ miles,

$\therefore\ e_{\text{♃}} = 23{,}662{,}166\cdot5$,,

as against Sir Lockyer's 23,338,000 ,,

or Chambers' 23,275,000 ,,

We know also that

$$\vartheta_{\text{♂}}/\vartheta_{\text{♁}} = 10\cdot4341755,$$

hence $\vartheta_{\text{♂}} = 20\cdot34547127 d_{\text{♁}}$.

Therefore

$$(t/\vartheta)_{\text{♂}} = 33\cdot765737,$$

$$\rho_{\text{♂}} = 0\cdot997841807\ S_{\text{♂}}$$

$$= 141{,}345{,}060 \text{ miles}$$

$$2\pi\rho_{\text{♂}} = 888{,}097{,}203 \text{ ,,}$$

$$\therefore\ e_{\text{♂}} = 13{,}150{,}863\cdot5 \text{ ,,}$$

as against Sir Lockyer's 13,210,000 ,,

or Chambers' 13,178,000 ,,

Accordingly $e_{\text{♃}}/e_{\text{♂}} = 1\cdot799286145$;

and as $\vartheta_{\text{♄}}/\vartheta_{\text{♁}} = 100$,

it follows that $e_{\text{♅}} = 179\cdot9286145 \times 458{,}298\cdot6$

$= 82{,}461{,}032$ miles

as against Sir Lockyer's 82,676,000 ,,

or Chambers' 83,163,000 ,,

γ. *The Evaluation of* $\sigma_{☿}$ *and* ϑ_{ψ}.

Seeing that Mercury and Neptune are to be grouped in contra-distinction against the planets falling in between them, the determination of the elementary evidence of their individuality should be independent of any reference to σ or ϑ of the other planets. Accordingly, it suggests itself to postulate a dialectically significant relation between their own σ and ϑ. And inasmuch as we must also take into account the qualitative distinction between them, it occurs to us further that the relation in question should, on the side of Mercury, concern σ and, on the side of Neptune, ϑ.

In so far as we need a middle term for the two determinations now in question, we cannot but fall back upon the In-itself of the whole planetary system. Therefore, we are meant to single out two features of the Earth exemplifying the same kind of dialectical relationship as that between Mercury and Neptune. The feature to be correlated with Mercury should represent the Earth's simple qualitativity apart from any reference to its individuality. In that case, we obviously refer to the Earth's mean distance from the Sun. A simple identification of the feature to be correlated with Neptune with $t_{♁}$ invites, however, doubt. Even though already pregnant with a reference to planetary individuality (by virtue of the return into self implied in one complete revolution round the Sun), the siderial period as such still concerns only the simple qualitativity of a planet, not yet the *return* into qualitativity to be emphasized in connection with Neptune. In so far as this kind of qualitativity completes, instead of merely hinting at, the standpoint of the planetary individuality, the orbital movement, implied in the present reference to the sidereal period, must be explicitly Earth-centred. That is to say, the Earth's implicit individuality, as embodied in its orbital movement round the Sun, must now be reflected upon as a hypostatized aspect of its concrete individuality, as a self-discerning unity of the orbital and axial movement. From the standpoint of our familiarity with facts, we cannot doubt that we thus refer to the Moon's revolution round the Earth; and it follows, therefore, that in connection with the other feature of the Earth, to be correlated with Neptune, we refer to $t_{☾}/d_{♁}$.

Inasmuch, then, as $\sigma_{☿}$ and $\vartheta_{♆}$ are to be determined from their correlatedness with $S_{♁}$ and $t_{☾}/d_{♁}$ respectively, and we are assuming, to begin with, that the relation between $S_{♁}$ and $t_{☾}/d_{♁}$ should be of the same kind as the relation between $\sigma_{☿}$ and $\vartheta_{♆}$, then, seeing that this relation implies qualitatively distinct moments and therefore is an *inverted* relation, we postulate that

$$\sigma_{☿} \times \vartheta_{♆} = S_{♁} \times t_{☾}/d_{♁} \qquad . \quad \text{(XXIII)}$$

Seeing, however, that the evaluation of this equation necessitates a separate evaluation of one of the two factors on the left side ($t_{☾}$ on the right side having, so far, the significance of a given datum), we proceed to transform the equation into

$$S_{♁} : \sigma_{☿} = \vartheta_{♆} : t_{☾},$$

with a view to determining the common value of the two direct ratios; and in so far as the clue to this determination can be sought only in the general nature of the exponent of a direct relation (seeing that the transformation of the equation XXIII concerns simply the standpoint of the Quantitative Relation), it suggests itself to identify the common value in question with the simplest display of the qualitative Quantum in 2π. Hence, we postulate further that

$$S_{♁}/\sigma_{☿} = \vartheta_{♆}/t_{☾} = 2\pi \quad . \quad . \quad \text{(XXIV)}$$

It follows that $e_{☿} = 7{,}397{,}953$ miles
as against Sir Lockyer's 7,399,000 ,,
or Chambers' 7,389,000 ,,
whilst, given Sir Lockyer's figure for

$$t_{☾} = 27{\cdot}321661418d_{♁},$$
$$\vartheta_{♆} = 2\pi t_{☾}$$
$$= 171{\cdot}6670614d_{♁}$$

as against $172{\cdot}2258d_{♁}$

computed from the standpoint of Sir Lockyer's figures for $t_{♆} = 60180{\cdot}86d_{♁}$ and $\sigma_{♆} = 50{,}239$ miles.

But as the ultimate object of the present determination of $\vartheta_{♆}$ is the evaluation of $2\pi\rho_{♆}$ in the equation (XX), and the correctness of the quoted figures for $t_{♆}$ and $\sigma_{♆}$ calls

for further vindication (Chambers' tables adducing only 60,126·71 for t_{ψ} and 48,688,000 miles for σ_{ψ}), it occurs to us to inquire into the possibility of expressing the ratio $(t/\vartheta)_{\psi}$ in terms of a feature derived from Mercury. Now, for Sir Lockyer's figure for t_{ψ} and our own figure for ϑ_{ψ}, the ratio $(t/\vartheta)_{\psi}$ amounts roundly to 350·5668, and we find that this amount is practically identical with the expression $4(t/\delta)_{☿}$, when evaluated from Sir Lockyer's figure for $t_{☿}$ (= $87·9692 d_{♁}$) and $\delta_{☿}$ (= 24 h. 5 m. 28 (?) s.), seeing that in that case $(t/\delta)_{☿} = 87·635$ and hence $4(t/\delta)_{☿} = 350·54$. In any case, from the standpoint of the equation,

$$(t/\vartheta)_{\psi} = 4(t/\delta)_{☿} \quad . \quad . \quad . \quad \text{(XXV)}$$

a quadrant of Neptune's orbit would represent the same amount of his σ as Mercury's sidereal period is of his sidereal day, and this relation unmistakably is of dialectical significance, as a further record of the qualitatively quantitative contrast between Mercury and Neptune.

3. *Evaluation of the Left Side.*

Inasmuch as we are now in a position to evaluate the right side of the equations resulting from the application of the general formula $2\pi r \mathrm{T} = \mathrm{S}$ to Uranus and Neptune—though in the case of Neptune, the evaluation remains still a matter of our own future results for $t_{☾}$ and $(t/\delta)_{☿}$—we turn our attention to a separate determination of the two factors on the left side. In this respect, we naturally look forward to a further exemplification of the standpoint of the Involved Inverted Relation. In so far, however, as we must also mark the distinction between Uranus and Neptune, as extremes of totality—in the case of the former with respect to the system of the proper planets, in that of the latter with respect to the whole planetary system: it suggests itself that it is only in reference to Neptune that the two factors on the left side of the equations in question admit of simple equalization. As regards the relation to be postulated between them in reference to Uranus: seeing that, in this respect, we should refer only to the system of the proper planets, it commends itself to us to identify the value of the ratio between t/δ and $2\pi r$ with the coefficient a

because in that case we at the same time are implying that

$$2\pi r_{\text{♅}} = \sqrt{2\pi\rho_{\text{♅}}} \quad . \quad . \quad \text{(XXVI)}$$

and this equation explicitly brings to the front the standpoint of the Involved Inverted Relation in a manner consistent with the modification to be postulated in connection with its exemplification in the case of Uranus as against its full display in the case of Neptune, when

$$\begin{aligned} 2\pi r_{\psi} &= (t/\delta)_{\psi} \quad . \quad . \quad . \quad . \quad \text{(XXVII)} \\ &= 2{\cdot}1624416327\sqrt{\rho_{\psi}}. \end{aligned}$$

Now, given our figure for

	$S_{\text{♅}}$	=	1,782,715,656 miles
and	$e_{\text{♅}}$	=	82,461,032 ,,
we find that	$\rho_{\text{♅}}$	=	1,781,761,827 ,,
or	$2\pi\rho_{\text{♅}}$	=	11,195,139,720 ,,
Therefore	$2\pi r_{\text{♅}}$	=	105,807·09 miles roundly by excess,
i.e.	$2r_{\text{♅}}$	=	33,680 ,,
(as against Sir Lockyer's			33,563 ,,
or Chambers'			30,875 ,,);
and	$(t/\delta)_{\text{♅}}$	=	73,560·183.

Seeing, further, that, calculated on the basis of our figures for $S_{\text{♅}}$, $S_{\text{♁}}$, and $t_{\text{♁}}$,

	$t_{\text{♅}}$	=	$30{,}674{\cdot}386523 d_{\text{♁}}$
(as against Sir Lockyer's			$30{,}688{\cdot}30 d_{\text{♁}}$
or Chambers'			$30{,}686{\cdot}82 d_{\text{♁}}$),
it follows, finally, that	$\delta_{\text{♅}}$	=	$0{\cdot}416995 d_{\text{♁}}$
		=	10 h. 0 m. 28·3 s.,
as against Sir Lockyer's			10 h. 0 m. 0 s.
or Chambers'			9 h. 30 m. ? s.

Were Sir Lockyer's figures for t_{ψ} and e_{ψ} quite correct,

then	ρ_{ψ}	=	2,793,961,802 miles
and hence	$2\pi r_{\psi}$	=	52,888 × 2·16244
		=	114,367 miles roundly
or	$2r_{\psi}$	=	36,407 ,,
as against Sir Lockyer's			36,620 ,,
or Chambers'			37,205 ,,
whilst	δ_{ψ}	=	60,180·86 : 114,367
		=	$0{\cdot}526208 d_{\text{♁}}$
		=	757·73 miles = 12 h. 37 m. 43·8 s.

Of course, as the results for Neptune remain subject to further correction, they serve, so far, only as a rough test of the objective validity of the equation (XXVII), and in that case, too, properly only in reference to $2\pi r_{\psi}$, seeing that δ_{ψ} still awaits its observational determination. But it is to be noticed that equatorial velocities of the planets on the side of totality decrease with the increasing distance from the sun, and that the resulting figure for δ_{ψ} does make the equatorial velocity of Neptune less than that of Uranus, the reduction amounting almost exactly to one seventh.

c. The Equator and Axial Rotation of Mercury.

In completion of the record of the established unity of the measurable Quality and Quantity in the concluding portion of the standpoint of the Involved Inverted Relation within the transcendentally displayed dialectic whole of Measurableness, we finally proceed to identify the Earth with Mercury. For, the accomplished return from the measurable Quantity into the measurable Quality ought to be exemplified by the negative unity of the two planets originally representative of the two standpoints, whose separate consideration culminates in their double transition into another. And it is obvious that the exemplification of this explicitly self-contradictory nature of the unity now in question, is meant to serve the practical purpose of determining the still unknown elements of Mercury.

1. $r_{♀}$ and $\delta_{♂}$.

Seeing that the Earth is now before us as the fully established In-itself of the whole planetary system, the right side of our formula $2\pi r\mathrm{T} = \mathrm{S}$ must, in application to Mercury, represent the orbit. Inasmuch, further, as $2\pi r$, or the qualitative factor on the left side, equally should refer to Mercury, his identity with the Earth must embody itself in terms of the idealistic factor T.

At first sight, it would suggest itself to identify the amount which Mercury's orbit is of his equator with the amount which the Earth's orbit is of its equator. In reflection, however, upon the negative, or explicitly self-contradictory, character of Mercury's identity with the Earth, the immediately postulated equality between the ratios

$(\rho/r)_{♁}$ and $(\rho/r)_{☿}$ is rather an inequality to be superseded. We are not meant simply to reject the inference immediately occurring from the application of our general formula to Mercury, but to explicate it in its contradictory Inwardness. The two ratios $(\rho/r)_{♁}$ and $(\rho/r)_{☿}$ must be treated, to begin with, as independent aspects of a fundamentally negative unity entering into Presence only by means of their specification in simultaneous record of the contradiction belonging now to the Earth's Inwardness.

As regards, then, the thus implied alteration of the ratio $(\rho/r)_{♁}$: if the Earth's orbit is not to be taken in the sense of an amount of its own equator, whilst a reference to another *proper* planet is now out of place, we cannot but treat the orbit as an amount of the equator of that planet, whose individual elements are determinable only by giving it the express sense of *another* Earth, i.e. of Venus.

As regards the other ratio $(\rho/r)_{☿}$, it commends itself to us to connect the postulated alteration with the orbit. Partly, we cannot be meant to replace Mercury's equator; partly, from the qualitative contrast between the two ratios, it also easily follows that the alteration should concern opposite moments. As a matter of fact, then, we seek to restate the product $\rho_{♁} r_{☿}$ in terms of $r_{♀}$ and the to be specified value of $\rho_{☿}$.

Now, seeing that our present concern is with the concrete nature of the planetary individuality, and the establishing of this its concrete nature must equally present itself by way of a conclusion with its elementary evidence in σ, it follows, and indeed is also quite plain at first sight, that the required alteration of the value corresponding to $2\pi\rho_{☿}$ must be sought by means of an alteration in the already derived value of $\sigma_{☿}$. For, to replace his actual orbit by the orbit of another planet is for the same reason out of place, as was the replacement of the Earth's equator by the equator of another proper planet.

The question is, therefore, as to what we are meant to put in the place of $\sigma_{☿}$. The equation (XXIV) confirms our immediate impression that the new value, meant to take its place, should have some connection with the Earth's immediate qualitativity. In any case, the clue to the

nature of the specification of either of the two original ratios $(\rho/r)_{\text{♁}}$ and $(\rho/r)_{\text{☿}}$ lies in the significance of this specification as a partial record of the contradiction implicitly belonging to the Earth's Inwardness and now at last explicitly coming to the front. As we know, the Earth steps from the very first on the scene only as a background of the immediately arising distinction between the aspects of its own, to begin with, only implicitly present immediacy; and it is certainly in place that the present concluding establishing of its concreted immediacy should recall to our mind its original Ideality. Therefore, in so far as the specification of the ratio $(\rho/r)_{\text{♁}}$ puts in the place of the Earth Venus, it commends itself to our dialectic instinct that the clue to the specification of the ratio $(\rho/r)_{\text{☿}}$ should, by contrast, be sought in reflection upon the originally equally implied identification of the Earth's immediacy with Mars. But as this identification is to concern the postulated specification of $\sigma_{\text{☿}}$, it suggests itself to replace, in the equation $S_{\text{♁}}/2\pi = \sigma_{\text{☿}}$, $S_{\text{♁}}$ by $S_{\text{♂}}$. On further reflection, however, it occurs to us that the specified value of $\sigma_{\text{☿}}$, which let us agree to symbolize by $\sigma^{\text{☽}}{}_{\text{☿}}$, should no longer remain the same part of $S_{\text{♂}}$ as $\sigma_{\text{☿}}$ is of $S_{\text{♁}}$: seeing, precisely, that, by specifying $\sigma_{\text{☿}}$, we are also effacing its original implication of the ratio $\vartheta_{\psi}/t_{\text{☾}}$. Now, inasmuch as the part which $\sigma^{\text{☽}}{}_{\text{☿}}$ is of $S_{\text{♂}}$ must be a record of the present identification of the Earth with Mars, *not* as with another proper planet, but only as with its own aspect of totality in the group on the side of unity, and totality is to be interpreted only in reference to the mean distance from the sun, we postulate that

$$\sigma^{\text{☽}}{}_{\text{☿}} = S_{\text{♂}}/10, \quad . \quad . \quad \text{(XXVIII)}$$

with the consequence that the postulated formula for the determination of $r_{\text{☿}}$ acquires the shape of the equation

$$\rho_{\text{♁}}/r_{\text{♀}} = \sqrt{S_{\text{☿}}{}^2 - \frac{(0\cdot 1 S_{\text{♂}})^2}{8}}\Big/ r_{\text{☿}} \quad . \quad \text{(XXIX)}$$

where the expression under the root sign may be symbolized by $\rho^{\text{☽}}{}_{\text{☿}}$.

On evaluating $r_{\text{☿}}$ on the basis of the given $S_{\text{☿}}$, we reach

a remarkable result: we find that $r_{☿} = 1477{\cdot}33$ miles: that is to say, almost exactly the *same amount of miles as there are minutes in Mars' sidereal day*, our result for $\delta_{♂}$ being 1477·210 m. Are we justified in postulating the equation:

$$r_{☿} \text{ in miles} = \delta_{♂} \text{ in minutes}, \quad . \quad \text{(XXX)}$$

when, for $\delta_{♂} = 1477{\cdot}210$ m.,
the resulting figure for $S_{☿} = 35{,}984{,}081$ miles,
and hence for $t_{☿} = 87{\cdot}959180 d_{♁}$,

as against Sir Lockyer's 35,987,000 miles and $87{\cdot}9692 d_{♁}$ respectively? The conclusive vindication of the correctness of the implied assumption must be sought from the standpoint of the interdependency of all our results. The figure for $S_{☿}$, $r_{☿}$ or $t_{☿}$ will undoubtedly receive further checks in connection with the determination of $\delta_{☿}$, or the elements of the Sun, or of the Moon—in short, in connection with our future results; but, in anticipation of these future tests, we propose immediately to examine the present suggestion, that one minute is not merely an arbitrary unit of time, on its own ground.

2. *A Suggestion of a Dialectic Background to the Current Subdivision of a Day.*

It seems from the very first appropriate that, in so far as the concluding stage of the system of the planetary individuality recapitulates the double transition concluding the dialectic whole of the measurable Quantity, this double transition should also find its exemplification in terms of the *à priori* determinable units of length and time. The correlation of the implied numerically identical totalities to these units with $r_{☿}$ and $\delta_{♂}$ ceases to perplex us, when we reflect that the double transition is meant to bring together the two standpoints originally represented by Mercury and the Earth; and that, in so far as the Earth's own Immediacy is immediately represented by Venus and Mars, the contrast against Mercury concerns on the side of the Earth its aspect of totality, hence Mars. The association of this planet with the sidereal day, as against the association of Mercury with his equatorial radius, may be traced to the necessity of reducing the features exemplifying the two aspects of the

planetary individuality to their simplest units. Inasmuch, however, as we are thus committing ourselves to the presupposition that one minute is an *à priori* determinable unit of time, we find ourselves under the dialectical necessity of vindicating this assumption also for its own sake, or independently of the practical demonstrableness of its tenability, as already implied in what precedes, and possibly also in what follows. In this way, we are faced with the dialectical background of the division of Time.

a. One Year—One Half Day—One Minute.

The sidereal period is naturally subdivisible into sidereal or mean solar days. Of course, in so far as, with respect to a system of time units, we leave aside the self-differentiated or plural aspect of the planetary individuality, the subdivision in question immediately concerns the *unit* sidereal period in terms of mean solar days. But whereas, so far, we have treated this subdivision as a matter of course and accepted the corresponding value of $t_{\text{♁}}$ as given to begin with, we are now meant to convert it into a product of our own thinking.

Accordingly, we aim at determining the number corresponding to $t_{\text{♁}}$ as a *self-differentiated* totality. Inasmuch as in connection with this totality, we reflect on the standpoint of Counting, we must just as much negate the latter's immediate meaning, or rather lack of meaning. Totality resulting from Self-differentiation is not representable by means of a number implying no real reason for breaking off the endless, and for that reason meaningless, flux of Counting. Further, in so far as the unit to the totality in question is a unit of *time*, the Reality of the implied reason for breaking off the Counting at a particulur number is simultaneously advanced to the stage of Ideality. Consequently, there is here no analogy to a counting up of a given number of things lying side by side or in a heap. Inasmuch as the limit to Counting refers to a self-differentiated number, it suggests itself to seek its mathematical significance in the *factorial n*.

But so we are faced with the problem as to the value of n. Now, in so far as $|\underline{n}$ ceases to be a simple number, but acquires the character of a qualitative Quantum, the value

to be assigned to n should be sought in reflection upon the spatial aspect of the Earth. In any case, inasmuch as Time and Space are bound up together in a negative unity, and the division of Time, consequently, ought to have an implicit connection with the element of space (as, indeed, we already presuppose is the case), the embodiment of this connection cannot but refer primarily to the value to be assigned to n. That, however, the implied spatial aspect of the Earth is not meant to be identified with $s_{☿}$ follows from the already made reflection that, in connection with the division of Time, the Earth counts as the Substrate rather than simply as one member of the planetary system. But, then, in so far as with respect to this its meaning, the Earth is properly reflected upon from the standpoint of its already established identity with Mercury, the spatial aspect in question should manifestly be embodied by the number 6, i.e. by that number which from the very first implied the present sense of the Substrate with respect to the series of planetary distances when the point of departure is identified with Mercury. In this way, $t_{☿}$ comes to be grasped as the factorial $6 = 720$, which is the *round* number of *days and nights* in one year. The self-differentiated totality is an amount of abstract unity, which is only one half of one mean solar day, because this unity is naturally subdivided into day and night from the standpoint of the fundamental unity of the orbital and axial movement.

With respect to one minute as a unit of time, one half-day becomes a middle term connecting it, the minute, with the unit sidereal period. Whilst counting on the one side as unit, one half-day becomes on the other side a totality, and the same totality as is indicated by $|\underline{6} = 720$. That is to say, one minute is the same part of one half-day as this latter is of one year:

$$t_{☿} : \tfrac{1}{2}d_{☿} = \tfrac{1}{2}d_{☿} : 1 \text{ m.} \qquad . \qquad . \quad \text{(XXXI)}$$

β. One Year—One Half-Month—One Hour.

But that the subdivision of one day into minutes leaves room for an intermediate unit, is obvious. If the fixation of this intermediate unit is not to remain a matter of arbitrary choice, we cannot but postulate at its back a necessarily alternative subdivision of the unit sidereal period.

In fact, we are aware that such an alternative subdivision there is ; namely, in reference to the Moon's sidereal revolution. But in so far as we should refrain from drawing upon our familiarity with facts in advance of the present stage of our progress in knowing, we must try to get at the meaning determinative of the alternative number for the self-differentiated $t_{♁}$ from the standpoint of our present attitude towards the Earth as the Substrate of the planetary system.

The alternative subdivision of the unit sidereal period can be dialectically necessary only in reference to the subject-matter involved in the already discussed subdivision. Now, seeing that the middle term in the fixation of one minute was sought in $\underline{|n}$, and n was then identified with the number 6 in reflection upon the implicit coincidence of the standpoint originally meant to be symbolized by it with the standpoint equally implied in the subdivision of $t_{♁}$ in terms of a unit bound up with the aspect of *axial rotation* : it suggests itself that, in so far as the alternative meaning of the self-differentiated unit sidereal period should, by contrast, emphasize the aspect of the *orbital movement*, and hence be connected with the standpoint of the measurable Quality, the alternative value of n ought to be identified with the number 4. In that case, as against the previous totality of 720 *half-days*, we have the alternative totality of 24,—obviously *half-months*, the unit being in this respect equivalent to the French *quinze jours*.

If we proceed to transform the immediately set up equation

$$t_{♁} : 15d_{♁} = 1d_{♁} : 1 \text{ h.}$$

into $$24d_{♁} : 1d_{♁} = 1d_{♁} : 1 \text{ h.} \qquad \text{(XXXII)}$$

then, since $24d_{♁} = 576 \text{ h.} = 360 \times 1{\cdot}6 \text{ h.},$

whilst $360 = t_{♁}$ and $1{\cdot}6 = s_{♂}/s_{♁},$

we get finally $$(1d_{♁} \text{ in hours})^2 = s_{♂} \Big/ \left(\frac{s}{t}\right)_{♁} \qquad \text{(XXXIII)}$$

As is seen, the intermediate unit, i.e. one hour, may be defined as a unit such that the square of the amount corresponding in its terms to one mean solar day represents roundly the amount of m. s. d. in which the Earth describes space equivalent to Mars' orbit, the roundness being

traceable to the fact that, conformably with our original symbol for the orbital velocity, the orbital radius is identified with the round comparative number for the mean distance from the Sun, and the unit sidereal period with the round figure 360.

$$\gamma.\ \delta_{\text{♁}} - \left(\frac{r}{r - \mathrm{P}(r)}\right)_{\text{♁}}^{\prime} - 1\ \text{s.}$$

When one minute is treated as a middle term between 1 h. and the next smaller unit, this latter unit is called one second. In so far as this further subdivision of one minute simply reproduces the subdivision of one hour into minutes, seeing that

$$1\ \text{h.} : 1\ \text{m.} = 1\ \text{m.} : 1\ \text{s.}, \qquad . \quad \text{(XXXIV)}$$

it opens at first sight a vista of a progress or regress towards an endless succession of time units decreasing in the same constant ratio. However, the principle of the subdivision in question concerns us only as an embodiment of the significance attaching to that unit of time which we must postulate in necessary conclusion of the dialectical whole prompting the fixation of our units of time. Seeing that in connection with the two already derived units we had to resuscitate the contrast between the two aspects of the planetary individuality, and this contrast must, for dialectical completeness' sake, be equally superseded, it becomes obvious that we are concerned in the derivation of only one further unit in explicit reflection upon the negative unity of the previously merely contrasted aspects, and hence upon the *relation* between hours and minutes.

In order, however, adequately to vindicate the character we are postulating for one second, as a *dialectically necessary* unit of time, we must seek the middle term to its derivation in some spatial reference. In any case, in so far as the spatial reference enters into the determination of the two preliminary units of time, it must *à fortiori* form part and parcel of the determination of the concluding unit. The two preliminary units occupying in the dialectical whole of their derivation a place analogous to Quality and Quantity, when viewed from the standpoint of their concluding unity in Measure, the appropriateness of giving

the finally adequate mode of determination for the concluding unit the form of the proportion

$$\mathrm{T} : \mathrm{S} = \mathrm{S} : 1 \text{ s.}$$

will be obvious. And in so far as the spatial reference in question is meant to serve in explicit record of the concluding unity of the measurable Quality and Quantity, and the immediate record of this concluding unity is already known to concern the determination of $\mathrm{P}(2\pi r)_{♁}$, it suggests itself to seek the value represented by S in reference to that measurable feature of the Earth which explicitly concerns its shape as an oblate spheroid. This feature is, of course, its polar compression, i.e. the difference between the mean equatorial and polar diameter.

Inasmuch, now, as this difference is meant to stand for a unit on the one side, and for a totality on the other (in inwardization, that is, of its significance as a middle term between the two units T and 1 s.), the totality to the difference obviously refers to the Earth's equatorial diameter, whilst the extreme of unity refers to the same part of the difference as this latter is of the equatorial diameter. Consequently, the amount represented by T in seconds represents the square of the amount which the Earth's equatorial diameter is of its polar compression; and when we avail ourselves of the figure resulting for the polar compression from Sir Lockyer's figures for

$$2r_{♁} = 41{,}852{,}404 \text{ feet,}$$
$$\mathrm{P}(2r)_{♁} = 41{,}709{,}790 \quad \text{"}$$

we find that T is meant to stand for $\delta_{♁}$. To be quite exact the ratio for the polar compression should in that case be 1/293·5372 . . . , instead of the ratio 1/293·46 resulting from the adduced figures for the two diameters; but the discrepancy is manifestly only a witness to the inevitable margin of indefiniteness surviving even the most painstaking attempt at exactness in connection with astronomical magnitudes. In any case, as the objective validity of our inference calls for no doubt, we are putting it on record in the form of the equation

$$\delta_{♁} \text{ in seconds} = \left[\frac{r}{r - \mathrm{P}(r)}\right]^2_{♁} \quad . \quad \text{(XXXV)}$$

In so far, by the way, as from the standpoint of the concluding unity of the two aspects of the planetary individuality it seems in place to exhaust the exchangeability of the two preliminary units of time, we may also interest ourselves in the alternative form of the equation (XXXIV) with one *hour* for middle term. In that case we are immediately concerned with the proposition

$$150 \text{ d.} : 1 \text{ h.} = 1 \text{ h.} : 1 \text{ s.}$$

Since, however, with respect to a further treatment of this proposition, it suggests itself to identify the extreme of totality with 1 d., the middle term reduces to 24 s. and the extreme of unity to 1/150 s. Curiously enough, this, the 3600th part of the 3600th part of one mean solar day represents, according to the Hindû Shâṣtras, the least unit of time under the name of one *Truti.*

3. $\delta_{☿}$.

The determination of $\delta_{☿}$ ought to have the same background as the determination of $r_{☿}$, because the two determinations must be envisaged in the light of an exemplification of the double transition between Quality and Quantity in the concluding phase of their dialectical whole. In so far, then, as, in connection with the determination of $r_{☿}$, the substitution for T of a spatial relation (i.e. of the ratio $\rho_{♁}/r_{♀}$ in the equation XXIX : $r_{☿} \times \rho_{♁}/r_{♀} = \rho^{☽}{}_{☿}$) may well be taken as the present manner of embodying the transition from Quantity into Quality, and the reverse sense of this transition should find its exemplication in connection with the determination of $\delta_{☿}$, we proceed now to concern ourselves with the ratio $\rho_{♁}/r_{♀}$ as indicative of an amount of Mercury's sidereal days. And that our immediate aim must be to convert the amount in question (= 24,122·41540) into an amount of Mercury's sidereal periods is obvious. That is to say, we are setting up the equation :

$$(t_{☿}/d_{♁})x = (\rho_{♁}/r_{♀})\delta_{☿} \quad . \quad (1)\ (\text{XXXVI})$$

But since this equation contains two unknowns, its solution calls for an alternative version of the given relationship between them. In so far, now, as this other relationship must be bound up with the equation already set up, let us,

first of all, clearly realize that this equation represents the present form of the general formula $2\pi r\mathrm{T} = \mathrm{S}$ in explicit application to the determination of $\delta_{☿}$, and hence that its difference from the equation (XXIX) consists solely in the substitution of $(t_{☿}/d_{♁})x/\delta_{☿}$ for $\rho_{♁}/r_{♀}$. For, in that case the said equation becomes

$$r_{☿} \times (t_{☿}/d_{♁})x/\delta_{☿} = \rho^{☽}{}_{☿}$$
$$\therefore (t/\delta)_{☿}x = (\rho^{☽}/r)_{☿} = \rho_{♁}/r_{♀}.$$

Of course, that, in so far as

$$(t/\delta)_{☿}x = 24{,}122{\cdot}4154,$$

this amount is equivalent to either of the two equivalent ratios $(\rho^{☽}/r)_{☿}$ and $\rho_{♁}/r_{♀}$, would be to us a matter of no further consequence but for the fact that we are under the necessity of finding an alternative relation between x and $\delta_{☿}$. Namely, in so far as the two equations needed for the evaluation of $\delta_{☿}$ are two dialectically bound up aspects or meanings of one and the same number, and the equation (1) (which may be simply written $(t/\delta)_{☿}x = 24{,}122{\cdot}4154$) connects this number primarily only with the ratio $\rho_{♁}/r_{♀}$: all we have to fall back upon for the needed clue to the complementary equation is the other meaning of the common middle term. True, this other meaning, the ratio $(\rho^{☽}/r)_{☿}$, is also already part and parcel of the first equation, but deliberately only one-sidedly, because the ratio $(\rho^{☽}/r)_{☿}$ is to be reflected upon only as an amount of $\delta_{☿}$, *not* in reference to its spatial aspect. Yet, if the second equation is to be set up in further explication of the subject-matter implicitly contained in the first equation, what else have we to fall back upon for further treatment except the alternative spatial significance of the number 24,122·41540 with respect to its particular connection with the ratio $(\rho^{☽}/r)_{☿}$?

But that, in so far as the spatial import of this ratio is to serve as a clue to the setting up of the second equation for x and $\delta_{☿}$, we are not meant to stop at its immediate sense, must be obvious. Just as, in order to set up the first equation, we transformed the implied amount of $\delta_{☿}$ into the corresponding amount x of $t_{♀}/d_{♁}$; so now we aim at an

analogous transformation of the amount which Mercury's specified orbit is of his equator. Or rather, since $\delta_{☿}$ is now no longer to be associated with this amount, and our object is yet to set up another relation between x and $\delta_{☿}$, we aim at a twofold transformation of the immediate spatial significance of the ratio $(\rho^{☽}/r)_{☿}$. And just because the element of space is, in this respect, reflected upon as blended with the element of time, it becomes further plain that the spatial significance in question must really be grasped dynamically. That is to say, Mercury's specified orbit is to be viewed as described with his orbital or equatorial velocity; for the direct unity of Space and Time yields the concept of Velocity, and the needed double specification of this concept, once in reference to x and another time to $\delta_{☿}$, cannot but refer to the specified orbit as traversed with the orbital or the equatorial velocity respectively.

The postulated alternative relation between x and $\delta_{☿}$ is now distinctly looming on our mental horizon. From the standpoint of our present interpretation of the ratio $(\rho^{☽}/r)_{☿}$, it is at once plain that x represents the amount of $t_{☿}/d_{♁}$, in which Mercury traverses the space equivalent to his specified orbit with his *equatorial* velocity. For, since

$$(t/\delta)_{☿} x = (\rho^{☽}/r)_{☿},$$

$$x = \left(\frac{\rho^{☽}}{rt/\delta}\right)_{☿}.$$

And it is also plain that, in so far as the evaluation of x must be attempted independently of $\delta_{☿}$, this task, i.e. the setting up of the second equation, coincides with the other aspect of the twofold re-interpretation of the spatial significance of the ratio $(\rho^{☽}/r)_{☿}$; when, that is, in contrast to the just embodied aspect, we bring into relief the standpoint of the *orbital* velocity. In this connection, however, we must duly realize that planets do not move with *either* the equatorial velocity alone *or* the orbital velocity alone, but simultaneously with *both*.

Even though, for analytical purposes, we may consider the spaces traversed with either of the two kinds of movement separately, and in that case commit ourselves to the statement that a planet traverses its orbit with its orbital

velocity *additionally* as many times as is indicated by the number of its own sidereal periods in which its equatorial velocity traverses its orbit only *once:* the implied summation on the side of Space is manifestly out of place on the side of Time. From the standpoint of the simultaneity of the two kinds of movement, the single description of the orbit with the equatorial velocity alone does not count as an additional unit to the number of sidereal periods actually elapsed, even though, when we are concerned with the number of times the orbit is traversed by the same equatorial point in the same time, the single description of the space equivalent to the orbit with the equatorial velocity must be added to the implied number of sidereal periods.

In the light of this necessary reflection upon the contradictory nature of the fundamental unity of the two aspects of the absolutely free planetary movement in connection with the setting up of the second equation for x and $\delta_☿$, it becomes now plain to us that, in immediately associating x with that amount of $t_☿$ in which Mercury traverses his specified orbit with his equatorial velocity *alone*, we would allow the single description of the specified orbit with the equatorial velocity to pass for an *additional* sidereal period. Therefore, the actual number of $t_☿$ elapsing, whilst the specified orbit is traversed with the equatorial velocity, is given by $x - 1$. And since the same time, in reference to the single description of $2\pi\rho^{☽}{}_☿$, is also given by $(t/\delta)_☿ x$, there follows the equation

$$t_☿(x - 1) = (t/\delta)_☿ x \quad . \quad . \quad . \quad (2)$$

or simply

$$\delta_☿ = x/(x - 1) \quad . \quad (\mathrm{XXXVII})$$

On combining this equation with the equation (XXXVI) or (1), we get:

$$t_☿(x - 1) = 24{,}122{\cdot}4150$$

so that, given

$$t_☿ = 87{\cdot}959180 d_♁,$$

$$x = 275{\cdot}245569;$$

$$\therefore \delta_☿ = 1{\cdot}0036464 d_♁$$

$$= 24 \text{ h. } 5 \text{ m. } 15 \text{ s.},$$

as against Sir Lockyer's 24 h. 5 m. 28 (?) s.,
or Chambers' 24 h. 5 m. 30 s.

In so far, by the way, as $(t/\delta)_{\text{☿}} = 87{\cdot}639645$, we find now from the standpoint of the equation (XXV) that

$$t_{\Psi} = 60{,}179{\cdot}361277 d_{\text{♁}},$$

as against Sir Lockyer's $60{,}180{\cdot}86 d_{\text{♁}}$
or Chambers' $60{,}126{\cdot}71 d_{\text{♁}}$
with the consequence that $s_{\Psi} = 300{\cdot}53832476$,
or $S_{\Psi} = 2{,}793{,}967{,}565$ miles,
as against Sir Lockyer's 2,794,000,000 ,,
or Chambers' 2,791,750,000 ,,
and since $\rho_{\Psi} = S_{\Psi} a\sqrt{8}/\sqrt{4\pi^2 + 8a^2}$,
where $a = 350{\cdot}558580$,

$$a\sqrt{8} = 991{\cdot}529396484,$$
$$4\pi^2 = 39{\cdot}4784175 14136,$$
$$\sqrt{4\pi^2 + 8a^2} = 991{\cdot}549304124,$$
$$\therefore \rho_{\Psi} = 0{\cdot}999979927 S_{\text{♁}}$$
$$= 2{,}793{,}911{,}470 \text{ miles},$$
$$\sqrt{\rho_{\Psi}} = 52{,}857{\cdot}4637 \quad \text{,,}$$
$$\therefore 2\pi r_{\Psi} = 114{,}301{\cdot}180 \quad \text{,,}$$
$$\therefore 2r_{\Psi} = 36{,}383{\cdot}196217 \quad \text{,,}$$

as against Sir Lockyer's 36,620 ,,
or Chambers' 37,205 ,,

$$\therefore \delta_{\Psi} = 0{\cdot}5264981 d_{\text{♁}}$$
$$= 12 \text{ h. } 38 \text{ m. } 10{\cdot}43 \text{ s.}$$

as against Sir Lockyer's ?
or Chambers' ?

CHAPTER IV.

TRANSITION INTO THE DEMONSTRABLENESS OF THE FULLY ESTABLISHED NEGATIVE UNITY OF THE TWO ASPECTS OF THE PLANETARY INDIVIDUALITY.

a. Polar Compression.

INASMUCH as the now established planetary individuality ought next also to be envisaged in present correspondence to the Nodal Line of Measure Relations, it is obvious that, in this respect, we must reflect upon it as a system of its completed shape, as a three-dimensional body. It suggests itself, therefore, that the number of parts, into which equatorial diameters are divided by their difference from polar diameters, should be of particular dialectical significance, in final reflection upon the systematic arrangement of the planets. In attempted derivation of the Polar Compression, we again contrast the two planets, representative of the standpoint of the measurable Quality, with the system of the proper planets, and assign to the two remaining planets the significance of a middle term, at once joining and disjoining the two groups.

Now, since this threefold subdivision is now made with a view to determining the number of parts into which equatorial diameters are divisible by polar compression, and this determination is part and parcel of the full establishing of the planetary individuality, we must conclude this latter with its elementary evidence in σ; that is to say, we propose to connect the numbers now in question with the corresponding σ by means of an appropriate reflection upon the meaning attaching to planets in their systematic subdivision into groups. With respect to Mercury and Neptune, σ ought to be specified in distinct reference to the orbital movement, in contrast to a distinct reference to the axial rotation in connection with the proper planets; whilst with respect to Venus and Uranus, the specification should

assume a character consistent with the contradictory position of these planets, in so far as they at once fall inside and outside the two main groups. But this outline of the specification must be further qualified in reflection upon the correlation of the planets within their groups.

1. *The System of the Proper Planets.*

To begin with, if, in connection with the group of the proper planets, σ is to be generally specified in distinct reference to the axial rotation, and hence in terms of the planetary equator, it is the ratio $\sigma/2\pi r$ that calls for further specification, consistently with their systematic inter-relation. As is the case in the original determination of σ, the further qualification should be of an identical character for Mars and Jupiter, whilst the mode of determination to be adopted for Saturn should be a simple modification of the mode adopted for the Earth. That is to say, there is here a question of only two fundamental modes of determination, —in reference to the Earth and Mars.

Inasmuch, now, as these modes are to concern the specification of the ratio $\sigma/2\pi r$, it suggests itself, first of all, to interpret this ratio in terms of either space or time: in such wise, that the exponent either retains its immediate significance or else comes to stand for an amount of days; but of *mean solar days*, because σ is not to be conceived as traversed by equatorial velocity alone. In so far as the immediate significance of the ratio undoubtedly should refer to Mars and the other significance to the Earth, and the ratio must be further also qualified with respect to the postulated correlation of Mars with Jupiter and of the Earth with Saturn, it suggests itself to treat the exponent resulting for Mars in the sense of a *totality to be decreased x times* and the exponent resulting for the Earth in the sense of a *unity to be increased y times.*

In this way, the expression for the determination of the amount which Mars' equatorial diameter is of his polar compression is $\sigma_{♂}/2\pi r_{♂} \,.\, x$, whilst in the case of Jupiter we should have $\sigma_{♃}/2\pi r_{♃} \,.\, x$. That is to say, we get for Mars $1986{\cdot}6/x$ and for Jupiter $170{\cdot}6/x$. As a matter of fact, the figures adduced by Sir Lockyer are respectively 200 and 17, so that $x = 10$.

We preferred to appeal at once to the results of observation, because it did not seem to us likely that x should be identified with the ratio $s_{♃}/s_{♂}$, on account of its indifference to the relation between Mars and Jupiter; and we hesitated to identify it with the ratio $s_{♄}/s_{♁}$, because the relation between Saturn and the Earth ought to concern the other significance of the ratio $\sigma/2\pi r$. But we see that we would have been, after all, justified in arguing that, just because x is to reflect upon the contrast between totality and unity, whilst yet remaining indifferent to such a correlation between Jupiter and Mars, its evaluation can concern only the ratio $s_{♄}/s_{♁}$; when it will be, of course, understood that s is to be identified with the comparative round numbers in the originally derived nodal line of the mean planetary distances from the Sun. Accordingly, the formulae for Mars' and Jupiter's polar compression are as follows:—

$$\left[\frac{r}{r - P_{(r)}}\right]_{♂} = \left(\frac{\sigma}{2\pi r}\right)_{♂} \times \frac{s_{♁}}{s_{♄}} \qquad \text{(XXXVIII)}$$

$$\left[\frac{r}{r - P_{(r)}}\right]_{♃} = \left(\frac{\sigma}{2\pi r}\right)_{♄} \times \frac{s_{♁}}{s_{♄}} \quad . \qquad \text{(XXXIX)}$$

In so far as the relation between Saturn and the Earth concerns also the determination of y, and, in that case, we must interpret this relation in terms of time, it easily follows that the formula for the Earth's polar compression is:—

$$\left[\frac{r}{r - P_{(r)}}\right]_{♁} = \left(\frac{\sigma}{2\pi r}\right)_{♁} \times \frac{d_{♁}}{d_{♄}} \quad . \quad . \qquad \text{(XL)}$$

Indeed, $(\sigma/2\pi r)_{♁} = 125{\cdot}2144$

$$d_{♁}/d_{♄} = \frac{1440}{614{\cdot}2516} = 2{\cdot}3443162$$

$$\therefore \left[\frac{r}{r - P^{(r)}}\right]_{♁} = 293{\cdot}541,$$

as against Sir Lockyer's 293·46,

or 293·537 postulated by (XXXV).

With respect, finally, to Saturn's polar compression: if we are right in assuming that it amounts to a specification

of the Earth's polar compression in terms of another unity, then since this specification involves comparison between the Earth and Saturn, and the feature to be compared should be σ, we infer that $\{r/(r - P_{(r)})\}_{♄}$ should be as many times smaller than $\{r/(r - P_{(r)})\}_{♁}$ as $\sigma_{♄}$ is greater than $\sigma_{♁}$. That is to say, we postulate that

$$\left[\frac{r}{r - P_{(r)}}\right]_{♄} = \left[\frac{r}{r - P_{(\,)}}\right]_{♁} \times \frac{\sigma_{♁}}{\sigma_{♄}} \quad . \quad \text{(XLI)}$$

Indeed $\sigma_{♄}/\sigma_{♁} = 100{,}890{,}207{\cdot}6 : 3{,}118{,}048 = 32{\cdot}337$,

$$\therefore 293{\cdot}541 : 32{\cdot}337 = 9{\cdot}077,$$

as against Sir Lockyer's $\qquad 9,$

these being, of course, results for the equatorial diameter in terms of the polar compression.

2. *Mercury and Neptune.*

The fact that our general assumption as regards the mode of determination to be adopted for the polar compression proves itself in agreement with the results of observational astronomy in reference to the proper planets, inspires us with confidence in its objective validity equally in reference to Mercury and Neptune. In this case, we are assuming that σ should be specified in reference to the orbital movement and hence converted into ϑ, i.e. treated with respect to the time, in which the space equivalent to it is traversed with orbital velocity. But just as the determination of the polar compression of the proper planets called for further qualification of the general assumption in more detailed embodiment of their systematic inter-relation; so now too we must, first of all, determine the distinctive character of Mercury and Neptune from the general import of their group in contradistinction to the opposite group of the proper planets. But since this group concerns the planetary individuality primarily with accent on its aspect of Self-differentiation, and the accent in connection with the group now under discussion falls, by contrast, on the aspect of Self-integration, it is easy to see that the present import of the distinction to be drawn between Mercury and Neptune refers to a double aspect of the planetary individuality as the already established Substrate of its systematic self-

discernment. In other terms, the distinctive character of Mercury and Neptune presents itself to us as an instance of the distinction falling within the true Being, as at once the direct and at once also the negative unity of its self-specification. Inasmuch, now, as these specifications refer to the two diameters of the planetary body, so that the distinction now in question at the same time acquires the sense of a contrast between their equality and inequality, it would follow that, in connection with one of the two members of the qualitative group, polar compression becomes conspicuous by its absence; and that, in that case, reference is made to Neptune should be obvious from the previous association of the negative unity of the two aspects of the planetary individuality with Mercury. Accordingly, we postulate that whilst

$$(2\pi r)_{♆} = \mathrm{P}(2\pi r)_{♆}, \quad . \quad . \quad \text{(XLII)}$$

$$\left[\frac{r}{r - \mathrm{P}_{(r)}}\right]_{☿} = \frac{\vartheta_{♆}}{\vartheta_{☿}} \quad . \quad . \quad . \quad \text{(XLIII)}$$

On computing $\vartheta_{☿}$ from our results for $\rho_{☿}$ and $t_{☿}$, we find that it amounts to $5{\cdot}792067 d_{♁}$; and as $\vartheta_{♆} = 2\pi t_{☾}/d_{♁}$ $= 171{\cdot}66706 14 d_{♁}$, it follows that

$$\left[\frac{r}{r - \mathrm{P}_{(r)}}\right]_{☿} = 29{\cdot}6382,$$

as against Sir Lockyer's 29.

3. *Venus and Uranus.*

Whereas Venus and Uranus appeared, with respect to the nodal line of the mean planetary distances from the Sun, to be included only for completeness' sake, and in that case presented themselves allied to the quantitative group of planets, they proved themselves further also as indispensable moments in the subsequent development of the system of the planetary individuality, and their intimate connection with the qualitative group equally became apparent. It is for this reason, then, that now that we deal with the self-differentiating aspect of the already self-integrated planetary individuality we are assigning to them the significance a of

middle term, and postulate that the mode of determination to be adopted for their polar compression should take the form of a third to the two already vindicated modes of determination.

In so far, then, as the mode of determination now in question is also qualitatively distinct from the two previous modes, we must negate the immediately arising suggestion that σ should now be specified once in reference to the orbital movement and another time in reference to the axial rotation. In reflection upon the distinction simultaneously to be drawn between Venus and Uranus within their present common significance, we would at first sight expect that the mode of determination to be adopted in connection with Venus should have the general form adopted in the case of the quantitative group of planets, and the mode to be adopted for Uranus' polar compression the general form applying to the qualitative group. But inasmuch as this correlation is just as much to be negated as affirmed, $\sigma_{♀}$ no longer can concern us simply as an amount of $2\pi r_{♀}$, and neither can the mode of determination in the case of Uranus imply the conversion of $\sigma_{♅}$ into $\vartheta_{♅}$.

In so far as in the latter case we must emphasize the extreme of qualitative significance, $\sigma_{♅}$ should be replaced by $s_{♅}$. By contrast, in so far as the emphasis on the extreme of quantitative significance, in connection with Venus, should be exemplified by way of a suitable modification of $\sigma_{♀}$ in negation of a reference to $2\pi r_{♀}$, but still also in analogy to its modification in the mode of determination adopted for the proper planets, and in that case more particularly for the two middle planets: it suggests itself to connect the determination of Venus' polar compression with $0{\cdot}1\sigma_{♀}$ in British statute miles; in reflection, of course, upon the established coincidence of this mile with the unit length postulated by our own standpoint. In so far, finally, as, in reflection upon the common significance of Venus and Uranus, we must equally postulate an abstractly common feature in the mode of determination for their polar compression, and the two results already obtained should, therefore, be subjected to one and the same treatment, it commends itself at once to our dialectic instinct to identify

this treatment with Evolution. Accordingly, our inferences acquire the form of the two equations,

$$\left[\frac{r}{r - P_{(r)}}\right]_{♀} = \sqrt{0 \cdot 1 \sigma_{♀}} \quad . \quad . \quad \text{(XLIV)}$$

$$\left[\frac{r}{r - P_{(r)}}\right]_{♅} = \sqrt{s_{♅}} \quad . \quad . \quad . \quad \text{(XLV)}$$

On putting them to the test, we find that, for our figure for $\sigma_{♀} (= 916,597 \cdot 2)$,

$$\sqrt{0 \cdot 1 \sigma_{♀}} = 302 \cdot 75,$$

as against Sir Lockyer's 303;

whilst $\sqrt{s_{♅}} = 14$,

as against Sir Lockyer's 14.

b. Polar Circumference.

In connection with the other determinant of the planetary volume, the Polar Circumference, we must bring home to ourselves that our results do not admit of empirical verification except in the case of the Earth. For, owing to its non-elliptical shape, the empirical determination of the Polar Circumference must rest on direct measurement, and this is, in connection with the rest of the planets, out of the question. It is true, that, in so far as the spheroidal form of the planets has its analogy in the shape acquired by any fluid and originally perfectly spherical mass, when set revolving round its axis in a liquid of exactly the same density, the determination of the polar circumference may be viewed also in the light of a physical problem; but the results would even in this case still call for final verification by means of the impossible direct measurement.

But that the absence of empirical measurableness of the polar circumference does not necessarily mean that our results must remain wholly *en l'air*, has already come to our notice in connection with our determination of Venus' polar circumference. As has been seen, even though our identification of $\sqrt{2\pi\rho_{♁}}$ with $P(2\pi r)_{♀}$ lacks conclusive evidence of its absolute correctness, the surviving element of uncertainity does not affect the value of this identification as an additional corroboration of the previously ascertained coincidence of the dialectically postulated unit length with

the British Statute Mile. May not the magnitude of the remaining polar circumferences be credited with a similar import? We know that the polar circumference is smaller than the ellipse having the two planetary diameters for its axes, and, for abstract completeness' sake, we may as well inquire into the assumed likelihood of striking relationships in connection with the smaller magnitudes in question.

In this way, we find that, in so far as Mars' polar circumference is less than 13,190 miles, it may well represent the ratio $0{\cdot}1\sigma_{♃}/360 = 13{,}145{\cdot}64$ miles, whilst Jupiter's polar circumference may, by contrast, be identified with the ratio $0{\cdot}1\sigma_{♂}/10 = 263{,}017{\cdot}27$ miles, the figure for its elliptical shape being roundly 270,000 miles. The likely relationship for Saturn's polar circumference (less than 220,370 miles) is $360\delta_{♃}$ *in minutes* $= 214{,}279{\cdot}75$; whilst, if $P(2\pi r)_{♅}$ is identified with $\sqrt{(0{\cdot}1\sigma\delta)_{♅}}$, we get 99,514·5 miles, as against 102,102 miles for the corresponding ellipse; in this case, of course, σ counts in *miles* and δ in *minutes*. Finally, for $P(2\pi r)_{☿}$ the likely expression is $\sqrt{2\pi r_{☿}\ .\ 100(t/\delta)_{☿}} =$ 9,019·06 miles as against 9126 miles for the corresponding ellipse.

c. Dimensions of the Sun.

In so far as, in transition into the concluding portion of the total dialectical whole of the Solar System, the contradictory unity between Mercury and the Earth must be equally reduced to the condition of Oneness, we refer to the planetary individuality in explicit correspondence to Absolute Indifference, and herewith as identified with the Sun.

Accordingly, the Sun becomes the object of our present consideration in the sense of an inert One of Space in contradistinction against the system of the planetary individuality, and hence in the sense of a distinct celestial body of a strictly spherical form. The mode to be adopted for the determination of his dimensions is not far to seek. Since we must aim at a record of the reduction of the contradictory unity between the Earth and Mercury to a condition of Oneness, it suggests itself to embody this unity in the form of an Inverted Relation with a view to the identification of $2\pi r_{\odot}$ with the root of the Exponent. As regards

the significance attaching to the two factors, since they must refer to two spaces embodying Mercury's and the Earth's individualities with respect to their contradiction against the Sun, we cannot but connect them with the aspect of the axial rotation. Again, in so far as we thus refer to spaces traversed with the equatorial velocity alone, i.e. to certain amounts of the respective planetary equators, these amounts cannot but be identified with the amount of sidereal days in the respective sidereal periods. For, all we have to fall back upon for the determination of the totalities in question, is only the simultaneous reference of the described spaces to the Sun's Being, and hence also to the aspect of the orbital movement. Therefore, we postulate that

$$\sqrt{(2\pi rt/\delta)_{☿}(2\pi rt/\delta)_{♁}} = 2\pi r_{\odot} \quad . \quad \text{(XLVI)}$$

Now, given our results:

$$(2\pi rt/\delta)_{♁} = 9{,}120{,}389{\cdot}735430 \text{ miles,}$$
$$(2\pi rt/\delta)_{☿} = 813{,}434{\cdot}739557 \text{ ,,}$$
$$\therefore 2\pi r_{\odot} = 2{,}723{,}755{\cdot}1003 \text{ ,,}$$

or $2r_{\odot} = 866{,}998{\cdot}176$,,

as against Sir Lockyer's 867,000 ,,

C.

THE PLANETARY INDIVIDUALITY AS MEDIATED WITHIN ITSELF WITH ITS HYPOSTATIZED ASPECTS.

The concluding portion of the total dialectical whole of the Solar system manifestly advances our previous recapitulation of the original dialectical whole of the measurable Quality and Quantity to its concluding stage under the head of Indifference and its mediation within itself. Accordingly, we are finally meant to concern ourselves with the true Being of the Solar System in supersession of its already discussed aspects, and hence in explicit vindication of its Absolute Ideality.

CHAPTER I.

THE STANDPOINT OF THE DIRECT RELATION: SIDEREAL REVOLUTION OF THE MOON.

INASMUCH as the mediation belonging to the Inwardness of the true Being presupposes, in its present application, a contrast between pure Self-centredness and pure Centrelessness, its two sides manifestly refer to the Axial Rotation and the Orbital Movement as distinctively displayed against one another and their fundamental unity in the Earth. That the thus hypostatized aspect of the Axial Rotation refers to the Sun and the opposite aspect to the Moon, is at once plain from the standpoint of our familiarity with facts.

Thus, the Axial Rotation of the Sun and the Orbital Movement of the Moon present themselves to us immediately only as sides of a Direct Relation. That is to say, even though we cannot avoid qualifying either of them by a reference to the other, we are not meant to seek an exemplification of the orbital movement as such in connection with the Sun, or of the axial rotation as such in connection with the Moon. In speaking of an orbit in connection with the Sun, we shall refer simply to a space equivalent to a certain amount of his equators, whilst axial rotation in connection with the Moon must simply refer to a conception of her sidereal revolution as an amount of a particular unit time. Because, however, the mediation has at first only the character of a simple contrast between its sides, there is immediately no call for a further quantitative specification of the Sun's equator. In so far as the contrast concerns a dual presentation of the same direct unity of the two aspects of the planetary individuality with an emphasis on either the one or the other in such wise, that the unemphasized aspect, whilst remaining in either respect purely implicit, is yet also directly identifiable with the emphasized aspect, we are, to begin with, justified in treating the Sun's

equator as an implicit exemplification of his orbit. Besides already in connection with the previous determination of the Sun's equator, we have implicitly correlated his axial rotation with a planetary sidereal period rather than with a sidereal day; in which way we may be said to have anticipated the opposite to the direct identification of the Moon's sidereal revolution with one axial rotation.

Abstractly considered, the contrast now in question may be specified indifferently as either the contrast between the Sun's orbital movement and the Moon's axial rotation, or the contrast between the Sun's axial rotation and the Moon's orbital movement. Since, however, with a view to the initial pure implicitness of the complementary aspects on either side, we must emphasize, in connection with the Sun, the aspect of the axial rotation and, in connection with the Moon, that of the orbital movement, and these two aspects must further be transformed into the sides of a Direct Ratio, the practical import of our present task being the determination of the Sun's and Moon's elements, and hence necessarily also a matter of the Quantitative Relation: it suggests itself to identify the side of the amount with $2\pi r_{\odot}$ and the side of unity with $t_{☾}/\delta_{♁}$. For, the side of the amount should have a spatial significance, which cannot but refer to the just determined magnitude of $2\pi r_{\odot}$; and that, in so far as the side of unity must, by contrast, have a temporal significance, we cannot but refer to $t_{☾}/\delta_{♁}$, follows from the very character of the two contrasted sides as hypostatized aspects of the Earth.

As regards the exponent of the just specified ratio, seeing that it is more particularly in this respect that the Earth is emphasized in contradistinction against its hypostatized aspects, and *eo ipso* also against its own orbital movement and axial rotation, it at once suggests itself to associate the exponent with the Polar Circumference. But a glance at the derived magnitude of $2\pi r_{\odot}$ and the given $t_{☾}$ leads at once to the inference that $P(2\pi r)_{♁}$ must be treated only in the sense of an abstract unit to a particular amount of simple qualitative significance. In so far as we have already found that $\vartheta_{♁} = 4\vartheta_{♀}$, or that $(t\vartheta)_{\psi} = 4(t/\delta)_{☿}$, in which case $2\pi\rho_{\psi}/(t/\delta)_{☿} = 4e_{\psi}$, it suggests itself to postulate that

$$\frac{2\pi r_{\odot}}{t_{☾}/\delta_{♁}} = 4\text{P}(2\pi r)_{♁} \quad . \quad . \quad \text{(XLVII)}$$

In fact, seeing that $2\pi r_{\odot} = 2{,}723{,}755{\cdot}1003$ miles,
and $\text{P}(2\pi r)_{♁} = 24{,}854{\cdot}9875$ miles
and $\delta_{♁} = 0{\cdot}99726967 d_{♁}$,
we find that $4t_{☾}/\delta_{♁} = 109{\cdot}58585677$,
$\therefore \quad t_{☾} = 27{\cdot}321661 50 d_{♁}$,
as against Sir Lockyer's $27{\cdot}321661418 d_{♁}$.

As is seen, in fixing our figure for $e_{♁}$ at 1,559,024 miles, we may indeed claim to have reduced the margin of indefiniteness to a negligible minimum. Of course, if we are right in assuming that the given figure for $t_{☾}$ is exact, the almost perfect agreement of our own result at the same time argues that the same degree of exactness is characteristic of the rest of our results, seeing that all of them stand or fall together by virtue of their dialectical interdependence. But that even the remaining margin of indefiniteness in connection with our $e_{♁}$ makes itself on occasion noticeable by means of a discrepancy running into a considerable number of miles, comes home to us on recalculating our figure for $2\pi\rho_{♁}$ from the standpoint of the figure resulting for $\text{P}(2\pi r)_{♁}$ on the basis of the given figure for $t_{☾}$ and our own figure for $2\pi r_{\odot}$. Namely, we find that $\text{P}(2\pi r)_{♁}$ becomes then 24,854·988827 miles, and as $\text{P}(2\pi r)_{♀}$ is then 24,167·71053 miles, our figure for $2\pi\rho_{♁}$ comes to be increased to 584,078,232·3 miles, as against, 584,078,145 miles. In this way, it would follow that our figure for $\text{S}_{♁}$ probably errs by about 15 miles by defect. But, then, as we are not minded to spend the rest of our lives on eliminating the otherwise all the same inevitable margin of indefiniteness, we must content ourselves with the degree of approximate exactness already reached.

CHAPTER II.

THE STANDPOINT OF THE INVERTED RELATION.

a.

1. $t_{\odot}$ *and* $2\pi r_{☾}$.

When we now proceed to concern ourselves with the Sun's axial rotation and the Moon's orbital movement in exemplification of the standpoint of the Inverted Relation, it commends itself to our dialectic instinct that the formula embodying this standpoint should be construed on the model of the equation originally representative of this standpoint in application to the Earth, i.e. of the equation $(2\pi rt/d)_{♁} = S_{♅}/196$. In so far as this equation represents the space traversed with the Earth's equatorial velocity in one year only upon the proviso that $2\pi r_{♁}$ stands for the equatorial velocity per one mean solar day, and we are now meant to correlate $2\pi r$ with the Sun's axial rotation and t with the Moon's orbital movement, the left side of the equation should now represent the space traversed with the Sun's equatorial velocity in the Moon's sidereal revolution, and hence have its embodiment in the expression

$$\left(\frac{2\pi r}{t}\right)_{\odot} \cdot t_{☾},$$

where $t_{\odot}$ stands, of course, for the Sun's day in terms of mean solar days. As regards the magnitude to be substituted for $S_{♅}$, we cannot be in doubt that it refers to the Earth's orbit, seeing that the right side of the formula $2\pi r T = S$ must now acquire the meaning we associated with it on the side of totality, in simultaneous reference to the Earth as the middle term of the present mediation. That the resulting equation

$$(2\pi r/t)_{\odot} t_{☾} = 2\pi\rho_{♁}/196 \qquad . \text{(XLVIII)}$$

serves the practical purpose of determining $t_{\odot}$, is obvious.

Inasmuch as this equation may also be read in the sense that the Earth's orbit is equivalent to the space traversed with the Sun's equatorial velocity in 196 Moon's sidereal revolutions, i.e. in

$$196 \times 27{\cdot}321661418 = 5355{\cdot}045638928 d_{\male},$$

then, seeing that for the improved figure for $2\pi\rho_{\male}$,

$$\rho_{\male}/r_{\odot} = 214{\cdot}438600,$$

we find that $t_{\odot} = 24{\cdot}972396 d_{\male}$,

as against $24{\cdot}971868 d_{\male}$,

resulting for it at the equator from the standpoint of Sir Lockyer's formula $865' \pm \sin \frac{7}{4} l$ for the Sun's rotation in 24 hours of mean solar time in reflection upon its variableness with latitude.

As regards the present correspondence to the alternative reading of our original formula for the standpoint of the Inverted Relation in application to Mars, it suggests itself that the right side is now meant to stand for $P(2\pi r)_{\male}/100$, seeing that the original reference to $S_{♄}$ was introduced in reflection upon the character of immediate concreteness which the Earth acquires through its identification with Mars, and this meaning is now fittingly symbolized by the polar circumference. In this way, it becomes plain that the left side is meant to be equivalent to 248·5498882 miles; and in so far as this space undoubtedly is, by contrast, to be interpreted as traversed with the Moon's equatorial velocity, we proceed to transform the preceding equation into

$$(2\pi r/t)_{☾} = P(2\pi r)_{\male}/100, \qquad \text{(XLIX)}$$

where $t_{☾}$ is an amount of $d_{\male}$, in reflection upon the direct unity of the Moon's axial rotation with her sidereal revolution as an aspect of the Earth's orbital motion.

Accordingly

$$\begin{aligned} 2\pi r_{☾} &= 248{\cdot}5498882 \times 27{\cdot}321661418 \text{ miles} \\ &= 6790{\cdot}795891 \text{ miles}, \\ \therefore\ 2r_{☾} &= 2{,}161{\cdot}577{,}470 \text{ ,,} \end{aligned}$$

as against Sir Locker's 2,162 miles,
or Chambers' 2,160 ,,

2. $\rho_{☾}$ *and* $\sigma_{☾}$.

In so far as we have also postulated the equation :

$$(2\pi rt/d)_{♁} = 2\pi r_{♀} t_{♁} / \delta_{♀},$$

it suggests itself to inquire what becomes of this equation, when $2\pi r_{☉}$ is substituted for $2\pi r_{♁}$ and $2\pi r_{☾}$ for $2\pi r_{♀}$.

In that case, of course, the sides cease to be equal. But since it is obvious that the resulting ratio is to us of no consequence except in so far as it bears on the completion of our task, it commends itself to us to treat the ratio in question as equivalent to the ratio between $2\pi\rho_{♁}$ and $2\pi\rho_{☾}$. Namely, it suggests itself to postulate that

$$2\pi r_{☉}(t/d)_{♁} : 2\pi r_{☾} t_{♁} / \delta_{♀} = 2\pi\rho_{♁} : 2\pi\rho_{☾} \quad . \quad \text{(L)}$$

$$\therefore\ 2\pi\rho_{☾} = 584{,}078{,}232 : 390{\cdot}00423936$$

$$= 1{,}497{,}620{\cdot}4690 \text{ miles,}$$

or $$\rho_{☾} = 238{,}353{\cdot}7008 \quad ,, \ .$$

If, in order to test this result, we proceed to calculate from it and $t_{☾}$ the space traversed by a falling body in the first second on the Earth's surface, we find ourselves indeed in almost perfect agreement with the results yielded by observation. Namely, the Moon traverses in one minute an arc of 32·9408960″. With the help of the cosine series we find that the cosine of this angle is 0·999999987247640. It follows that the space traversed by a falling body in the first minute at the distance of 238,353·7008 miles from the Earth's centre amounts to 16·0489412 feet. Consequently, at the distance of 3963·5 miles the space traversed in the first second is 16·12241 feet, as against the given 490·5 cm. = 16·09266 feet. As regards the discrepancy of about one third of an inch, it seems to us more plausible to trace its source to the empirical rather than to our own result. In any case, we do not see our way to reducing our figure for $\rho_{☾}$ to 238,207 miles, which is the magnitude resulting for it on the supposition that the given figure for acceleration is exact. The only way to settle our doubts is to determine $S_{☾}$, i.e. the Moon's mean distance from the *Earth*, with the help of our present figure for $\rho_{☾}$ and the still to be ascertained figure for $\sigma_{☾}$, because then we shall have in the

given figure for $S_{☾}$ an additional test ready to hand for our previous results.

As regards, now, a likely clue to the determination of $\sigma_{☾}$, it occurs to us at once to reflect upon the dialectical Inwardness of the already vindicated relation $\vartheta_{\psi} = 2\pi t_{☾}$. Inasmuch as the Earth has thus been, by anticipation, credited with its present meaning, as a third to its hypostatized aspects, and this its meaning must equally belong to the dialectical Inwardness of the now attempted determination of $\sigma_{☾}$, it commends itself to us to contrast ϑ_{ψ} and $\sigma_{☾}$ in reference to the double aspect of the common middle term in their determination. Accordingly, we are assuming that whereas ϑ_{ψ} refers to the space traversed with Neptune's *orbital* velocity, $\sigma_{☾}$ refers to the space traversed *in the same time* with the Moon's *equatorial* velocity. Seeing, then, that, in the time $2\pi t_{☾}$, the Moon performs 2π axial rotations, we postulate that

$$\sigma_{☾} = 2\pi r_{☾} \times 2\pi, \quad . \quad . \quad . \quad \text{(LI)}$$

an equation, by the way, going together with the proportion

$$\rho_{☾} : r_{☾} = \vartheta_{\psi} : \vartheta_{☾},$$

when $\vartheta_{☾}$ represents the time in which the Moon's *orbital* velocity traverses the space equivalent to her σ.

On evaluating the expression $\sqrt{\rho_{☾}^2 + \sigma^2/8}$, we find,

given	$\sigma_{☾}$ =	42,667·829	miles,
and	$\rho_{☾}$ =	238,353·7008	"
that	$S_{☾}$ =	238,830·58	"
as against Sir Lockyer's		238,840	"
or Chambers'		238,833	" .

As is seen, our figure for $\rho_{☾}$ errs by defect rather than excess.

b. The Mean Revolution of the Line of Apsides.

Inasmuch as the two aspects of the total planetary individuality have so far been contrasted only with respect to

the immediately characteristic feature of their distinctiveness against one another, whilst the full scope of the mediation between them must bring to the front the transitory character of this their immediate distinctiveness, and, in this way, lead to the latter's final idealization : we must further also postulate, in connection with the Moon, an exemplification of the standpoint which is immediately associated with the Sun, and, in connection with the Sun, by contrast, an exemplification of the standpoint immediately associated with the Moon.

Because, in this respect, we no longer reflect upon the complementary aspects on the two contrasted sides in their original significance, i.e. from the standpoint of their *direct* unity with the primarily emphasized aspects, we must postulate for them an embodiment in *qualitative* distinction from the features originally denoted by them. In this way, we are concerned with an alternative exemplification of the axial rotation in connection with the Moon, and with an alternative exemplification of the orbital movement in connection with the Sun.

In so far as the implied alternativeness is now at the same time bound up with a reflection upon the original primary aspects, yet a simple return to these latter is out of the question, it suggests itself to connect the sought double embodiment with a suitable modification of the Moon's orbital movement on the one side and of the Sun's axial rotation on the other. Our familiarity with facts leads us to suppose that the modification in question, on the side of the Moon, can only refer to the mean revolution of her line of Apsides which we propose to symbolize by $A_☾$. As regards the corresponding modification of the Sun's axial rotation, seeing that we thus negate its previous identification with the orbital movement, we refer to the Earth's orbital movement in the sense of axial rotation, and hence to the mean revolution of its line of Apsides : $A_♁$.

It is true that the thus postulated interpretation of the present embodiment of the two complementary aspects remains pictorially obscure; but we are meant to vindicate it by showing that $A_☾$ and $A_♁$ are, indeed, determinable along the implied line of reasoning.

1. $A_{☾}$.

If we are right in our assumption that $A_{☾}$ represents an alternative exemplification of the standpoint of the axial rotation, then, seeing that alternativeness implies, in this respect, *qualitative* distinction, the determination of this period should involve a reference to the Sun's axial rotation. The simplest way to incorporate this reference would be to identify $A_{☾}$ with $t_{\odot}$. But as this identity must immediately be negated, and negation brings in the element of space, it suggests itself to connect the determination of $A_{☾}$ with the ratio $2\pi S_{☾}/t_{\odot}$, i.e. with the mean velocity per $d_{♁}$, with which the end-points of the major axis of the Moon's orbit would rotate, if $A_{☾} = t_{\odot}$.

In so far, now, as the space thus indicated becomes, with respect to the sought determination of $A_{☾}$, a totality of a particular unit, our next task consists in the determination of this unit. All we have to fall back upon in this respect is the reflection that the determination of $A_{☾}$ should just as much involve a reference to the Moon's own axial rotation. Since, however, we must in that case at the same time negate the direct unity of this rotation with her own orbital movement, the sought unit must not be identified simply with $(2\pi r/t)_{☾}$ but rather with $2\pi r_{☾}/t_{♁}$, i.e. with the Moon's equatorial velocity per one $d_{♁}$ on the understanding that $t_{☾} = t_{♁}$.

As a matter of fact, on evaluating $A_{☾}$ from the equation

$$\frac{2\pi s_{☾}}{t_{\odot}} = \frac{2\pi r_{☾} A_{☾}}{t_{♁}}, \quad . \quad . \quad . \quad \text{(LII)}$$

we find that $A_{☾} = 3{,}232{\cdot}115 d_{♁}$,
as against Sir Lockyer's $3{,}232{\cdot}37543 d_{♁}$.

2. $A_{♁}$.

In order to vindicate our assumption that $A_{♁}$ admits of being interpreted in the sense of an alternative exemplification of the orbital movement in connection with the Sun's axial rotation, in negation, that is, of the latter's primary grasp in the sense of orbital movement, we must, to begin

with, set aside the already preconceived objective significance of this alternative exemplification.

Therefore, we are simply concerned with the form to be postulated for the present embodiment of the orbital movement in *qualitative* distinction from its primarily *direct* unity with the Sun's axial rotation. Now, inasmuch as the orbital movement remains thus qualified by a reference to the axial rotation, or still is to be grasped in terms of axial rotation, the orbit does not refer to a space actually traversed by an orbital movement, but stands only for a particular amount of $2\pi r_{\odot}$. In so far, further, as the implied embodiment of the orbital movement must now be also correlated with the Moon, and should then be equally grasped in terms of the axial rotation, it suggests itself to treat the ratio $(2\pi r/t)_{☾}$, i.e. the equatorial velocity of the Moon per one $d_{♁}$ $= 248{\cdot}5498882$ miles, as a unit to the space symbolizing the Sun's orbit.

As regards the amount which this space is of $2\pi r_{\odot}$, it suggests itself to interpret it in the sense of a record of the present significance of the Sun as another Earth. For, owing to the present exemplification of the orbital movement, the Sun is no longer simply grasped as the hypostatized aspect of the Earth's axial rotation alone, but as a phase of the total planetary individuality. Still, in so far as the orbital movement in connection with the Sun nevertheless remains implicit, having to be interpreted in terms of his axial rotation, the Sun becomes now a phase of the Earth only with respect to the latter's embodiment of the standpoint of the Inverted Relation in the previously developed system of the planetary individuality. That is to say, the Sun presents itself now as another Earth in analogy to the original relationship between the Earth and Venus.

Consequently, the amount which the space symbolizing the Sun's orbit is of his equator, i.e. T in the equation $2\pi r\mathrm{T} = \mathrm{S}$, should be simply $t_{♁}/\delta_{♀}$; seeing that the equation $196(2\pi rt/d)_{♁} = \mathrm{S}_{♅}$ becomes in application to Venus $196\ 2\pi r_{♀} t_{♁}/\delta_{♀} = \mathrm{S}_{♅}$. But since it is equally true that the stated analogy implies now also an element of negation (the particular amount of $2\pi r_{\odot}$ standing for an exemplification of the standpoint of the orbital movement), it suggests

itself, in reflection upon the fact that the reference to the orbital movement takes at the standpoint of the Inverted Relation the form of a reference to the comparative number in the series of the mean planetary distances, and hence in the present case to the number 10,—it suggests itself that the sought amount of $2\pi r_{\odot}$ is represented by the expression $10t_{\text{♁}}/\delta_{\text{♀}}$.

Now, on dividing $(2\pi r/t)_{\text{☾}}$ into $10 \times (2\pi r)_{\odot} t_{\text{♁}}/\delta_{\text{♀}}$, we get for the sought period

$$41{,}165{,}208{\cdot}195418 d_{\text{♁}} = 112{,}702{\cdot}234589 t_{\text{♁}}.$$

If this period represents $A_{\text{♁}}$, then

$$112{,}702{\cdot}234589 t_{\text{♁}} = 112{,}701{\cdot}234589 a_{\text{♁}},$$

where $a_{\text{♁}}$ stands for the mean anomalistic year, given by Sir Lockyer as equal to

$$365 \text{ d. } 6 \text{ h. } 13 \text{ m. } 49{\cdot}3 \text{ s.} = 365{\cdot}2595983 d_{\text{♁}}.$$

As a matter of fact, our result for $a_{\text{♁}}$ is

$$365 \text{ d. } 6 \text{ h. } 13 \text{ m. } 49{\cdot}69 \text{ s.} = 365{\cdot}2596029 d_{\text{♁}}.$$

Accordingly, the equation serving for the determination of the mean revolution of the Earth's line of Apsides in terms of $d_{\text{♁}}$ has the form:

$$10(2\pi r)_{\odot} t_{\text{♁}}/\delta_{\text{♀}} = (2\pi r/t)_{\text{☾}} A_{\text{♁}} \quad . \quad \text{(LIII)}$$

Seeing that $100(2\pi r/t)_{\text{☾}} = P(2\pi r)_{\text{♁}}$ the just derived equation may be transformed into

$$1000(2\pi r)_{\odot} t_{\text{♁}}/\delta_{\text{♀}} = P(2\pi r)_{\text{♁}} A_{\text{♁}},$$

or, in still further reflection upon the fact that

$$P(2\pi r)_{\text{♁}} = P(2\pi r)_{\text{♀}}/\delta_{\text{♀}} = \sqrt{2\pi\rho_{\text{♁}}/\delta_{\text{♀}}},$$

into

$$1000(2\pi r)_{\odot} t_{\text{♁}} = A_{\text{♁}} \sqrt{2\pi\rho_{\text{♁}}},$$

$$\therefore \frac{2\pi r_{\odot}}{\sqrt{2\pi\rho_{\text{♁}}}} = \frac{A_{\text{♁}}}{1000 t_{\text{♁}}}, \quad . \quad . \quad \text{(LIV)}$$

an equation bringing to our notice that the aspect of the orbital movement in connection with the Sun may also be treated as in Involved Inverted Relation with the original

embodiment of this aspect in connection with the Earth. Since in that case we are also returning to the direct unity of the orbital and axial movement, the Sun's orbit presents itself as identified with his equator, whilst the moment of unity stands for the root of the Earth's orbit. In this way, we come to reflect again upon the coincidence of the British statute mile with the dialectically postulated unit length; but in so far as the ratio $2\pi r_{\odot}/\sqrt{2\pi\rho_{\text{♁}}}$ concerns $A_{\text{♁}}$, the unit of time becomes one *Millennium.*

In reflection upon our suggested mode of determination for $P(2\pi r)_{\text{☿}}$, it is interesting to notice that since

$$2\pi r_{\odot}^{2} = (2\pi rt/\delta)_{\text{♁}}(2\pi rt/\delta)_{\text{☿}} = (A_{\text{♁}}/1000t_{\text{♁}})^2\, 2\pi\rho_{\text{☿}},$$

if

$$P(2\pi r)_{\text{☿}} = \sqrt{100(2\pi rt/\delta)_{\text{☿}}},$$

$$\therefore \left(\frac{A_{\text{♁}}}{100t_{\text{♁}}}\right)^2 = \frac{P(2\pi r)^2_{\text{☿}}}{2\pi\rho_{\text{♁}}}(2\pi rt/\delta)_{\text{♁}},$$

$$\text{i.e.}\quad \frac{P(2\pi r)_{\text{♀}}}{P(2\pi r)_{\text{☿}}} = \frac{\sqrt{(2\pi rt/\delta)_{\text{♁}}}}{(A/100t)_{\text{♁}}} \quad . \quad . \quad \text{(LV)}$$

In this case, as is seen, the unit of time is one *Century.*

CHAPTER III.

THE STANDPOINT OF THE INVOLVED INVERTED RELATION.

a. Polar Rotation.

IN so far as the unity resulting from the consideration of the previous standpoint becomes the immediate object of further consideration only in the sense of a third to the factors, with which it is to be subsequently established as just as much mediated within itself, the Earth acquires now once again the significance of a Substrate to its double aspect. But as the Substrate is thus before us in explicit supersession of its immediate character, as Absolute Indifference to its own Inwardness, its present embodiment must be sought in connection with the Earth's own Being. It is only now, in the concluding portion of the concluding part of the total dialectical whole of the solar system, that we are meant properly to explicate the character of the Earth, as a mediation within itself with its hypostatized aspects; and the Substrate refers, therefore, at present to that aspect of the Earth which is to be postulated for it in immediate reflection upon this its Inwardness.

Now, that this aspect must be connected with the Polar Circumference, follows at once from the fact that we are now recapitulating that stage in the previous system of the planetary individuality, at which the third to its two aspects acquired the sense of Polar Circumference. But inasmuch as the Substrate is now to be emphasized in immediate reflection upon its entanglement with the mediation meant to explicate its contradictory Inwardness, and hence in the distinct sense of the finitized Infinite or the infinitized Finite, the Polar Circumference must now be credited with Self-movement. That is to say, the postulated third

aspect of the Earth, in idealistic contradistinction against its two primary aspects, refers to a *movement involving inversion of the poles;* a movement whose direction should be *clockwise*, on account of its *qualitative* distinction from the two primary *anti-clockwise* movements.

As a matter of fact, observation shows that the inclination of the equator to the ecliptic, i.e. the obliquity of the ecliptic, has been for the last 2000 years *decreasing* at the rate of about 48″ per century. "The actual motion of the pole can be calculated with tolerable precision by Spherical Trigonometry. This shows that there is an actual motion of the pole amongst the fixed stars of *about* 3 minutes of arc per century, or sufficient to carry the poles through a complete revolution in 720,000 years. *The supposed limits of the obliquity, therefore, are without any foundation in fact, since they have been obtained from erroneous premises, and there is so far no evidence that the inclination of the equator to the ecliptic has any limits whatever;* but for ought we can prove to the contrary the poles may, every 360,000 years, be *in the plane of the ecliptic*, when the tropics would extend to the polar regions, whilst once also in the same period, the poles would be *perpendicular to the ecliptic*, and we should have perpetual spring. If this be so, the occurrence of glacial and tropical periods is at once explained, and the problem which has puzzled the brains of Astronomers and Geologists for a century or more has found a solution." [1]

Of course, we accept Mr. Sutcliffe's conclusions simply because they are in agreement with our own inference. Carried along by our own dialectical momentum, we cannot but postulate the Polar Rotation; and, in view of the previous agreement of our deductions with astronomical facts, we have no reason to doubt the objective validity of our present inference. The dialectical significance of the period adduced for the Polar Rotation only strengthens our confidence. The number 720 stands for the unit siderial period as a self-differentiated totality; and we have also found that one Millennium, too, is a unit of time of dialectical significance in connection with vast periods. Clash of opinion founded on mere *amour-propre* does not attract us.

[1] "La Place's Gigantic Hoax," G. E. Sutcliffe, Bombay, 1906, pp. 9, 10.

b. The Entanglement of the Polar Rotation with the Orbital Movement and the Axial Rotation.

In so far, now, as the Polar Rotation coexists with the Orbital Movement and the Axial Rotation in such wise that it is at the same time to be grasped as mediated within itself with the Sun and the Moon, we must postulate for it a *three-dimensional* movement. That is to say, the polar diameter cannot simply rotate in a plane, as its rotation presents itself to our pictorial consciousness, so long as we immediately reflect only upon its mere coexistence with the two primary aspects of the planetary individuality. Just because coexistence must not, in this respect, be interpreted in the sense of Indifference, but of a mediated relationship having for its middle term the hypostatized aspects of the orbital and the axial movement, i.e. the now to be properly inwardized relation of the Earth with the Sun and Moon, we must postulate for the polar radius a movement in three dimensions.

a. Precession of the Equinoxes or Annus Maximus.

In so far as Physical Science is not a science of pure thought, the relationship now in question is described by it in terms appropriate to its own standpoint. The Sun and the Moon are said to *attract*, and in turn to be attracted by, the Earth ; and, consistently with the now implied reflection upon the shape of the Earth, the influence of the Sun's and Moon's attraction is connected with the equatorial protuberance.

"645. Let the equatorial protuberance be represented by a ring, supported by two points at the extremities of a diameter on a horizontal concentric ring, and inclined to its support as the Earth's equator is inclined to the ecliptic. Let a long string be attached to the highest portion of the ring, and let the string be pulled horizontally, at right angles to the two points of suspension, and away from the centre of the ring. This pull will represent the Sun's attraction on the protuberance. The effect on the ring will be that it will at once take up a horizontal position ; the highest part of the ring will fall as if it were pulled from below, the lowest part will rise as if pulled from above.

"646. The Sun's attraction on the equatorial protuber-

ance in certain parts of the orbit is exactly similar to the action of the string on the ring, but the problem is complicated by the two motions of the Earth. In the first place—in virtue of the yearly motion round the Sun—the protuberance is presented to the Sun differently at different times, so that twice a year (at the solstices) the action is greatest, and twice a year (at the equinoxes) the action is *nil;* and, in the second place, the Earth's rotation is constantly varying that part of the equator subjected to the attraction.

"647. If the Earth were at rest, the equatorial protuberance would soon settle down into the plane of the ecliptic: in consequence, however, of its two motions, this result is prevented, and the attraction of the Sun on a particle situated in it is limited to causing that particle to meet the plane of the ecliptic earlier than it otherwise would do if the Sun had not this special action on the protuberance. If we take the presentation of the Earth to the Sun at the winter solstice (Fig. 18), and bear in mind that the Earth's rotation is from right to left [anti-clockwise], it will be clear, that while the particle is mounting the equator, the Sun's attraction is pulling it down; so that the path of the particle is really less steep than the equator is represented in the diagram: towards the east the particle descends from this less height more rapidly than it would otherwise do, as the Sun's attraction is still exercised: the final compound result therefore is, that it meets the plane of the ecliptic sooner than it otherwise would have done.

"648. What happens with one particle in the protuberance happens with all; one half of it, therefore, tends to fall, the other half tends to rise, and the whole Earth meets the strain by rolling on its axis: the inclination of the protuberance to the plane of the ecliptic is not altered, but in consequence of the rolling motion, the places in which it crosses that plane precede those at which the equator would cross it were the Earth a perfect sphere: hence the term *precession.*" [1]

Ultimately, of course, the precession of the equinoxes is traced to the attraction of both the Sun and Moon, as is implied in its usual qualification as *luni-solar;* and from

[1] "Astronomy," Sir N. Lockyer.

the standpoint of the resulting motion, it becomes obvious, to our pictorial consciousness, that the Polar Rotation consists in a spiral-like slow revolution of the Earth's axis round the axis of the celestial sphere with an alternately increasing and decreasing angular distance therefrom. Or rather, in reflection upon the simultaneously to be postulated contrast between the Polar Rotation and the other two motions of the Earth, the delineated revolution of the latter's axis ought to proceed round the axis perpendicular to the Invariable Plane of the Solar System instead of to the variable ecliptic.

As regards the conflict, in which we find ourselves entangled with the modern theory of the variation of the obliquity, we find sufficient explanation in the following letter, originally addressed by Mr. G. E. Sutcliffe to the Editor of "Nature" and only after its rejection included in the pamphlet already referred to—a pamphlet, by the way, which drew upon him an appalling storm of vituperation, though all the same worthy of careful notice, seeing that the possibility of the existence of the Polar Rotation has since been demonstrated by M. Beziau, whose apparatus was exhibited at the Louisiana Purchase Exhibition at Saint Louis, and was warmly endorsed by M. Camille Flammarion :[1]

"My object in issuing this small pamphlet" says Mr. Sutcliffe, "is to draw the attention of a few scientific men to an important error of La Place which is seriously hindering the proper development of scientific thought. With this view sometime ago I addressed a letter to the Editor of "Nature" which was returned to me with a note regretting he was unable to find space.

"The following is the text of the letter with a few explanatory words added in brackets :—

'THE VARIATION OF THE OBLIQUITY.

'*An Important Error in La Place's Equations.*

'To the Editor of "Nature".

'Sir,

'Some time ago, on examining La Place's equations for the variation of the obliquity, I was astonished to

[1] "Rosicrucian Cosmo-Conception," Max Heindel, 3rd ed., p. 513.

find that in obtaining his results he has throughout given a *positive* value to the motion of precession, that is, he has given to precession the same direction as the planets. This mistake is involved in all his equations for the changes of the obliquity, and vitiates his proof that the limit of its variation amounts to no more than three degrees of arc. The error in question will be found in the "Mécanique Céleste," Livre vi, No. 31. My attention was first drawn to it by discovering that when the theory of La Place was tested by spherical trigonometry it proved that the obliquity of the ecliptic ought to be *increasing*, and not diminishing as observation shows.

'This theory implies that the mean angular distance of the terrestrial pole, and the pole of the invariable, or mean plane of our Solar System, does not change ("Ency. Brit.," Vol. II., p. 794). The invariable plane makes an angle with the plane of the ecliptic of about one and a half degrees, and its ascending node is about longitude 106° on the celestial sphere. The pole of the invariable plane is in Right Ascension 273° 53′ 35″, and its declination is N. 66° 56′ 26″. The pole of the ecliptic for A.D. 1900 is in Right Ascension 270°, and declination N. 66° 32′ 52″.

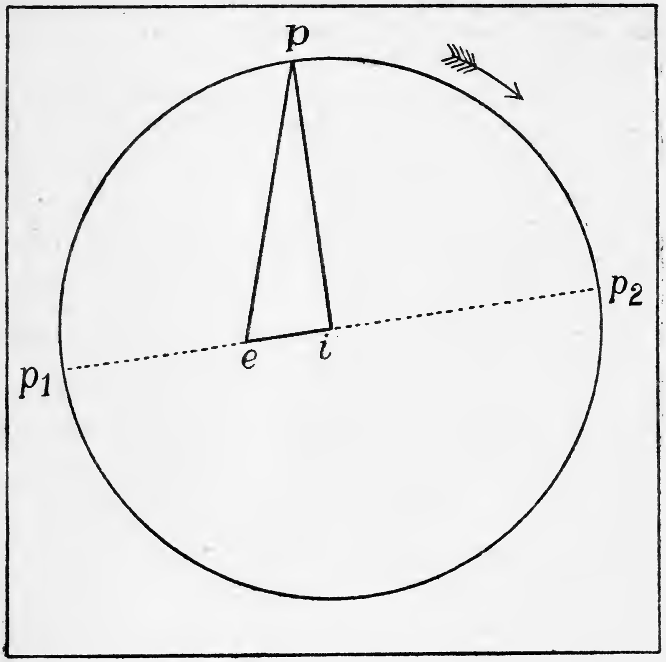
'The relative position of the three poles is shown in the annexed figure, where i is the pole of the invariable plane, e the pole of the ecliptic, and p the terrestrial pole. The pole e moves very slowly, and without material error may be considered stationary for short periods of a few thousand years. Modern theory requires that the side ip should be constant, whilst for short periods ei is also constant.

'By the motion of precession p is carried towards p_2 by a clockwise motion indicated by the arrow. Now it is evident by mere inspection that as p moves towards p_2, the side ep must increase, since in the position p_2 its length is $ip + ei$. But the lengthening of ep implies that the obliquity is increasing, which is contrary to observation. The only way to make the accepted theory agree with observation is to make p move in the opposite direction [towards p_1] or *contrary to its actual motion* and this is what La Place has done.

'It will be noticed, however, that for the purpose of merely determining the theoretical limits of the obliquity [which is twice the side ei, that is $2 \times 1\frac{1}{2} = 3$ degrees], as has been

done by Stockwell and others, it is immaterial whether p be moved clockwise or anticlockwise, so that such investigations could be made *without discovery of the original error;* and since in practice the variation of the obliquity is determined by observation, and not by the use of La Place's equations, the non-discovery of the error is still further explained.

'From the known motion of p and the observed shorten-



ing of the side *ep* [which is a measure of the observed variation of the obliquity] I have calculated that there must be a shortening of the side *ip* of 1·844 seconds per annum, or about three minutes of arc per century. This is a *real motion of the pole* on the celestial sphere quite independent of the ecliptic, which, if continued indefinitely, would cause a complete inversion of the poles in about 360,000 years.

'The projection of this polar motion on a fixed equator would be represented by a proper motion in Right Ascension

of $-0{\cdot}1229°$ [so that a star that was really stationary near to the pole would be given a fictitious proper motion of $+0{\cdot}1229°$], and on referring to the "Nautical Almanack" for A.D. 1900 (p. 294) I find that Polaris [the pole star] is given a proper motion of $+0{\cdot}1221°$ or the same as the projection of the polar motion above deduced, but of opposite sign. This proper motion of Polaris, therefore, appears to have been deduced *from the false basis* that the distance between *p* and *i* is invariable.

'It will be seen that *the proper motions of the stars are affected by this mistake on the part of La Place*, so that stars like Polaris, which may be nearly stationary, are given large proper motions, whilst on the other hand the real proper motions of other stars may be largely masked.

'The discovery of this error may throw light on many problems at present not satisfactorily solved, such as the causes of glacial and tropical epochs, and since it fundamentally affects the accuracy of astronomical measurements it is important that the attention of scientific men should be drawn to its significance.

'J. E. SUTCLIFFE.

'THE HERMITAGE, KURLA,
'BOMBAY, 14*th August*, 1905.'"

Mr. Sutcliffe finds it difficult to see how La Place could escape discovering that he had used the wrong sign in his expression for precession, "in view of what he says towards the end of No. 31, Livre VI. For he there first shows that 4004 B.C. the longitude of the equinox was $-87{\cdot}853°$, and that the longitude of perihelion of the earth's orbit at the same date was $+89{\cdot}17°$, and then goes on to state that they were only $1{\cdot}317°$ apart. Now the difference between $-87{\cdot}853$ and $+89{\cdot}17$ is not $1{\cdot}317$ but $177{\cdot}023$." Accordingly, he concludes that La Place simply desired to hoax his reader:

"It is well known that mathematicians are in the habit of copying each other's errors, but the fact of a blunder of this kind running the gauntlet of a whole century of mathematicians, is, I think, quite unique, and will occasion, perhaps, in some quarters, no little searching of heart. If such careless reading has been the practice, there is no telling what other important blunders future investigations may disclose.

"Of course, La Place knew quite well that many who professed to read his works, *did not really read them.* He once said that Mary Somerville was the only woman he had met who actually understood his writings; yet although Mrs. Somerville gives nearly the whole of La Place's 'Mécanique Céleste' in her 'Mechanism of the Heavens,' *she omits the equation for the obliquity, merely stating verbally La Place's results* (p. 396).

"Was it the necessity of finding some outlet for an over-repressed sense of humour, that led the great French Mathematician to play such a ponderous joke upon the weaknesses of the Mathematical public?"

Of course, we aim at the determination of the cycle of precession; of the so-called *Annus Maximus:* $T_{♁}$. In order to get a clear conception of our subject-matter, let us preliminarily recapitulate its import. Thus, the Earth completes one journey round the Sun when its centre finds itself again on a straight line connecting the centre of the Sun with the same fixed star. If the equator of the Earth lay in the plane of the ecliptic, day and night would be always equal. But as the equatorial plane is inclined to the plane of the ecliptic, day and night are equal only when the straight line connecting the centres of the Sun and the Earth at the same time intersects the equator and hence lies in the equatorial plane. This happens twice a year; at the vernal and autumnal equinoxes. If the inclination of the Earth's axis to the plane of the ecliptic were absolutely fixed, the interval between either of the two equinoxes would be identical with the sidereal period. But the inclination of the Earth's axis is not absolutely fixed. Consequently, there arises a distinction between the Mean *Sidereal* Year and the Mean *Solar* or *Tropical* Year; the former in reference to the Earth's journey round the Sun, the latter in reference to the interval between two successive vernal or autumnal equinoxes. But this distinction arises only upon the proviso that the Earth's axis, the polar diameter, moves three-dimensionally: *clockwise* towards the axis of the celestial sphere and at the same time also *clockwise* round it. If these two component movements were always in the same fixed relation, the resulting motion of the Earth's axis would be descriptive of the surface of a double cone, and *Annus Maximus* would be an exact

number of the Mean Sidereal Year. But Absolutely Free Movement does not admit of absolute fixedness in connection with its aspects. The relation between the two component motions in question is variable on *à priori* grounds. The contradiction characteristic of the analytic attitude towards the planetary motion round the Sun is bound to crop up again in connection with the Polar Motion. This motion, too, must be alternately accelerated and retarded, not only from the standpoint of its whole sweep, of which the *Annus Maximus* is only a part, but right through with respect to any interval; in evidence of the moment of Discreteness implied in Continuity of any kind. "We have mentioned in Arts. 168-9 that the direction of the Earth's axis is, roughly speaking, always the same. It has recently been discovered, however, that the poles of the Earth's axis have a motion round a certain point in a period of about 364 days. This causes a slight *Variation of Latitude* for all places, but the greatest deviation from the mean position only amounts to a few seconds of arc. The matter, however, is considered so important that an international organization has been formed to arrange for the systematic determination of latitude variations at numerous stations throughout the world."[1] To our mind the implied vibratory unrest of the Earth's axis connects itself at once with its total sweep in one Polar Rotation, the projection of the curve traced out by its endpoints on the *invariable plane* yielding a spiral made up of a particular number of turns starting at the centre and extending outwards up to the circumference described by the Earth's axis in order to wind back to the centre, the single turns standing at the same time for the *Annus Maximus.*

Now, that we are not meant to determine this period from the Polar Rotation is plain already from the fact that our figure for this Rotation is only approximately correct—just as is the case with the round number 360 for $t_{♁}$. A reflection on the Polar Rotation serves now only to convince us that the difference between the Mean Sidereal Year and the Mean Solar Year, i.e. $t_{♁} - \tau_{♁}$ does not necessarily yield an exactly round number of $t_{♁}$ for $T_{♁}$. Of course, we see at once that

[1] "Astronomy," Lockyer, art. 174*a*.

$$(\mathrm{T}/t)_{♁} = (\mathrm{T}/\tau)_{♁} - 1, \quad . \quad . \quad . \quad (1)$$

because $\tau_{♁} < t_{♁}$ and the difference amounts in $(\mathrm{T}/t)_{♁}$ years to one $t_{♁}$. That is to say, the first mean solar year elapses *before* the first mean sidereal year, the second $\tau_{♁}$ *still sooner* than the second $t_{♁}$, . . . the *n*th $\tau_{♁}$, say, *half a year* before the *n*th $t_{♁}$, and finally the number of $\tau_{♁}$ becomes just *by one greater* than the number of $t_{♁}$ elapsed in *Annus Maximus.*

But the equation set up is to us of little use by itself. We are meant to set up an independent equation for one of the two unknown magnitudes; undoubtedly for $\mathrm{T}_{♁}$, since this is the period we are mainly concerned with. In this respect, we ought to turn our attention to the Moon. The equation (1) may be said to concern only the present inwardization of the relation between the Earth and the Sun, whilst the determination of the *Annus Maximus* implies the inwardization of the relation of the Earth with the Sun *and* the Moon. Inasmuch, now, as the Earth is already negatively identified with the Sun, we refer to the Moon only as a middle term of this identification. As regards a suitable symbol of this her significance, we find it plainly in her *Synodical Revolution*, in which she keeps regaining the same position with regard to the Sun in distinction from her sidereal revolution. Obviously

$$t_{♁}/t^{s}_{☾} = (t_{♁}/t_{☾}) - 1, \quad . \quad . \quad . \quad (2)$$

where $t^{s}_{☾}$ symbolizes, of course, the synodical revolution.

Now, although there is noticeable a formal analogy between the right sides of the two equations, there is clearly no direct connection between them. Still, they must be aspects of one whole, because they embody the only clues we have to the determination of the cycle of precession. On further thought, it becomes plain that the equations concern us only in mathematization of the definition of the *Annus Maximus* in years and of the Synodical Revolution of the Moon in mean solar days. The dialectically to be postulated connection can concern only these two magnitudes. In short, $t^{s}_{☾}/d_{♁}$ is to serve as a unit to $(\mathrm{T}/t)_{♁}$; quite conformably with the fact that the Moon has now the

sense of a middle term to the identification of the Sun with the Earth. To begin with, therefore, we have before us the inequality

$$(\mathrm{T}/t)_{♁} \neq t^{s}{}_{☾}/d_{♁} \quad . \quad . \quad . \quad (3)$$

and we postulate that the two sides admit of being equalized by means of a particular operation. Obviously, the left side must be decreased, and the right side increased. The suggestion nearest to hand is that their equalization implies proportionateness in the decrease on the one side and the increase on the other. If they are indeed connected in such wise, that the right side is the extreme of unity and the left side the extreme of totality, we cannot but postulate that, by decreasing the magnitude represented by $(\mathrm{T}/t)_{♁}$ *as many times* as the magnitude represented by $t^{s}{}_{☾}/d_{♁}$ is increased, we remove the inequality between them. But at the same time it is also plain, in reflection upon the *qualitative* distinctiveness of the two correlated sides, that the left side must be a Power of the right side. Consequently, there is no further difficulty about the coefficient by means of which the two sides are reduced to equality: that this coefficient is to be identified with $t^{s}{}_{☾}/d_{♁}$ itself follows already from the coincidence of its meaning, as the middle term of equalization, with the significance attaching to the synodical revolution in the present mediation, there being here really nothing else to fall back upon for its specification except this identity of meaning. Accordingly, we are committing ourselves to the at first sight rather surprising inference that

$$(\mathrm{T}/t)_{♁} = (t^{s}{}_{☾}/d_{♁})^{3};$$

surprising, we say, because we do not expect a determination of Time to be cubed. But the cube is here really a consequence of the final formal transformation of the equation which properly represents the result of our dialectically prompted mode of determination for $\mathrm{T}_{♁}$; namely, of the equation:

$$\mathrm{T}_{♁}/t_{♁}\,t^{s}{}_{☾} = (t^{s}{}_{☾})^{2} \quad . \quad . \quad (\mathrm{LVI})$$

This equation shows that the dialectical mode of determination concerns properly $\mathrm{T}_{♁}/t_{♁}$ as a totality of as many

parts as there are mean solar days in the synodical revolution of the Moon. In so far as the *Annus Maximus* is in turn a unit to the Polar Rotation ($P_{♁}$), it suggests itself to postulate that

$$P_{♁} : T_{♁} = T_{♁} : T_{♁}/t^{s}_{☾},$$

i.e. that $$(P/t)_{♁} = (t^{s}_{☾}/d_{♁})^4 = (T_{♁}/t_{♁}t^{s}_{☾})^2 \quad . \quad \text{(LVII)}$$

In this way, we would remove the anomaly of the cube in connection with $t^{s}_{☾}/d_{♁}$. Anyhow, there is, so far as we can see, no other clue to the determination of $P_{♁}$, whose round value, 720,000 years, rests with us still properly only on an empirical basis, even though it undoubtedly is also of dialectical significance. But, then, we must, in the first place, test the objective validity of our result for $T_{♁}$:

$$\frac{t_{♁}}{t_{☾}} = \frac{365{\cdot}256362}{27{\cdot}321661418} = 13{\cdot}36874637,$$

$$\therefore t^{s}_{☾} = \frac{t_{♁}t_{☾}}{t_{♁} - t_{☾}} = \frac{365{\cdot}256362}{12{\cdot}36874637} = 29{\cdot}5305887d_{♁},$$

$$\therefore (t^{s}_{☾}/d_{♁})^3 = (T/t)_{♁} = 25{,}752{\cdot}3172838,$$

$$\therefore \left(\frac{t}{T/\tau}\right)_{♁} = t_{♁} - \tau_{♁} = 0{\cdot}0141834d_{♁},$$

$\therefore \tau_{♁} = 365{\cdot}2421786d_{♁}$ = 365 d. 5 h. 48 m. 44·2310 s.,
as against Sir Lockyer's 365 d. 5 h. 48 m. 46·054 s.

As is seen our result is in almost exact agreement with the result of observation. Accordingly, it would follow that the correct figure for the mean value of the Polar Rotation is

$$25{,}752{\cdot}3172838 \times 29{\cdot}5305887 = 760{,}481{\cdot}09 \text{ years},$$

though the objective validity of this result remains, of course, an open question.

β. Mean Revolution of the Nodes of the Moon, or the Cycle Saros.

We are familiar with the fact that the orbital plane of the Moon is not in the plane of the ecliptic, but that it is subjected to the same kind of rolling movement as we have just now determined in connection with the Earth's equa-

torial plane. If the Moon moved round the Earth in the plane of the ecliptic, i.e. in the same plane as the Earth moves round the Sun, there would be in every sidereal revolution of the Moon a total eclipse of both the Sun and the Moon, because the Moon would find herself twice in line with the Earth and the Sun. Such a monotonous repetition of the same relationships is on *à priori* grounds unthinkable. Alterableness, as we know, belongs to the very In-itself of all that is; it is for this reason that we had to deny Newton's first and second law of motion—as *laws of motion*. Consequently, the ecliptic and the Moon's orbital plane must be distinguishable from one another and the distinguishing must be a matter of a process. That is to say, it would not be enough simply to postulate a fixed angle between them, but an angle such that its magnitude is equally subject to change. Inertia of any kind *non est. Plus cela change, plus c'est la même chose; mais, plus c'est la même chose, plus cela change!* We agree only to disagree, and *vice versâ.*

We have referred already to the newly discovered fact that the Earth's axis is subject to a vibratory movement in the course of a year, and we saw in this movement only an evidence on a small scale of what is happening on a large scale in the course of a Polar Rotation. In so far as this Rotation must have the nature of the Nodal Line of Measure Relations, and the implied alternating increasing and decreasing of the angular distance between the Earth's axis and the axis perpendicular to the Invariable Plane of the Solar System must in that case remain constitutive of every member of the Nodal Line, hence also of every rotation which the Earth's axis describes during one precession of the equinoxes: we must postulate on *à priori* grounds the so-called *Nutation*, i.e. the fact that the spiral-like motion of the pole of the equator is additionally also *waved.* From physical standpoint this additional waviness presents itself as due to the attraction of the Moon in addition to the general effect of the Sun and the Moon in luni-solar precession, *on the understanding* that the Nodes of the Moon, i.e. the points in which her orbit cuts the plane of the ecliptic, are not stationary, but move backwards in further exemplification of the precessional movement of the equinoxes.

The implied analogy between the Moon's orbit and the Earth's equator is indeed in a way embodied already in the equation for $T_{♁}$. If we write this equation in the form

$$T_{♁} = t_{♁} \,.\, t^s_{☾} \times t^s_{☾} \,.\, t^s_{☾},$$

we may say that $T_{♁}$ in $d_{♁}$ is a fixed product of $t_{♁}$ and $t^s_{☾}$ *upon the proviso* that the unit of time, $d_{♁}$, is at the same time in either respect identified with $t^s_{☾}$: as if, that is to say, the Earth axial rotation were simultaneously replaced by the Moon's orbital movement.

As regards, now, our search for a clue to the determination of the mean revolution of the nodes of the Moon ($T_{☾}$), we undoubtedly refer to the Earth as the middle term to the present identification of the Moon with the Sun—of course, in so far as a reference to the present treatment of the Moon's orbital movement in analogy to the Earth's axial rotation is to us symbolical of the identity of the Moon with the Sun. The fact that the Earth is thus before us as an Other to its own self, leads us further to suppose that we must reflect upon it from the standpoint of its contradictory identity with Venus. Accordingly, it suggests itself at first sight to connect $(T/t^s)_{☾}$ with $(t/d)_{♀}$; for so we bring together the two extreme aspects of the Earth as the present middle term—on the one side, its identification with the Moon and on the other with Venus. We refer to $d_{♀}$, i.e. to Venus' *mean solar day*, because $t^s_{☾}$ has been previously implied as correlated with $d_{♁}$, and this correlation must persist even when Venus takes the place of the Earth. But just because Venus is not meant simply to take the place of the Earth, but to be grasped as in contradictory unity therewith, we really postulate, to begin with, the inequality

$$(T/t^s)_{☾} \neq (t/d)_{♀} \quad . \quad . \quad . \quad (1)$$

As a matter of fact, but for the dialectical background to our insistence on an inequality of the two correlated sides, we would feel tempted to put it down to the account of the margin of indefiniteness clinging to our own as well as to observational results. On evaluating $(t/d)_{♀} = (t/\delta)_{♀} - 1$, we get 230·090807 for $t_{♀} = 224{\cdot}7008 d_{♁}$. Accordingly

$$t^{s}_{☾} \times t_{♀}/d_{♀} = 29{\cdot}5305887 \times 230{\cdot}090807$$
$$= 6794{\cdot}7169851 d_{♁}$$

as against $\quad T_{☾} = 6793{\cdot}39108 d_{♁}$,

adduced by Sir Lockyer—though, on evaluating $T_{☾}$ from his figure for the Nodical Revolution of the Moon, according to the relation :—

$$t_{☾} - t^{n}_{☾} : t_{☾} = t_{☾} : T_{☾},$$

we find that $T_{☾} = 6820{\cdot}6 d_{♁}$, $t^{n}_{☾}$ being given as equal to

$$27 \text{ d. } 5 \text{ d. } 5 \text{ h. } 35{\cdot}6 \text{ s.} = 27{\cdot}212217569 d_{♁},$$

In order to vindicate our dialectically founded conviction that $T_{☾}$ is not simply equal to $t^{s}_{☾} \times (t/d)_{♀}$, we propose an alternative mode of determination from the standpoint of the Earth's identity with Venus. If we substitute for $t^{s}_{☾}$ the unit sidereal period, the alternative mode of determination is symbolized, to begin with, by the equation :—

$$T_{☾}/t_{♁} = y/x \quad . \quad . \quad . \quad . \quad (2)$$

where x is equally meant to bring in a restatement of $t_{♁}$ in terms of $d_{♀}$, but with emphasis on that contradictory unity between the Earth and Venus which has its original record in the determination of $\delta_{♀}$. That is to say, $d_{♀}$ is now to be identified with that value of $d_{♁}$ which it would have, if $\delta_{♁}$ were identical with $\delta_{♀}$. In so far, then, as $t_{♁}/\delta_{♀} = (3\sigma/2\pi r)_{♁}$, it follows that

$$t_{♁}/d_{♀} = (3\sigma/2\pi r)_{♁} - 1 = 374{\cdot}64345978.$$

But since this figure does not imply the actual value of $d_{♀}$, we must refrain from symbolizing it by $t_{♁}/d_{♀}$, but confine ourselves only to the expression $(3\sigma/2\pi r)_{♁} - 1$.

As regards the significance of y, the fact that the alternative mode of determination takes now the shape of the inverted relation :

$$T_{☾}\{(3\sigma/2\pi r)_{♁} - 1\} = t_{♁} y, \quad . \quad . \quad (3)$$

suggests that y must itself be grasped in the sense of a product, whose factors admit of dialectical determination. In

this respect, we cannot but postulate their connection with $t_{♁}$ and $(3\sigma/2\pi r)_{♁} - 1$ or $t_{♁}/\delta_{♀} - 1$. The postulated connection is obviously of a qualitative nature. We must, however, equally postulate a qualitative distinction between the two factors. In so far, then, as their connection with $t_{♁}$ and $t_{♁}/\delta_{♀} - 1$ respectively takes the shape of the involved inverted relation, it should be only in connection with one of them that this standpoint should come to the front; and that we then refer to the expression $\sqrt{(3\sigma/2\pi r)_{♁} - 1}$, should follow from this, that we are thus at the same time reflecting upon the substratum of the contradictory unity of the Earth with Venus. Since the other factor shares the same meaning negatively and hence becomes just as much a third to $t_{♁}$ and $t_{♁}/\delta_{♀} - 1$, it plainly can refer only to the notionally round figure for $t_{♁}$ in 360.

Accordingly, we postulate that

$$\begin{aligned} T_{☾}/t_{♁} &= 360/\sqrt{(3\sigma/2\pi r)_{♁} - 1}, \quad . \quad \text{(LVIII)} \\ &= 360 : 19{\cdot}35570871, \\ &= 18{\cdot}599163, \\ \therefore T_{☾} &= 6793{\cdot}462136 d_{♁}, \end{aligned}$$

as against Sir Lockyer's $6793{\cdot}39108 d_{♁}$.

That the discrepancy of 102·32 m. between the two results is most probably due to an incorrectness in Sir Lockyer's figure, is to be inferred from the already indicated lack of its agreement with the figure adduced by him for the nodical revolution of the Moon. Still, as his figure for the Synodic Revolution of the Node is, on the other hand, in fairly close agreement with the result postulated for it by the relation :

$$T_{☾}/t_{♁} + 1 = T_{☾}/\tau_{☾}$$

where $\tau_{☾}$ symbolizes the synodic revolution of the Node, we are interested in placing our formula for $T_{☾}$ beyond doubt in the light of a further possible test.

In this respect, it suggests itself at once to inquire into the nature of the specification, by means of which the ratio $(t/d)_{♀}$ becomes equal to the ratio $(T/t^{s})_{☾}$. The obvious necessity to reduce $t_{♀}$ for the purpose of this equalization should bring to light a particular fact of dialectical import.

Now, should not the fact in question serve to vindicate the dialectical necessity of a particular unit of time, and in that case refer to one *hour*, seeing that this is the only unit of time still lacking a due incorporation in our series of equations?

In fact, given Sir Lockyer's result for $T_{☾}$, we find that

$$(T/t^s)_{☾} = 230{\cdot}045913,$$

as against $$(t/d)_{♀} = 230{\cdot}090807,$$

when $$t_{♀} = 224{\cdot}7008 d_{♁}.$$

Therefore, the amount, by which $t_{♀}$ must be decreased for the purpose of the equalization now under treatment, is represented by x in the equation:

$$t_{♀} - x = 230{\cdot}045913 d_{♀},$$

where $d_{♀}$ for our figures for $t_{♀}$ and $\delta_{♀}$ amounts to 1406·26718 m.;

$$\therefore x = 63{\cdot}335 \text{ m.}$$

By contrast, given our own figure for $T_{☾}$, we find that

$$(T/t^s)_{☾} = 230{\cdot}048319,$$

$$\therefore t_{♀} - x = 230{\cdot}048319 d_{♀},$$

$$\therefore x = 59{\cdot}951 \text{ m.}$$

The discrepancy of less than 3 s. between this result and the postulated 1 h. is easily accounted for by the margin of indefiniteness clinging to our figures. But that this margin is, indeed, practically negligible comes home to us, if, in reflection upon the connection of the equation

$$(t_{♀} - 1 \text{ h.})/d_{♀} = (T/t^s)_{☾} \quad . \quad . \quad \text{(LIX)}$$

with the equation (LVIII), we put the present discrepancy to the account of the margin of indefiniteness clinging to our, as well as Sir Lockyer's, figure for $(2\pi r)_{♁}$ alone. Namely, we find that the margin of indefiniteness would in this respect amount to less than 70 yards. Yet that the discrepancy in question is also due to a slight error by defect in our figures for $t_{♀}$ and $t_{♁}$ may be seen from the fact that G. W. Hill's figure for $t_{♀}$ is $224{\cdot}700806 d_{♁}$ and Hansen's figure for $t_{♁} = 365{\cdot}256363 d_{♁}$.

The fact that our last equation supplies a conclusive test

of the coherency of all previous results, disposes of any lingering doubt as to its dialectical necessity. Its conclusive nature would come home to us more particularly, if we undertook to evaluate the system of our equations quite independently of any data of observational astronomy.

On closely inspecting the series of our equations, when written out consecutively on a piece of paper, it becomes obvious that their primarily purely tentative evaluation reduces to an adequate fixation of four principal data, i.e. of $(2\pi r)_{♁}$ and $t_{♁}$ on the one side [with the help of equations 3-6, 13, 14, 17, 18 and 35, as well as the dialectically derived round figures for $t_{♁}$ and the mean planetary distances from the Sun as given in the series of s] and on the other side of $\delta_{♂}$ and $S_{♀}$ [with the help of the equations bearing on the evaluation of $2\pi r_{♃}$ and $2\pi r_{♄}$]. Now, as these data present themselves at first sight as subdivided into two couples, at once independent of, and yet just as much also bound up with, one another, the series of equations should, if only for completeness' sake, equally imply an equation involving the four significant data all at once. But this is precisely the case with our last equation. The evaluation of $T_{☾}$ is bound up with $2\pi r_{♁}$ and $t_{♁}$, that of $t^{s}_{☾}$ with $\delta_{♂}$ and that of $t_{♀}$ of course with $S_{♀}$. Consequently, the smaller the final discrepancy between the resulting values of the two sides of the last equation, the nearer are we to the actual true values of all the magnitudes involved in our equations. Or rather, in reflection upon the alterableness belonging to the own Inwardness of the In-itself, and forming, as we may now say, the *essential* nature of all that is, the actual values in question should be conceived as only oscillating round their *à priori* determined magnitudes.

In so far as Being and Thought are in *negative* unity, it is only consistent with our own standpoint that the objectification of *à priori* results should imply a margin of indefiniteness which, as having its Being in Negation, as the Finite, cannot be credited with fixedness. Accordingly, we are not surprised to find, for instance, that the sidereal day is shortening at the rate of $\frac{1}{86}$th part of a second in 2500 years. "At all events, if the sidereal day be assumed to be invariable, it is impossible to represent the Moon's

true place at intervals 2000 years apart by the theory of gravitation. On this assumption the Moon, looked upon as a time-piece, is *too fast* by 6″ or 12 s. (nearly) at the end of each century."[1]

It should be really superfluous to remark, finally, that in availing ourselves straightway of observational results for the purpose of an immediate evaluation of our equations, we were not simply actuated by impatience to see our reasoning objectively vindicated. For, it did not occur to us to make ourselves absolutely independent of the need of the results of observation, just because our fundamental standpoint emphasizes the negative unity of Thought and Being. After all, the otherwise possible independent evaluation of our equations is itself also already a matter of experimentalism, and insistence on it is no longer prompted by pure thought, but only by our mathematical consciousness. However gratifying it is to know that the measurable features of the solar system admit of a purely mental computation apart from any appeal to observation, the task of pure thought ends with the setting up of the formulae embodying their dialectically systematic inter-relation.

c. The Earth as the Concreted Individuality of the Solar System: Physical Earth iu Transition into the Standpoint of Essence.

The idea of the Earth is now before us in result of its mediation within itself with its hypostatized aspects, and hence no longer in the sense of a merely planetary individuality, but in the sense of the concreted individuality of the whole solar system: in that sense in which it becomes the object of our further consideration in connection with the demonstrableness of the next higher department of the system of pure thought.

The trend of the self-exposition of the true Being in the course of this department is easily foreseen in reflection upon its dialectic past. In this respect, we already know that the dialectic whole of Quality and Quantity presents itself in the light of an elementary swing of the dialectic pendulum which continues to reproduce itself within the

[1] Sir Lockyer's "Astronomy," Art. 667.

sphere of Measurableness. But as ushered on the scene in return from Quantity into Quality, the immediate reproduction of the dialectic whole of Quality and Quantity within this sphere acquires at the same time the significance of a higher reproduction of the dialectic whole of Quality, i.e. only of the first half of the next wider swing of the dialectic pendulum, the second half, in higher reproduction of the dialectic whole of Quantity, having then to be correlated with the system of Mathematics. The Solar System having an analogous relation to the second swing as the immediate sphere of Measurableness has to the first one, it easily follows that its own completed dialectical whole becomes in turn the first half of the third, still wider swing, the second half referring to the ensuing dialectical whole of the Earth in that sense which it acquires as the fully established In-and-for-Itself, or Essence, of the whole Solar System and herewith also of the whole Starry System—a sense obviously investing it with *physical* nature, in supersession of its primarily only astral or planetary character.

In connection with the possible objective significance of the Many present Ones, we referred to the occult, as well as current scientific, conception of the stars as solar systems on their own: we are now in a position to deny such a conception any rational foundation. And neither can we find any justification for the further conception of inhabitableness of the remaining members of our solar system. In so far as the idea of physical Earth, of the Earth as the ground, on which we tread, as the home of the physical embodiment of Spirit in Man, is ushered on the scene in result of our dialectical attitude towards the solar system, this latter must hereafter be treated only in the sense of an elementary aspect of the Earth's own Inwardness.

It is self-understood, of course, that, in speaking of the dialectical relationship between the Earth and the Solar System, we really refer to the Demonstrableness of the nature of the true Being. If we are right in our present grasp of our Earth as the conclusive representative of the true Being at the level of Essence, then there must be only One physical Earth, because the true Being is immediately qualified as the One Being. In so far as the One Being is immediately a self-differentiating One, the already from the very first realized inappropriateness of interpreting Self-

differentiation in the sense of an actual manifolding of the one One calls now for no further argument.

We are aware that the further explication of our present idea of physical Earth implies a detailed recapitulation of its dialectic past; and that, in this respect, everything necessarily acquires a new meaning. That, in the pursuit of this task, we may have to refer once again to the solar system, is probable from the fact that, so far, many interesting features of this latter, e.g. the satellites, did not come up for treatment. That, however, the Earth acquires now a meaning definitely raising it above its simple planetary character and hence above its position in the Solar System, as one of many, or even as a kind of *primus inter pares*, is now certain.

In so far, however, as the ensuing recapitulation of the whole course of our previous progress in knowing may be taken at least to equal, if not to exceed, this latter in length, we shall make it the subject-matter of a separate volume, under the title of "Creation of Man on Earth". For, the self-individualization of the true Being in our physical Earth is obviously only an intermediate stage in the ascent of this self-individualization to the stage of Man. An explication of this culminating stage, again in systematically circumstantial recapitualization of the whole preceding progress in knowing, becomes the subject-matter of our concluding volume, or possibly volumes.

The First is dialectically the Last, and *vice versâ;* consequently, our task will reach its completion only until the true Being, God Himself, will step upon the scene in the explicit sense of a fully self-conscious or pure Thinker—in that sense which applies to our own Self at the outset of our present task. Finally it must become fully plain to us that the very conception of our task, as well as its prosecution, is nothing but the own Self-revelation of the very Essence of God as the only true Being.

Omnia ad majorem Dei gloriam!

END OF VOLUME I

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